

# Stat 333 HW2

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## **Q3.7.1**

The null hypotheses to which the p-values given in Table 3.4 correspond are that advertising budgets of TV, radio, or newspaper does not have an effect on sales. The p-values are high-significant for TV and Radio but not significant for Newspaper, which means TV and Radio are related to the Sales, while the Newspaper shows no evidence that it is related to the Sales in the presence of the other two.

## **Q3.7.3**

a)

ii, males tend to earn higher starting salaries given high GPA.

b)

$$\hat{y} = 50 + 20GPA + 0.07IQ + 0.01GPA \cdot IQ$$

Plugging in the above equation,  $\hat{y} = 137.1$  which gives a salary of \$137100

c)

False. To show that GPA/IQ has an impact on the model we need to test the hypothesis  $H_0 : \hat{\beta}_4 = 0$  and look at the p-value associated with the t statistic to draw a conclusion.

## **Q3.7.4**

a)

Since the true relationship is linear, we could expect that in any given training data that the RSS would be lower for a linear model than it would be with the cubic.

b)

test RSS for a cubic regression will be higher, (with higher flexibility) because the model will probably have been overfitted to the training data.

With such a high degree of flexibility, the model does its best to account for every single training point, which lead to overfitting on another data set.

c)

If we were to fit a cubic model onto a training set, regardless of how far the true relationship is from linearity, we could probably overfit it nicely with some polynomial and have a lower RSS than we could with a linear regression model.

d)

There is not enough information to tell which test RSS would be lower for either regression given the problem statement is defined as not knowing “how far it is from linear”.

