Client Specification (Fictitious)

The client is interested in investing in cryptocurrency due to its growth in market cap and return during the past years following COVID-19. Bitcoin, for instance, saw growth of 60% during the year 2021.

Last year [2022], however, showed a downfall of many cryptos, such as bitcoin falling from 35k USD to around 15k USD. Some cryptocurrencies were involved in rug pulls (a case where the cryptocurrency team announces a fake project to attract the investor money, only to suddenly close the project and essentially steal the money). Others, such as FTT token, a cryptocurrency issued by the crypto-exchange company FTX, also had a huge scandal.

This volatile nature of cryptocurrency made this risk averse investor reluctant to invest his money. He tasks the team with trying to analyze the predictability of the return of cryptocurrency, starting with the FTT token due to its prevalence in the news, using an academic paper he found, a three-factor pricing model for cryptocurrencies (Shen, D., Urquhart, A. and Wang, P., 2020). He also asks the team to find if there is any other method for predicting the return.

Additionally, the investor wanted to extend this predictability of return topic further. Following the news of FTX incident, the investor wondered about the integrity of cryptocurrency, i.e., if there was insider information circulating prior to the announcement of FTX bankruptcy. He then proposes to do an event study similar to the one done by Fama, Fisher, Jensen, and Roll (1969) to detect if there is any abnormal return.

As a result, this project's aim is two-fold. The first is to find a proper three factor model for predicting the return of FTT token. The second is to find if there exists any abnormal return prior to the bankruptcy of FTX.

Overview

Project overview

This project aims to find the suitable model between the CAPM or three factor model for predicting the return of the FTT token, a cryptocurrency that is issued by FTX company. This suitable model will be decided by using error metrics of RMSE and r-square.

After the suitable model has been obtained, this particular model will be used as expected return model to perform event study on the FTT token to detect any abnormal return and cumulative abnormal return. If the abnormal return is not zero, it suggests that the price may have the same trend in future similar incidents of crypto bankruptcy.

Event overview

(timeline based on and summarized from https://www.protocol.com/fintech/ftx-collapse-timeline)

The FTX incident was an event where the FTX company, one of the largest crypto trading platform, announced that it would file for bankruptcy on 11th November 2022.

The event started on 2nd November 2022 after a leak report showed that Alameda Research, a crypto-trading firm, held much of its asset in FTT token, a currency that allows the holders to gain various benefits, including discount when trading on the FTX platform. The amount held was larger than that in trading, potentially signaling a liquidity issue. Moreover, Sam Bankman-Fried, the CEO of FTX, was also a founder for the Alameda Research, which sparked some controversies.

To exacerbate the matter further, there were sell-off announcements from FTT big holders, including from the CEO Binance, Changpeng "CZ" Zhao. This worrying matter made FTX users start to withdraw their money. After a while, FTX faced liquidity issue and while there were bailout plans, they never materialized.

FTX, failing to find enough cash for the withdrawers, had to announce bankruptcy. The FTT token price saw a sharp decrease from approximately 20 USD to 1 USD during that day.

CITE THIS LATER PIC FROM COIN METRIC



1. Literature review

1.1 Return predictability

In the crypto market, Urquhart (2016) has found that the bitcoin market itself is inefficient when analyzing the return over the whole period. He noted, though, that the return in the later time period, when viewed separately, starts to exhibit efficiency.

This inefficiency that still exists in the market was later explored by Shen et al.(2020). In their work, "A Three-factor Pricing Model for Cryptocurrencies", they apply the concept from Fama French five factor model (1993). They find that the proposed three factor model, which contains market, size and reversal factors has better predictability in weekly return compared to the CAPM model, which only contains the market factor.

Other kind of prediction model was an ensemble model, a model where predictions are made from a combination of models. This kind of model was explored by Their model include linear regression, random forest, and support vector machine. They found that the prediction made by the ensemble model outperforms each individual model.

