ASSIGNMENT 2

Graduate Seminar – MA693

SUBMITTED TO
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(a). For deterministic SEIR model,

$$S(t+\Delta t) = S(t)(1 - \beta I(t)\Delta t)$$

$$E(t+\Delta t) = \beta S(t) I(t)\Delta t + (1 - \kappa \Delta t) E(t)$$

$$I(t+\Delta t) = \kappa E(t)\Delta t + (1-\alpha\Delta t) I(t)$$

$$R(t+\Delta t) = \alpha I(t)\Delta t + R(t)$$

For Stochastic SEIR Model,

$$E[\Delta S(t)] = -\beta S(t) I(t) \Delta t$$

from $S(t)$ take $S^* \sim BIN(S(t), \beta I(t)\Delta t)$

Similarly for E & I

$E^* \sim BIN(E(t), \kappa \Delta t)$ & $I^* \sim BIN(I(t), \alpha \Delta t)$

$\therefore$

$$S(t+\Delta t) = S(t) - S^*(t)$$

$$E(t+\Delta t) = E(t) + S^*(t) - E^*(t)$$

$$I(t+\Delta t) = I(t) + E^*(t) - I^*(t)$$

$$R(t+\Delta t) = R(t) + I^*(t)$$

So, by using Binomial distribution the expected value of changes in stochastic models will be equal to that of deterministic model.

b)

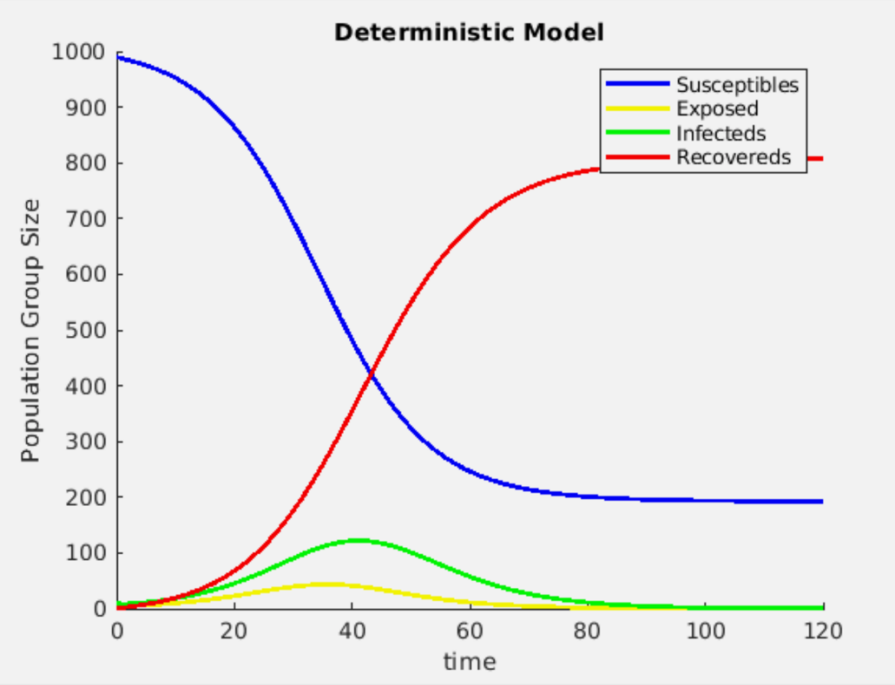

Figure: 1

In figure 1, we can see the deterministic SEIR Model. In contrast to that we can see in figure 2, the stochastic vs deterministic SEIR Model shows us that stochastic model consider the worst case and best case scenario of the epidemic. We can see that sample paths are spread around the lines of deterministic model. The variation in the starting of time period is less while it expands when the time increases.