### Q3.7.15

```
library(MASS)
attach(Boston)
```

```
fit.zn <- lm(crim ~ zn)
summary(fit.zn)
```

```
##
## Call:
## lm(formula = crim ~ zn)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.429  -4.222  -2.620   1.250  84.523
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.45369    0.41722  10.675 < 2e-16 ***
## zn          -0.07393    0.01609  -4.594 5.51e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.435 on 504 degrees of freedom
## Multiple R-squared:  0.04019,    Adjusted R-squared:  0.03828
## F-statistic: 21.1 on 1 and 504 DF,  p-value: 5.506e-06
```

```
fit.indus <- lm(crim ~ indus)
summary(fit.indus)
```

```
##
## Call:
## lm(formula = crim ~ indus)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.972  -2.698  -0.736   0.712  81.813
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.06374    0.66723  -3.093  0.00209 **
## indus        0.50978    0.05102   9.991 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.866 on 504 degrees of freedom
## Multiple R-squared:  0.1653, Adjusted R-squared:  0.1637
## F-statistic: 99.82 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
chas <- as.factor(chas)
fit.chas <- lm(crim ~ chas)
summary(fit.chas)
```

```
##
## Call:
```

```
## lm(formula = crim ~ chas)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.738 -3.661 -3.435  0.018 85.232
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.7444     0.3961   9.453  <2e-16 ***
## chas1         -1.8928     1.5061  -1.257   0.209
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.597 on 504 degrees of freedom
## Multiple R-squared:  0.003124,    Adjusted R-squared:  0.001146
## F-statistic: 1.579 on 1 and 504 DF,  p-value: 0.2094
```

```
fit.rm <- lm(crim ~ rm)
summary(fit.rm)
```

```
##
## Call:
## lm(formula = crim ~ rm)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6.604 -3.952 -2.654  0.989 87.197
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   20.482     3.365   6.088 2.27e-09 ***
## rm            -2.684     0.532  -5.045 6.35e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.401 on 504 degrees of freedom
## Multiple R-squared:  0.04807,    Adjusted R-squared:  0.04618
## F-statistic: 25.45 on 1 and 504 DF,  p-value: 6.347e-07
```

```
fit.age <- lm(crim ~ age)
summary(fit.age)
```

```
##
## Call:
## lm(formula = crim ~ age)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -6.789 -4.257 -1.230  1.527 82.849
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.77791     0.94398  -4.002 7.22e-05 ***
## age          0.10779     0.01274   8.463 2.85e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.057 on 504 degrees of freedom
## Multiple R-squared:  0.1244, Adjusted R-squared:  0.1227
## F-statistic: 71.62 on 1 and 504 DF,  p-value: 2.855e-16
```

```
fit.rad <- lm(crim ~ rad)
summary(fit.rad)
```

```
##
## Call:
## lm(formula = crim ~ rad)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-10.164	-1.381	-0.141	0.660	76.433

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-2.28716	0.44348	-5.157	3.61e-07 ***
rad	0.61791	0.03433	17.998	< 2e-16 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.718 on 504 degrees of freedom
## Multiple R-squared:  0.3913, Adjusted R-squared:  0.39
## F-statistic: 323.9 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
fit.dis <- lm(crim ~ dis)
summary(fit.dis)
```

```
##
## Call:
## lm(formula = crim ~ dis)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-6.708	-4.134	-1.527	1.516	81.674

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	9.4993	0.7304	13.006	<2e-16 ***