1.2 Event study

Event study is a study of how new information, such as an occurrence of an event, affect the target. One prominent example of event study is from Fama et al.(1969), where they studied how stock price reacted to announcement of either stock splits or dividends. The event study for the FTX incident was conducted by Yousaf et al. (2023) to show that the effect of FTX collapse on traditional effect was minor, even though the event itself had major effects on price of other cryptocurrencies. To the knowledge of the project author, there is no research on the FTT token price reaction itself.

2. Data

The data was downloaded directly from coinmetrics.com. It contains the estimating supply in circulation and price, both in USD, for 374 cryptocurrencies, spanning from 2018 to 2023. Since the data coverage for some cryptocurrencies only start in the later years and that the event occurred in 2022, the timespan for calculating and finding out the best model has been reduced to later half (August onward) of 2022 only. The data for the appendix A for the study of the factor effect, however, starts from 2021.

The risk free rate is taken from Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity from FRED. Since there is no holiday for cryptocurrency but there are days when the bond market is closed, the missing data is backfilled from the previous nearest day.

3. Methodology

The methodology is summarized as follows

3.1 Calculation of Daily Return

The daily return for each cryptocurrency, including the FTT token is calculated from the price of each day as

$$Ret_t = \frac{Rate_t}{Rate_{t-1}} - 1$$

where

 Ret_t is the return of the cryptocurrency at day t

 $Rate_t$ is the price in USD of the cryptocurrency at day t

 $Rate_{t-1}$ is the price in USD of the cryptocurrency at day t-1 (the prior day)

3.2 Factor formation

The factors are constructed using the return of the cryptocurrency. The factors included for this regression are market, size, and reversal factors. These factors are the same factors that were represented in the work of Shen et al.(2020). The difference is that the factor is calculated on a daily basis instead of weekly basis so that FTX event study can be achieved.

3.2.1 Market Factor

The market factor represents the overall condition of the market at the time. This market factor is first used in CAPM model by William Sharpe (1964), where it is argued that more risks necessitate higher level of returns.

The market factor is calculated by

$$Mkt_{t} = \left(\sum_{i=1}^{N} \frac{Cap_{i,t}}{TotalCap_{t}} * Ret_{i,t}\right) - Rf_{t}$$

where

 Mkt_t is the market return at day t

 $Cap_{i,t}$ is the market cap of the cryptocurrency i in circulation at day t

 $Total Cap_t$ is the total market cap of all cryptocurrencies at day t

 $Ret_{i,t}$ is the return of cryptocurrency i at day t

 Rf_t is the risk free rate at day t

3.2.2 Size Factor

In the work of Liu et al.(2019), they found that a portfolio which short on the bitcoin (considered as the big size) and long on the smallest cryptocurrency (the small size) on a weekly basis generates a significant 3% return. One possible explanation could be that since the market cap for the small cryptocurrency is low, it is easier to push up the price to attract crowd attention.

The size factor is calculated by

$$SMB_t = Small_t - Big_t$$

where

 $Small_t$ is the return at day t of portfolio containing small market cap cryptocurrencies from day t-1

 Big_t is the return at day t of portfolio containing big market cap cryptocurrencies from day t-1 Small/ Big market cap is obtained by first ranking all the cryptocurrency market cap. Then the small ones are the bottom 10% and the big ones are the top 90%.

3.2.3 Reversal Factor

In the work of Shen et al.(2020), they found that by buying the cryptocurrency with the lowest return in past week and selling the cryptocurrency with highest return, positive return can be achieved.

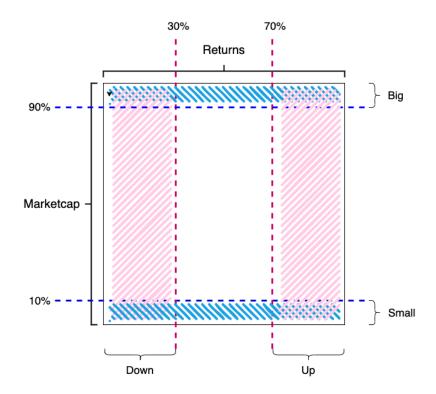
The reversal factor is calculated by

$$DMU_t = Down_t - Up_t$$

 $Down_t$ is the return at day t of portfolio containing lowest return cryptocurrencies from day t-1 Up_t is the return at day t of portfolio containing highest return cryptocurrencies from day t-1 Down/ Up is obtained by first ranking all the cryptocurrency returns. Then the down ones are the bottom 30% and the up ones are the top 70%.

