# Abelian Sandpile Basics

2018

Abelian Sandpile Model

- Rough model of a pile of sand, based on a finite directed (multi-) graph G, with vertices V and edges E.

- Chips (grains of sand) are stacked on the vertices.

- Chips can flow to other vertices via edges of G.

- Originally studied on uniform grids.

- Also known as the chip-firing game.

Chip-Firing

- Let n_i be the number of chips on vertex v_i

- Let $e_i = \{e \in E : e$ starts at $v_i\}$

- If a vertex $n_i \geq |e_{v_i}|$ it can **fire**.

- When a vertex fires, it transfers a chip down each of its **outbound** edges.
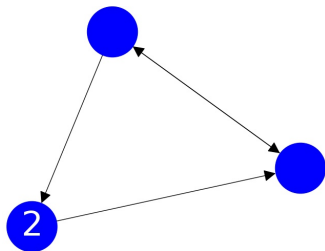
(a) Before firing: Lower left vertex has one outbound edge and two chips.
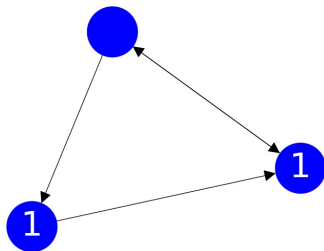
(b) After firing: One chip has been transferred to the far right vertex.

Figure: Firing example.

Chip Configuration

Stable Configuration

"Sink"

Reduced Laplacian

Chip Configuration Revisited

Stabilizing

Firing "History"

Chip Addition Operator