dis	-1.5509	0.1683	-9.213	<2e-16 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.965 on 504 degrees of freedom
## Multiple R-squared:  0.1441, Adjusted R-squared:  0.1425
## F-statistic: 84.89 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
fit.nox <- lm(crim ~ nox)
summary(fit.nox)
```

```
##
## Call:
## lm(formula = crim ~ nox)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.371  -2.738  -0.974   0.559   81.728
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -13.720      1.699   -8.073 5.08e-15 ***
## nox           31.249      2.999   10.419 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.81 on 504 degrees of freedom
## Multiple R-squared:  0.1772, Adjusted R-squared:  0.1756
## F-statistic: 108.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
fit.tax <- lm(crim ~ tax)
summary(fit.tax)
```

```
##
## Call:
## lm(formula = crim ~ tax)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.513  -2.738  -0.194   1.065   77.696
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -8.528369   0.815809  -10.45  <2e-16 ***
## tax          0.029742   0.001847   16.10  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.997 on 504 degrees of freedom
## Multiple R-squared:  0.3396, Adjusted R-squared:  0.3383
## F-statistic: 259.2 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
fit.ptratio <- lm(crim ~ ptratio)
summary(fit.ptratio)
```

```
##
## Call:
## lm(formula = crim ~ ptratio)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -7.654  -3.985  -1.912   1.825  83.353
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -17.6469      3.1473  -5.607 3.40e-08 ***
## ptratio       1.1520      0.1694   6.801 2.94e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 8.24 on 504 degrees of freedom
## Multiple R-squared:  0.08407,    Adjusted R-squared:  0.08225
## F-statistic: 46.26 on 1 and 504 DF,  p-value: 2.943e-11

fit.black <- lm(crim ~ black)
summary(fit.black)

##
## Call:
## lm(formula = crim ~ black)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.756  -2.299  -2.095  -1.296   86.822
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 16.553529   1.425903  11.609  <2e-16 ***
## black       -0.036280   0.003873  -9.367  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.946 on 504 degrees of freedom
## Multiple R-squared:  0.1483, Adjusted R-squared:  0.1466
## F-statistic: 87.74 on 1 and 504 DF,  p-value: < 2.2e-16

fit.lstat <- lm(crim ~ lstat)
summary(fit.lstat)

##
## Call:
## lm(formula = crim ~ lstat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.925  -2.822  -0.664   1.079   82.862
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.33054    0.69376  -4.801 2.09e-06 ***
## lstat        0.54880    0.04776  11.491  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.664 on 504 degrees of freedom
## Multiple R-squared:  0.2076, Adjusted R-squared:  0.206
## F-statistic: 132 on 1 and 504 DF,  p-value: < 2.2e-16

fit.medv <- lm(crim ~ medv)
summary(fit.medv)

##
## Call:
## lm(formula = crim ~ medv)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.071 -4.022 -2.343  1.298 80.957
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.79654    0.93419   12.63  <2e-16 ***
## medv        -0.36316    0.03839   -9.46  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.934 on 504 degrees of freedom
## Multiple R-squared:  0.1508, Adjusted R-squared:  0.1491
## F-statistic: 89.49 on 1 and 504 DF,  p-value: < 2.2e-16
```