The breakpoints are set in a similar fashion to Fama and French (1993)

Reasons for using the day t-1 and day t for forming these factors are given in Appendix A. The picture below illustrate how the cryptocurrency is grouped into the size and reversal factors.



3.3 Model Construction and Comparison

3.3.1 Construction

After building the factors, the models are built to predict return at day t+1 from the factors at day t. There are three considerations when building the model: the time period for fitting the model, the factors to be used, and the type of model.

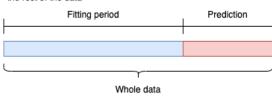
3.3.1.1 Time period

The time period for constructing the model ranges from 1 week to 8 weeks before prediction, incrementing in 1 week. The prediction period is fixed at 15 days in total. The reason for choosing the small time window for fitting is due to the fast changing nature of cryptocurrency.

The prediction period is at 15 days since for the event study, abnormal return needs to be calculated a week prior and a week after the event. Furthermore, two types of prediction are employed. The first is a case where all models are fit using the whole data in the timespan to predict the 15 days return. The second employs a rolling window prediction, where the fitting data is moved 1 day ahead each time to make a new prediction. The picture below illustrates this concept.

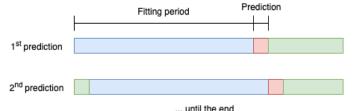
No Rolling window

Without the rolling window, the fitting is done only once to predict the rest of the data



With Rolling window

With the rolling window, the fitting is first done to predict the first data point. Then the fitting period is moved by one timestep (in this case it is a day) to predict the second data point.



3.3.1.2 Factors

There are 2 cases for the factors selection. The first selection follows the CAPM model follows Fama and French (2004), where the model only contains the market factor. The second selection follows the recent factor model by Urquhart (2016), where the model contains market, size, and reversal factors.

3.3.1.3 Type of regression model

There are four types of model to be used: linear regression, random forest regression, support vector regression, and ensemble model.

3.3.1.3.A Linear Regression

The linear regression model fits the returns at day t+1 and factors at day t according to the equation:

$$Ret = Factors * \beta + \epsilon$$

$$Ret = \begin{bmatrix} Ret_2 \\ Ret_3 \\ \dots \\ Ret_t + 1 \end{bmatrix} \text{, a matrix containing the returns for the next day}$$

$$Factors = \begin{bmatrix} 1 & Mkt_1 \\ 1 & Mkt_2 \\ \cdots & \cdots \\ 1 & Mkt_t \end{bmatrix} \text{ and } \beta = \begin{bmatrix} intercept \\ MktCoeff \end{bmatrix} \text{ for the CAPM model or }$$

$$Factors = \begin{bmatrix} 1 & Mkt_1 & SMB_1 & DMU_1 \\ 1 & Mkt_2 & SMB_2 & DMU_2 \\ \dots & \dots & \dots \\ 1 & Mkt_t & SMB_t & DMU_t \end{bmatrix} \text{ and } \beta = \begin{bmatrix} intercept \\ MktCoeff \\ SMBCoeff \\ DMUCoeff \end{bmatrix} \text{ for the three factor model }$$

and ϵ is the error term.

t is the time period that is picked for fitting and the subscript 1, 2,... denotes the day sequence.

This means that the first row of Factors is the data from day 1 and so on.

In this form of regression, the intercept and coefficients are found such that the square error

$$\frac{1}{2} \parallel Ret - Factors * \beta \parallel^2$$
, where $\parallel x \parallel^2$ means the dot product of x itself, is minimized.