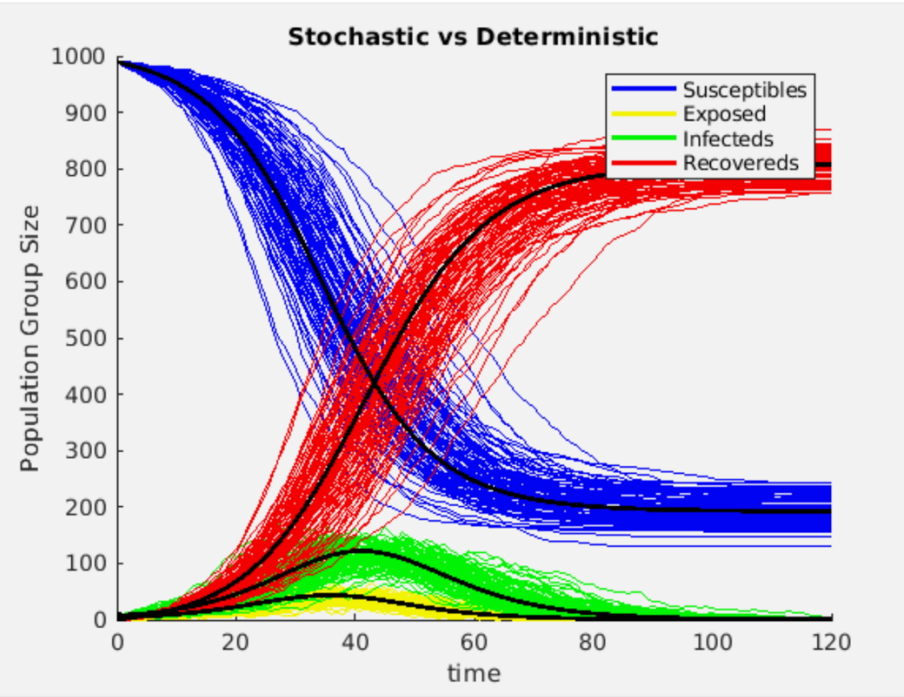

Figure: 2

c).