All predictors have a p-value less than 0.05 except “chas”, so we may conclude that there is a statistically significant association between each predictor and the response except for the “chas” predictor.

b)

```
fit.all <- lm(crim ~ ., data = Boston)
summary(fit.all)

##
## Call:
## lm(formula = crim ~ ., data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.924 -2.120 -0.353  1.019 75.051
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 17.033228    7.234903   2.354 0.018949 *
## zn           0.044855    0.018734   2.394 0.017025 *
## indus       -0.063855    0.083407  -0.766 0.444294
## chas        -0.749134    1.180147  -0.635 0.525867
## nox        -10.313535    5.275536  -1.955 0.051152 .
## rm           0.430131    0.612830   0.702 0.483089
## age          0.001452    0.017925   0.081 0.935488
## dis         -0.987176    0.281817  -3.503 0.000502 ***
## rad          0.588209    0.088049   6.680 6.46e-11 ***
## tax         -0.003780    0.005156  -0.733 0.463793
## ptratio     -0.271081    0.186450  -1.454 0.146611
## black       -0.007538    0.003673  -2.052 0.040702 *
## lstat        0.126211    0.075725   1.667 0.096208 .
## medv       -0.198887    0.060516  -3.287 0.001087 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.439 on 492 degrees of freedom
## Multiple R-squared:  0.454, Adjusted R-squared:  0.4396
## F-statistic: 31.47 on 13 and 492 DF,  p-value: < 2.2e-16
```

We may reject the null hypothesis for “zn”, “dis”, “rad”, “black” and “medv”.

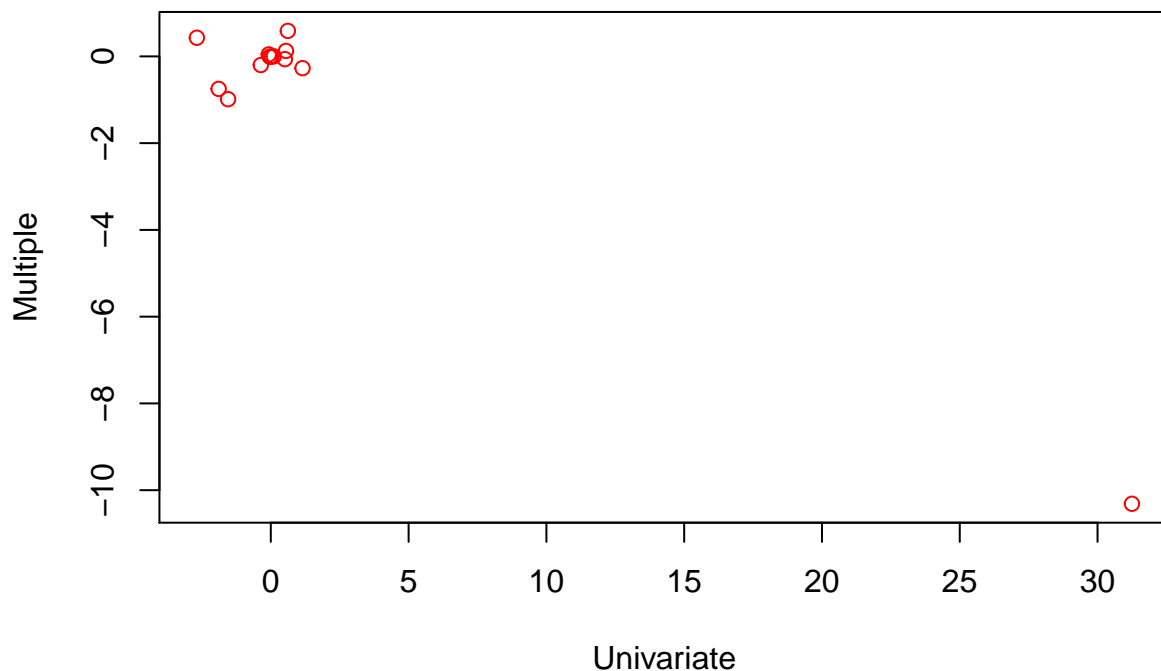
c)

```
univcof <- lm(crim ~ zn, data = Boston)$coefficients[2]
univcof <- append(univcof, lm(crim ~ indus, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ chas, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ nox, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ rm, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ age, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ dis, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ rad, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ tax, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ ptratio, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ black, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ lstat, data = Boston)$coefficients[2])
univcof <- append(univcof, lm(crim ~ medv, data = Boston)$coefficients[2])
fooBoston <- (lm(crim ~ . - crim, data = Boston))
fooBoston$coefficients[2:14]
```

```
##          zn          indus          chas          nox          rm
##  0.044855215 -0.063854824 -0.749133611 -10.313534912  0.430130506
##          age          dis          rad          tax          ptratio
##  0.001451643 -0.987175726  0.588208591 -0.003780016 -0.271080558
##          black          lstat          medv
## -0.007537505  0.126211376 -0.198886821
```

```
plot(univcof, fooBoston$coefficients[2:14], main = "Univariate vs. Multiple Regression Coefficients",
     xlab = "Univariate", ylab = "Multiple", col = "red")
```