3.3.1.3.B Support Vector Regression

(adapted from INSERT NAME)

In support vector regression, it is supposed similarly that

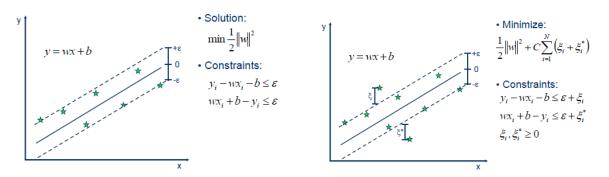
$$Ret = Factors * \beta + \epsilon$$

However, instead of minimizing the error term through minimizing the square error alone, the β is

found such that
$$\frac{1}{2} \| \beta \|^2 + C \sum_{i=1}^t (\xi_i + \xi_i^*)$$
 is minimized subject to
$$\left(Ret_i - Factors_i * \beta < \epsilon + \xi_i \right)$$

$$\begin{cases} Ret_i - Factors_i^* \beta \leq \epsilon + \xi_i \\ Factors_i^* \beta - Ret_i \leq \epsilon + \xi_i^* \text{ where } i \text{ denotes the day from day 1 to day } t \\ \xi_i, \xi_i^* \geq 0 \end{cases}$$

The addition of the slack variables ξ, ξ^* would allow for additional rooms of error in the case that the linear model cannot be fitted. This concept is illustrated by the picture below.



https://www.saedsayad.com/support_vector_machine_reg.htm

3.3.1.3.C Random Forest Regression

(adapted from https://www.geeksforgeeks.org/decision-tree/)

In random forest regression, multiple decision trees are built and the final prediction is done by averaging across the predictions from those trees. Since single decision tree is prone to overfitting, random forest aims to solve that problem.

Its algorithm can be summarized as follows:

- 1. A total of number of *N* decision trees are built.
- 2. These tree predictions are then average to obtain the final prediction.
- 3. To build a single tree
 - 1. One must pick some subsample with repetition from the whole fitted period
 - 2. At each node starting from the root, start splitting at the node m which has n_m amount of data Q_m , using the factor t on threshold t_m for left and right separation such that

$$G(Q_m,t_m) = \frac{n_m^{Left}}{n_m} MSE(Q_m^{Left}(t_m)) + \frac{n_m^{Right}}{n_m} MSE(Q_m^{Right}(t_m)) \text{ is minimized.}$$

Note that MSE is the mean square error for the left and right split.

Note that
$$MSE$$
 is the mean square error for the left and right $MSE(Q_m^{Left}(t_m)) = \sum_{i=1}^{n_i^{Left}} (Ret_i - Re\bar{t}_{Left})^2, Re\bar{t}_{Left} = \sum_{i=1}^{n_i^{Left}} Ret_i$ $MSE(Q_m^{Right}(t_m)) = \sum_{i=1}^{n_i^{Right}} (Ret_i - Re\bar{t}_{Right})^2, Re\bar{t}_{Right} = \sum_{i=1}^{n_i^{Right}} Ret_i$

The split would be continued until a certain depth is reached or the MSE cannot be further minimized.

4. After building all the trees, their predictions are averaged for the final results.

3.3.4.D Ensemble Model

The ensemble model in this case works similarly to the one by NAME. It is equally weighted average from the three prior model.

3.4 Comparison

Two criteria are employed to pick the best model. They are the root mean square and the rsquare.

$$\begin{split} RMSE &= \sqrt{\sum_{i=1}^{T} \frac{(Ret_i - \hat{Ret_i})^2}{T}} \\ R^2 &= 1 - \frac{SS_{RES}}{SS_{TOT}} = 1 - \frac{\sum_{i=1}^{T} (Ret_i - \hat{Ret_i})^2}{\sum_{i=1}^{T} (Ret_i - \bar{Ret_i})^2} \end{split}$$

where

 Ret_i is the real return at day i

 \hat{Ret}_i is the prediction return from the model at day i \hat{Ret}_i is the mean of the real return from day i=1 to day T

