Numerical Software Lab
Prof. Ulrich Kleinekathöfer
Spring term 2023
Project 4, due April 20, 2023 at 11:59 pm
to be uploaded to https://elearning.jacobs-university.de

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4. Project [100 points]

Write a script which has the following functionality inside:

a) [60 points] Write a function that solves the second order differential equation

$$\frac{\partial^2 x}{\partial t^2} - a(1-x)\frac{\partial x}{\partial t} + bx^2 = c\cos(\omega t) \ .$$

This second-order differential equation can be re-written into two first-order equations for which $v = \partial x/\partial t$ is actually one of the two first-order equations. Arguments of the function should be the parameters a, b, c, and ω as well as the initial values for x and v with the default values $x(t=0) = 1$ and $v(t=0) = 0$. Moreover, vector of times t needs to be supplied at which the solution is requested. Additional input values should be the starting and end values of x and v for a plot. The function should solve the differential equation and make a sensible plot of the results, i.e., x and v as a function of time, including axes labels, legend and title. The plot shall not be displayed on the screen but be exported as a pdf file. Moreover, write the vectors t and the resulting vector x into one comma-separated file as two rows.

b) [40 points] Write three function which numerically determine the volume and moments of inertia of a torus of uniform density, unit mass, average radius R and cross-sectional radius r. Use `scipy.integrate.tplquad` to numerically determine

$$\begin{aligned}
V &= 2\int_0^{2\pi}\int_{R-r}^{R+r}\int_0^{\sqrt{r^2-(\rho-R)^2}} \rho\, dz d\rho d\theta \ , \\
I_z &= \frac{2}{V}\int_0^{2\pi}\int_{R-r}^{R+r}\int_0^{\sqrt{r^2-(\rho-R)^2}} \rho^3\, dz d\rho d\theta \ , \\
I_x = I_y &= \frac{2}{V}\int_0^{2\pi}\int_{R-r}^{R+r}\int_0^{\sqrt{r^2-(\rho-R)^2}} (\rho^2\sin^2\theta + z^2)\rho\, dz d\rho d\theta \ .
\end{aligned}$$

Evaluate these integrals for the torus ($R > r$) and compare to the exact values given by

$$\begin{aligned}
V &= 2\pi^2 R r^2 \ , \\
I_z &= R^2 + \frac{3}{4}r^2 \ , \\
I_x = I_y &= \frac{1}{2}R^2 + \frac{5}{8}r^2 \ .
\end{aligned}$$

In the main part of the script call the functions with arbitrary test values and print input as well as output.

Happy coding!