## Univariate vs. Multiple Regression Coefficients





In simple regression, the slope term represents the average effect of an increase in the predictor, ignoring other predictors. In contrast, in the multiple regression case, the slope term represents the average effect of an increase in the predictor, while holding other predictors fixed.

d)

```
summary(lm(crim ~ zn + I(zn^2) + I(zn^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ zn + I(zn^2) + I(zn^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.821 -4.614 -1.294  0.473 84.130
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.846e+00  4.330e-01  11.192 < 2e-16 ***
## zn          -3.322e-01  1.098e-01  -3.025  0.00261 **
## I(zn^2)       6.483e-03  3.861e-03   1.679  0.09375 .
## I(zn^3)      -3.776e-05  3.139e-05  -1.203  0.22954
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.372 on 502 degrees of freedom
## Multiple R-squared:  0.05824, Adjusted R-squared:  0.05261
## F-statistic: 10.35 on 3 and 502 DF, p-value: 1.281e-06
```

```
summary(lm(crim ~ indus + I(indus^2) + I(indus^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ indus + I(indus^2) + I(indus^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8.278 -2.514  0.054  0.764 79.713
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.6625683  1.5739833   2.327  0.0204 *
## indus        -1.9652129  0.4819901  -4.077 5.30e-05 ***
## I(indus^2)    0.2519373  0.0393221   6.407 3.42e-10 ***
## I(indus^3)   -0.0069760  0.0009567  -7.292 1.20e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.423 on 502 degrees of freedom
## Multiple R-squared:  0.2597, Adjusted R-squared:  0.2552
## F-statistic: 58.69 on 3 and 502 DF, p-value: < 2.2e-16
```

```
summary(lm(crim ~ chas + I(chas^2) + I(chas^3), data = Boston))
```

```
##
```

```
## Call:
## lm(formula = crim ~ chas + I(chas^2) + I(chas^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.738 -3.661 -3.435  0.018 85.232
##
## Coefficients: (2 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.7444     0.3961   9.453  <2e-16 ***
## chas          -1.8928     1.5061  -1.257   0.209
## I(chas^2)         NA          NA      NA      NA
## I(chas^3)         NA          NA      NA      NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.597 on 504 degrees of freedom
## Multiple R-squared:  0.003124, Adjusted R-squared:  0.001146
## F-statistic: 1.579 on 1 and 504 DF, p-value: 0.2094
```

```
summary(lm(crim ~ nox + I(nox^2) + I(nox^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ nox + I(nox^2) + I(nox^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.110 -2.068 -0.255  0.739 78.302
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   233.09      33.64   6.928 1.31e-11 ***
## nox          -1279.37     170.40  -7.508 2.76e-13 ***
## I(nox^2)       2248.54     279.90   8.033 6.81e-15 ***
## I(nox^3)      -1245.70     149.28  -8.345 6.96e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.234 on 502 degrees of freedom
## Multiple R-squared:  0.297, Adjusted R-squared:  0.2928
## F-statistic: 70.69 on 3 and 502 DF, p-value: < 2.2e-16
```

```
summary(lm(crim ~ rm + I(rm^2) + I(rm^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ rm + I(rm^2) + I(rm^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.485  -3.468  -2.221  -0.015  87.219
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept) 112.6246    64.5172    1.746    0.0815 .
## rm          -39.1501    31.3115   -1.250    0.2118
## I(rm^2)       4.5509     5.0099    0.908    0.3641
## I(rm^3)      -0.1745     0.2637   -0.662    0.5086
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.33 on 502 degrees of freedom
## Multiple R-squared:  0.06779,    Adjusted R-squared:  0.06222
## F-statistic: 12.17 on 3 and 502 DF,  p-value: 1.067e-07
summary(lm(crim ~ age + I(age^2) + I(age^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ age + I(age^2) + I(age^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -9.762 -2.673 -0.516  0.019  82.842
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.549e+00  2.769e+00  -0.920  0.35780
## age          2.737e-01  1.864e-01   1.468  0.14266
## I(age^2)     -7.230e-03  3.637e-03  -1.988  0.04738 *
## I(age^3)      5.745e-05  2.109e-05   2.724  0.00668 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.84 on 502 degrees of freedom
## Multiple R-squared:  0.1742, Adjusted R-squared:  0.1693
## F-statistic: 35.31 on 3 and 502 DF,  p-value: < 2.2e-16
summary(lm(crim ~ dis + I(dis^2) + I(dis^3), data = Boston))
```

```
##
## Call:
## lm(formula = crim ~ dis + I(dis^2) + I(dis^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.757  -2.588    0.031    1.267   76.378
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  30.0476     2.4459  12.285 < 2e-16 ***
## dis         -15.5543     1.7360  -8.960 < 2e-16 ***
## I(dis^2)      2.4521     0.3464   7.078 4.94e-12 ***
## I(dis^3)     -0.1186     0.0204  -5.814 1.09e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.331 on 502 degrees of freedom
## Multiple R-squared:  0.2778, Adjusted R-squared:  0.2735
```