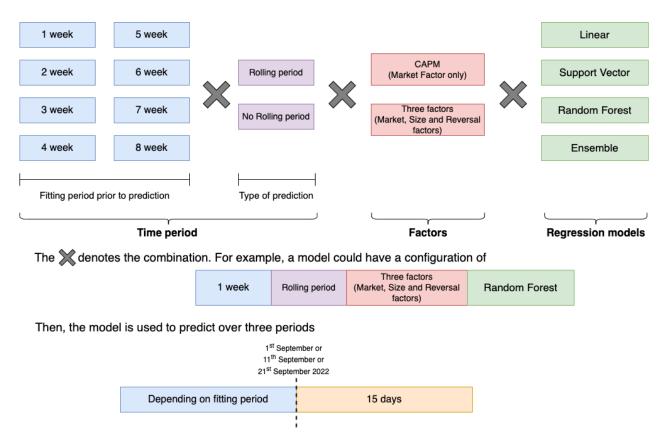
The root mean square and r-square perform different functions. The root mean square measure the deviation in prediction from the real value. The lower the root mean square, the better the prediction model is. In contrast, the r-square explains how well the factors of the model can explain the real return. The higher the r-square value, the better the model is.

Chicco et al. (2021) argues that r-square is a better criteria than the root mean square since it has better interpretability. The reason is that root mean square can range from 0 to infinity which makes it hard to understand it alone. R-square value ranges from 1 to negative infinity. However, when the R-square is negative, it means that the model performs worse than a single horizontal line (i.e. no model is needed to perform the prediction) when the model is non-linear. When the model is linear, it means the slope or intercept is constrained. As a result, the consideration of rsquare lies in 0 to 1 domain, where the closer to 1 value means the better model.

Even so, since there are several models to be compared with different constructions, both RMSEand R^2 are employed here.

Using these two criteria, the prediction will be performed on the return of FTT token. There will be 3 prediction periods in total starting from beginning of September, 2 months before the FTX incident. The next prediction period will then be incremented on a 10 days. The model with the smallest average RMSE and highest average R square during these 3 periods will be picked. (The 3 period with 10 days increment would mean that the model is tested for around one month)

Overall the construction and comparison period can be summarized using the picture below.



and the model with lowest average RMSE/ highest average r-square is picked.

3.5 Event Study

(adapted from Name)

After the model is selected, the event study is performed. An event study aims to find abnormal return by first defining an expected return model. Normally, a period called estimation period is used to fit for parameters of the expected return model. After that, during the event period, abnormal return for each day is found by

$$AR_t = Ret_t - \hat{Ret_t}$$

where

 AR_t is the abnormal return at day t

 Ret_t is the market return at day t

 Ret_t is the model predicted return at day t

After the abnormal return is calculated, cumulative abnormal return CAR from time t_1 to t_2 by

$$CAR_{t} = \sum_{i=t_{1}}^{t} AR_{i}, t = t_{1}, \dots, t_{2}$$

where

 CAR_t is the cumulative abnormal return at day t

In this FTX event study, the time t_1 is set to 4^{th} November and t_2 is set to 18^{th} November, which is one week prior and one week after the event. If the CAR is not zero, it suggests that there is cumulative abnormal return. The illustration of estimation and event period is given below. Picture

4. Findings

Over the prediction period, it is found that the 8 week- rolling period prediction- three factor-random forest model performs the best compared to other models with average RMSE of 0.000783 and r-square of 0.082597 when predicting the return of FTT token. Other models that perform comparatively well are also 8 week- three factors- random forests model but with no rolling window prediction. However, in the work of Sebastião and Pedro Godinho (2021), they argue that the ensemble model performs better. This discrepancy could be explained by the different in forecasting frequency, since this project focuses on daily return instead of weekly return. Similar to the work of Shen et al., the factor model performs better than the CAPM model.