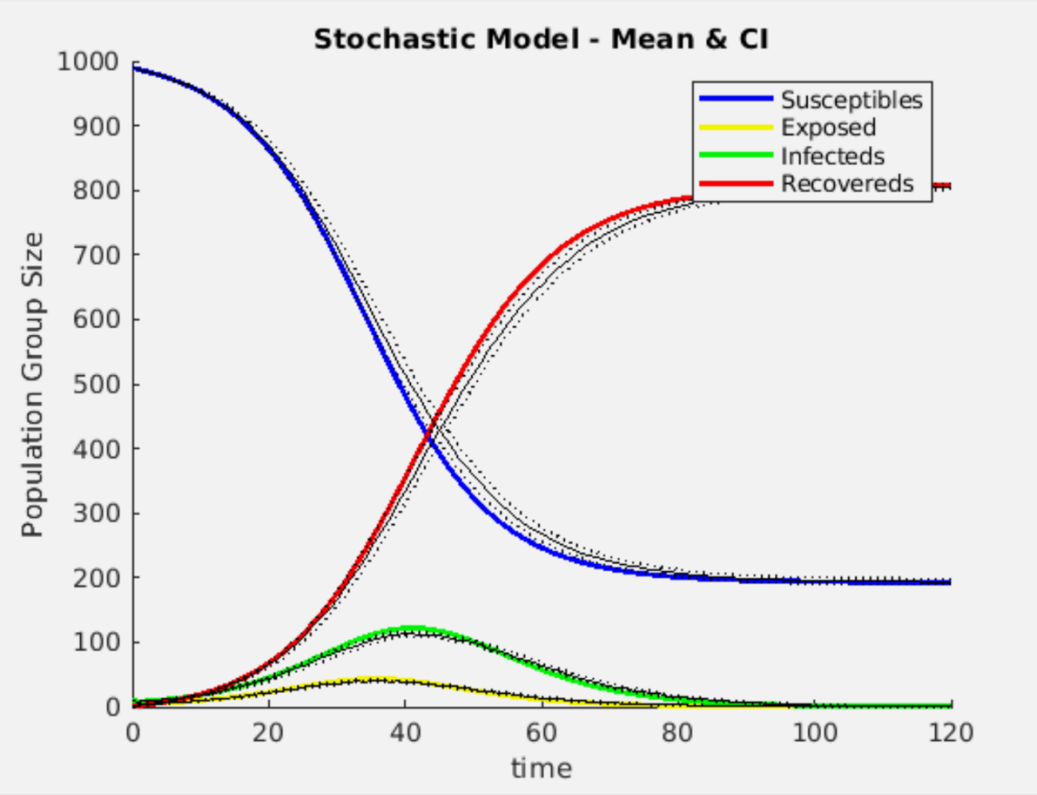

Figure: 3

After plotting the Mean and Confidence Interval of the stochastic SEIR model we can see that (Figure:3) the mean of the stochastic model is closely following the deterministic model for susceptible, exposed, infected and recovered. Also, the upper and lower confidence interval for the mean of stochastic model are narrow.

d). For the deterministic model,

Maximum value of $I_{max} = 122.0434$

Time at which maximum value occurs $T_{max} = 42$

e).

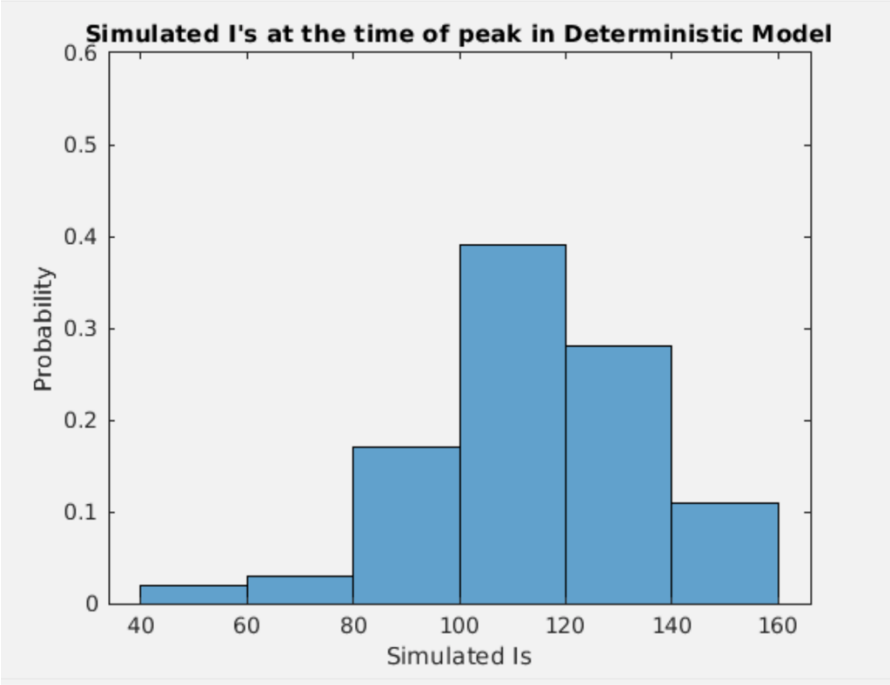

Figure: 4

In the figure 4, we can see that simulated I's have the mean nearby 120 which is close to the maximum value of deterministic model. Hence, we can say that average of stochastic model is following the deterministic model. We can also observe that at the time of peak of deterministic model, simulated I's range from low as 40 to high as 160.

f). For the deterministic model,

The value of $\tau_0 = 36$,

The value of $\tau_1 = 48$

And

The length of stressed period $\tau_1 - \tau_0 = 12$

g). For the stochastic model,

The mean of $\tau_0 = 48.44$,

The mean of $\tau_1 = 59.17$

And

The mean of the length of stressed period $\tau_1 - \tau_0 = 10.73$

The observed difference in the mean of the values of τ_0, τ_1 and length of the stressed period is due to the fact that where I never exceeded the $0.9 * I_{max}$, there we assumed their values to be equal to Time Horizon which is 120 in our case.