```
## F-statistic: 64.37 on 3 and 502 DF, p-value: < 2.2e-16
```

```
summary(lm(crim ~ rad + I(rad^2) + I(rad^3), data = Boston))
```

```
##
```

```
## Call:
```

```
## lm(formula = crim ~ rad + I(rad^2) + I(rad^3), data = Boston)
```

```
##
```

```
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max  
## -10.381  -0.412  -0.269   0.179  76.217
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)  
## (Intercept) -0.605545   2.050108  -0.295   0.768  
## rad          0.512736   1.043597   0.491   0.623  
## I(rad^2)     -0.075177   0.148543  -0.506   0.613  
## I(rad^3)      0.003209   0.004564   0.703   0.482
```

```
##
```

```
## Residual standard error: 6.682 on 502 degrees of freedom
```

```
## Multiple R-squared:  0.4, Adjusted R-squared:  0.3965
```

```
## F-statistic: 111.6 on 3 and 502 DF, p-value: < 2.2e-16
```

```
summary(lm(crim ~ tax + I(tax^2) + I(tax^3), data = Boston))
```

```
##
```

```
## Call:
```

```
## lm(formula = crim ~ tax + I(tax^2) + I(tax^3), data = Boston)
```

```
##
```

```
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max  
## -13.273  -1.389   0.046   0.536  76.950
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)  
## (Intercept)  1.918e+01  1.180e+01   1.626   0.105  
## tax          -1.533e-01  9.568e-02  -1.602   0.110  
## I(tax^2)      3.608e-04  2.425e-04   1.488   0.137  
## I(tax^3)     -2.204e-07  1.889e-07  -1.167   0.244
```

```
##
```

```
## Residual standard error: 6.854 on 502 degrees of freedom
```

```
## Multiple R-squared:  0.3689, Adjusted R-squared:  0.3651
```

```
## F-statistic: 97.8 on 3 and 502 DF, p-value: < 2.2e-16
```

```
summary(lm(crim ~ ptratio + I(ptratio^2) + I(ptratio^3), data = Boston))
```

```
##
```

```
## Call:
```

```
## lm(formula = crim ~ ptratio + I(ptratio^2) + I(ptratio^3), data = Boston)
```

```
##
```

```
## Residuals:
```

```
##      Min       1Q   Median       3Q      Max  
## -6.833  -4.146  -1.655   1.408  82.697
```

```
##
```

```
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)
```

```

## (Intercept)  477.18405  156.79498   3.043  0.00246 **
## ptratio     -82.36054   27.64394  -2.979  0.00303 **
## I(ptratio^2)  4.63535   1.60832   2.882  0.00412 **
## I(ptratio^3) -0.08476   0.03090  -2.743  0.00630 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.122 on 502 degrees of freedom
## Multiple R-squared:  0.1138, Adjusted R-squared:  0.1085
## F-statistic: 21.48 on 3 and 502 DF,  p-value: 4.171e-13
summary(lm(crim ~ black + I(black^2) + I(black^3), data = Boston))

##
## Call:
## lm(formula = crim ~ black + I(black^2) + I(black^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.096  -2.343  -2.128  -1.439   86.790
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.826e+01  2.305e+00   7.924  1.5e-14 ***
## black        -8.356e-02  5.633e-02  -1.483   0.139
## I(black^2)    2.137e-04  2.984e-04   0.716   0.474
## I(black^3)   -2.652e-07  4.364e-07  -0.608   0.544
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.955 on 502 degrees of freedom
## Multiple R-squared:  0.1498, Adjusted R-squared:  0.1448
## F-statistic: 29.49 on 3 and 502 DF,  p-value: < 2.2e-16
summary(lm(crim ~ lstat + I(lstat^2) + I(lstat^3), data = Boston))

##
## Call:
## lm(formula = crim ~ lstat + I(lstat^2) + I(lstat^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.234  -2.151  -0.486   0.066   83.353
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.2009656  2.0286452   0.592  0.5541
## lstat       -0.4490656  0.4648911  -0.966  0.3345
## I(lstat^2)    0.0557794  0.0301156   1.852  0.0646 .
## I(lstat^3)   -0.0008574  0.0005652  -1.517  0.1299
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.629 on 502 degrees of freedom
## Multiple R-squared:  0.2179, Adjusted R-squared:  0.2133

```

```
## F-statistic: 46.63 on 3 and 502 DF,  p-value: < 2.2e-16
summary(lm(crim ~ medv + I(medv^2) + I(medv^3), data = Boston))

##
## Call:
## lm(formula = crim ~ medv + I(medv^2) + I(medv^3), data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -24.427  -1.976  -0.437   0.439   73.655
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 53.1655381  3.3563105  15.840 < 2e-16 ***
## medv        -5.0948305  0.4338321 -11.744 < 2e-16 ***
## I(medv^2)     0.1554965  0.0171904   9.046 < 2e-16 ***
## I(medv^3)    -0.0014901  0.0002038  -7.312 1.05e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.569 on 502 degrees of freedom
## Multiple R-squared:  0.4202, Adjusted R-squared:  0.4167
## F-statistic: 121.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

With the variables `indus`, `nox`, `age`, `dis`, `ptracio`, and `medv`, there is evidence of a non-linear relationship, as each of these variables squared and cubed terms is found to be statistically significant (we reject the null hypothesis that the coefficients on these exponentiated variables are zero). For `zn`, `rm`, `rad`, `tax` and `lstat` as predictor, the p-values suggest that the cubic coefficient is not statistically significant