This 8 week- rolling period prediction- three factor- random forest model will then be used as an expected return model. The table providing full results of RMSE and r-square can be found in Appendix B. Note that the r-square here is considerably less than 1, which means that the three factors may not be enough to explain the return of the FTT token.

After the expected return model (the best model) is picked, the event study is performed. It is found that in the week leading to the event, the abnormal return remains around 0 until two days before the event on 9 November. This is reasonable because it was when Binance announced that it will not bail out the FTX company and US regulators began to investigate, which meant that the outlook should be negative.

Surprisingly, on the day of the event when FTX eventually announces the filing for bankruptcy, the abnormal return becomes positive. This could be because the market starts to factor in the information at the time.

After the event, the abnormal return fluctuates somewhat between the positive and negative region, but this could possibly be because of the value of the cryptocurrency drops to 2 USD, which makes it easy to fluctuate.

Even so, the cumulative abnormal return remains negative throughout the period, which suggests that the bankruptcy event could be perceived as a negative reaction. This is similar to filing bankruptcy incident in stock as studied by https://www.mdpi.com/2227-9091/9/3/56.

The graph below compares the prediction between the predicted return and the real return, the abnormal return, and the cumulative abnormal return.

5. Conclusion and afterthoughts

This project first calculates the market, size and reversal factors for prediction of the cryptocurrency. After trying out the regression model between linear, support vector, random forest and the ensemble model, it has found that the random forest with the fitting period of 8 week and rolling window prediction best explains the daily return of the FTT token. This random forest model is later used as the expected return model for the event study.

The event study of this FTX bankruptcy case found that there is negative cumulative abnormal return throughout the period. The abnormal return, meanwhile, was mostly negative during the period except for the day of announcement of bankruptcy. This suggested that, in expectation of a bankruptcy event, one potentially go short on that cryptocurrency, similar to what one do in the stock market. However, the timing of entry is important since as illustrated in the FTT token, the abnormal return becomes positive on the day of bankruptcy announcement.

Since the r-square from the best model in this project is still much below 1 or for some other models, even negative. Further development of this project may include additional factors such as the momentum factor like in the work of NAME or including sentiment index such as that provided by bittsanalytic. Additional improvement to his project may include changing the frequency of return to week for the event study, implementation of other regression model, or extending the study to other cryptocurrency failure cases such as the collapse of UST and LUNA token in 2022.

Appendix

Appendix A The reversal and size effects

Reversal effect

Similar to the work of Jegadeesh and Titman, J-K portfolio has been constructed for the selection and holding period of 1, 7, 14, 21 and 28 days. During the selection period, the return is calculated and the portfolio is constructed to buy (long) the cryptocurrency with previous return in the upper 90% decile and sell (short) the cryptocurrency with previous return in the lower 10% decile. The table below reports the excess return along with the p value(denoted under each excess return on the second line) and significance (denoted by the color).

