Backprop for a 3 layer NN

$$l_1 = \sigma(w_1 x + b_1)$$
$$l_2 = \sigma(w_2 l_1 + b_2)$$
$$l_3 = \sigma(w_3 l_2 + b_3)$$

$$e = Y - l_3$$

$$\frac{de}{dw_3} = (Y - l_3)\frac{dl_3}{dw_3} = \underbrace{e \cdot \sigma'(w_3 l_2 + b_3)}_{O_3} l_2 = O_3 l_2$$

$$\frac{de}{dw_2} = (Y - l_3)\frac{dl_3}{dw_2} = e \cdot \sigma'(w_3 l_2 + b_3) \cdot \frac{dl_2}{dw_2} \cdot w_3$$

$$l_3 = \sigma(w_3 \sigma(w_2 l_1 + b_2) + b_3)$$

$$= e \cdot \sigma'(w_3 l_2 + b_3) \cdot w_3 \cdot \sigma'(w_2 l_1 + b_2) \cdot l_1$$
$$= \underbrace{O_3 \cdot w_3 \cdot \sigma'(w_2 l_1 + b_2)}_{O_2} l_1 = O_2 l_1$$

$$\frac{de}{dw_1} = (Y - l_3)\frac{dl_3}{dw_1} = e \cdot \sigma'(w_3 l_2 + b_3) \cdot w_3 \cdot \frac{dl_2}{dw_1}$$
$$= e \cdot \sigma'(w_3 l_2 + b_3) \cdot w_3 \sigma'(w_2 l_1 + b_2) \cdot w_2 \cdot \frac{dl_1}{dw_1}$$
$$= e \cdot \sigma'(w_3 l_2 + b_3) w_3 \sigma'(w_2 l_1 + b_2) \cdot w_2 \cdot \sigma'(w_1 x + b_1) \cdot x$$
$$= \underbrace{O_2 \cdot w_2 \sigma'(w_1 x + b_1)}_{O_1} \cdot x$$
$$= O_1 x$$

Update Eqns

$$w_3 = w_3 + \alpha \cdot \frac{de}{dw_3} = w_3 + \alpha O_3 l_2$$

$$w_2 = w_2 + \alpha \frac{de}{dw_2} = w_2 + \alpha O_2 l_1$$

$$w_1 = w_1 + \alpha \frac{de}{dw_1} = w_1 + \alpha O_1 x$$