Q5: a) base case: for $n = 1 \Rightarrow \frac{n^3+2n}{3} = \frac{3}{3} = 1 \Rightarrow$ remainder $= 0$.

induction: assuming that $\frac{(n-1)^3+2(n-1)}{3}$ has no remainder $(n \geq 2)$.

$$\Leftrightarrow (n-1)^3+2(n-1) = 3k \text{, } (k \text{ is a certain integer}).$$

$$\Rightarrow n^3+3n-3n^2-1+2n-2 = 3k$$

$$\Rightarrow n^3-3n^2+5n-3 = 3k.$$

$$\Rightarrow n^3+2n = 3n^2-3n+3+3k.$$

$$= 3(n^2-n+1+k).$$

since $n^2-n+1+k$ is a integer, so $\frac{n^3+2n}{3}$ has no remainder.

b). base case: for $n=2$: $n = 2 = 1 \times 2$ (it is a product of prime numbers).

strong induction: assuming for a certain number n, any number $k<n$ is a product of prime numbers.

1) if n is prime number.

$n = 1 \times n \Rightarrow n$ is a product of prime numbers.

2) if n is no prime number.

then $\exists x, y \, [(x<n, y<n) \wedge (x \cdot y = n)]$.

since x, y are both products of prime numbers.

$n = x \cdot y$ is also product of prime numbers