| Reversal effect | Strategy | 1 | 7 | 14 | 21 | 28 | K | | |
|-----------------|----------|----------|----------|----------|----------|----------|---|-----|--------|
| 1 | Buy | -0.00159 | 0.001871 | 0.001315 | 0.000817 | 0.000587 | | * | < 0.1 |
| | | -0.96912 | 1.115095 | 0.818863 | 0.526171 | 0.381852 | | ** | < 0.05 |
| 1 | Sell | 0.001557 | 0.002042 | 0.001637 | 0.002066 | 0.003238 | | *** | < 0.01 |
| | | 0.911577 | 1.261053 | 1.042853 | 1.320259 | 2.017371 | | | |
| 1 | Buy-Sell | -0.00314 | -0.00017 | -0.00032 | -0.00125 | -0.00265 | | | |
| | | -2.70925 | -0.17347 | -0.36833 | -1.43203 | -2.8622 | | | |
| 7 | Buy | 0.17365 | 0.016273 | 0.01043 | 0.010326 | 0.005142 | | | |
| | | 22.29098 | 3.011762 | 2.385236 | 2.503954 | 1.277681 | | | |
| 7 | Sell | 0.015897 | 0.009141 | 0.016226 | 0.015518 | 0.024823 | | | |
| | | 2.96209 | 2.013957 | 3.173451 | 3.016594 | -4.42557 | | | |
| 7 | Buy-Sell | 0.157753 | 0.007131 | -0.0058 | -0.00519 | -0.01968 | | | |
| | | 30.10038 | 1.806675 | -1.74706 | -1.55557 | -5.64806 | | | |
| 14 | Buy | 0.238034 | 0.402751 | 0.030492 | 0.025942 | 0.014413 | | | |
| | | 17.35361 | 24.38582 | 3.880398 | 3.737616 | 2.183497 | | | |
| 14 | Sell | 0.08564 | -0.15974 | 0.03541 | 0.036747 | 0.056094 | | | |
| | | 8.480767 | -32.7488 | 3.490156 | 3.647272 | 4.993649 | | | |
| 14 | Buy-Sell | 0.152394 | 0.562492 | -0.00492 | -0.0108 | -0.04168 | | | |
| | | 15.77215 | 39.63055 | 0.586686 | -1.4164 | -5.06459 | | | |
| 21 | Buy | 0.290295 | 0.496515 | 0.332641 | 0.04664 | 0.027559 | | | |
| | | 16.86019 | 21.52648 | 18.78417 | 4.647384 | 3.139327 | | | |
| 21 | Sell | 0.144896 | -0.07818 | -0.07465 | 0.056315 | 0.081289 | | | |
| | | 9.820994 | -9.99099 | -7.31623 | 4.329684 | 5.525088 | | | |
| 21 | Buy-Sell | 0.145399 | 0.574697 | 0.407292 | -0.00968 | -0.05373 | | | |
| | | 12.57659 | 31.6237 | 27.52663 | -0.99574 | -5.37834 | | | |
| 28 | Buy | 0.353739 | 0.568839 | 0.672563 | 0.315661 | 0.049064 | | | |
| | | 15.77367 | 20.5091 | 22.44637 | 15.54815 | 4.188786 | | | |
| 28 | Sell | 0.196877 | -0.0112 | -0.20092 | -0.03722 | 0.099126 | | | |
| | | 10.85371 | -0.86653 | -23.029 | -3.05555 | 5.851997 | | | |
| 28 | Buy-Sell | 0.156862 | 0.580034 | 0.873486 | 0.352886 | -0.05006 | | | |
| | | 10.94136 | 27.66404 | 33.09389 | 22.03326 | -4.28775 | | | |
| | | | | | | | | | |
| | | | | | | | | | |

It has been found that with holding period (J) =1, there has been pervasive negative return during the 2021 onwards period for the buy-sell portfolio. This suggests a reversal effect, since negative return is achieved by buying the past winner (high return cyptocurrency) and selling the past loser (low return cryptocurrency). The portfolio with the J-K of 1-1 is then picked to represent the reversal effect as it has the smallest negative return.

Size effect

| | 1 (Up) | 2 | 3 | 4 | 5 (Down) | Ret | Down-Up |
|-------------|------------|------------|------------|------------|------------|-----|------------|
| 1 (Big) | 0.003606 | 0.003084 | 0.002096 | 0.002114 | 0.001214 | | -0.0023921 |
| 2 | 0.005128 | 0.003818 | 0.003658 | 0.001919 | 0.000706 | | -0.0044219 |
| 3 | 0.001623 | 0.003984 | 0.005156 | 0.00319 | 0.002649 | | 0.00102534 |
| 4 | -0.0028 | 0.002982 | 0.002513 | 0.00382 | 0.002445 | | 0.00524999 |
| 5 (Small) | -0.00541 | -0.00051 | 0.002386 | 0.002918 | 0.003422 | | 0.0088368 |
| Size | | | | | | | |
| Small - Big | -0.0090206 | -0.0035895 | 0.00028976 | 0.00080427 | 0.00220826 | | |