## MATRIX FACTORIZATION APPROACH FOR IMPLICIT RATING OF LAST.FM DATASET

As given in the Last.fm paper, the frequency for a given user i and artist j can be normalized as follows:

$$\operatorname{freq}_{i,j} = \frac{\operatorname{count}(i,j)}{\sum_{j'} \operatorname{count}(i,j')} (1)$$

$$\mathbf{r}_{i,j} = 4 \cdot \left(1 - \sum_{k'=1}^{k-1} freq_{k'}(i)\right) (2)$$

To remove the skewness problem in the dataset because of the above formulas they suggested binning method:

$$B_i^1 = \{j : 4 \ge r_{i,j} > 3\}$$

$$B_i^2 = \{j : 3 \ge r_{i,j} > 2\}$$

$$B_i^1 = \{j : 2 \ge r_{i,j} > 1\}$$

$$B_i^1 = \{j : 1 \ge r_{i,j} > 0\} \ (3)$$

and to normalize the error of each Bin they proposed following:

$$e_{i,j}^{norm} = \frac{r_{i,j} - u_i^T p_j}{\sqrt{\|B_i^t\|}}$$
 (4)

For estimating the user and the product matrices with the use of this error normalization can be written as following, the updation done by:

$$\mathbf{u}_{i} \leftarrow u_{i} + \gamma \cdot \left(e_{i,j}^{norm} \cdot p_{j} - \lambda \cdot u_{i}\right)$$
$$p_{j} \leftarrow p_{j} + \gamma \cdot \left(e_{i,j}^{norm} \cdot u_{i} - \lambda \cdot p_{j}\right) (5)$$

Here,  $p_j$  denotes  $Artist \times LatentFactor$  and  $u_i$  shows  $User \times LatentFactor$ . Parameter  $\gamma$  here, denotes the learning rate of the model which should be less like 0.001.  $\lambda$  parameter is used for regularization so that model should not be overfitt over the training dataset and it will be between 0.1 to 0.2. The updation continues till convergence or maximum number of iterations of the program.

## COMBINE SEMANTIC INFORMATION

Dbpedia is the source from where we have fetched data regarding each Artist or we can say items. In our case  $R = User \times Artist$  and  $S = Artist \times Categories$  are the Rating (R) and Semantic Information (S) matrix respectively. Because we want to add the information into Artist or item feature vector, so the updation equation will be changed as below:

$$u_{i} \leftarrow u_{i} + \gamma \cdot \left(e_{i,j}^{norm} \cdot p_{j} - \lambda \cdot u_{i}\right)$$

$$p_{j} \leftarrow p_{j} + \gamma \cdot \left(e_{i,j}^{norm} \cdot u_{i} + 2\alpha \left(S - p_{j}c_{n}\right) - \lambda \cdot p_{j}\right) (6)$$

where  $c_n$  denotes  $LatentFactor \times Category$  and S denotes  $Artist \times Category$ . Here,  $\alpha$  term decide the importance of the combination of Semantic information.

## COMBINE ARTIST TAG INFORMATION

$$u_{i} \leftarrow u_{i} + \gamma \cdot \left(e_{i,j}^{norm} \cdot p_{j} - \lambda \cdot u_{i}\right)$$

$$p_{j} \leftarrow p_{j} + \gamma \cdot \left(e_{i,j}^{norm} \cdot u_{i} + 2\alpha \left(S - p_{j}c_{n}\right) + 2\beta \left(T - p_{j}g_{k}\right) - \lambda \cdot p_{j}\right) (7)$$

Where,  $g_k$  denotes  $Tag \times LatentFactor$  and  $p_j$  denotes  $Artist \times LatentFactor$ . Other then that,  $t_{j,k} \in T$  is calculated using the following formula:

$$t_{j,k} = \frac{1}{a}log\left(\frac{N}{b}\right) (8)$$

where;

N= total no. of tags presents in the dataset

a= no. of times users given this tags (how often this tag has been used by other Artist)

b= no. of users given same tag to the same Artist

## ADDING TIME INFORMATION FOR TAGGED ARTIST

to combine tag information we refer the paper "Time based Tag Recommendation using Direct and Extended Users Sets" by:Tereza Iofciu and Gianluca Demartini.

In which they have introduced time decaying factor with tag speceficity, two formulas given in that paper is as follows:

$$postScore_i = \lambda^{\triangle Time_i} \tag{9}$$

where  $\lambda$  is the time decaying function which is smaller than '1',  $\lambda$ =0.9. other formula is related to Tag speceficity, or we can say how often the tag has been rated using same tag by other users,

$$TagSpeceficity = log(50 + tagCount_i)$$
(10)

to measure the overall tag score they have used following formula:

$$TagScore_{i}orc_{i,j} = \frac{\sum_{i}^{n}(PostScore)}{TagSpeceficity_{i}}$$
(11)

We can say Tag Score as  $c_{i,j}$ , or the confidence coefficient for the Matrix T  $(Tag \times LatentFactor)$ , so to include this coefficient we will modify the Equation.7 as follows:

$$u_{i} \leftarrow u_{i} + \gamma \cdot \left(e_{i,j}^{norm} \cdot p_{j} - \lambda \cdot u_{i}\right)$$

$$p_{j} \leftarrow p_{j} + \gamma \cdot \left(e_{i,j}^{norm} \cdot u_{i} + 2\alpha \left(S - p_{j}c_{n}\right) + 2\beta \times c_{i,j} \left(T - p_{j}g_{k}\right) - \lambda \cdot p_{j}\right) (12)$$