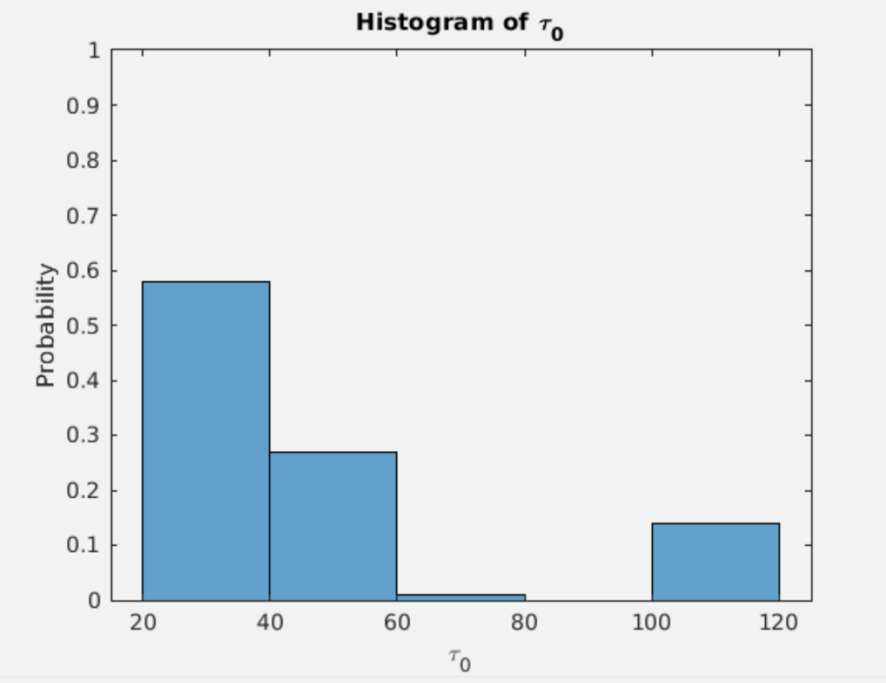

Figure: 5

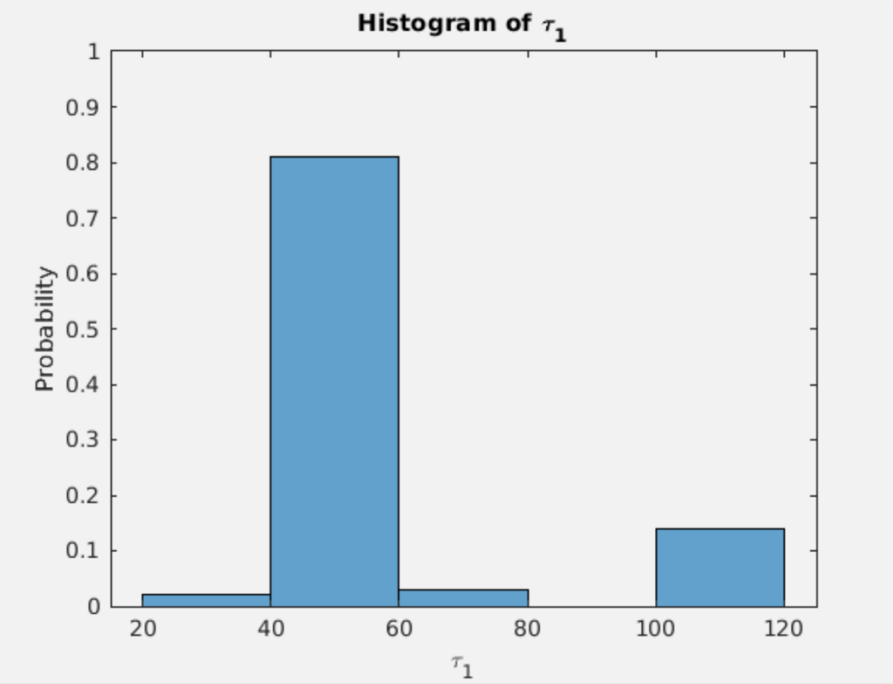

Figure: 6

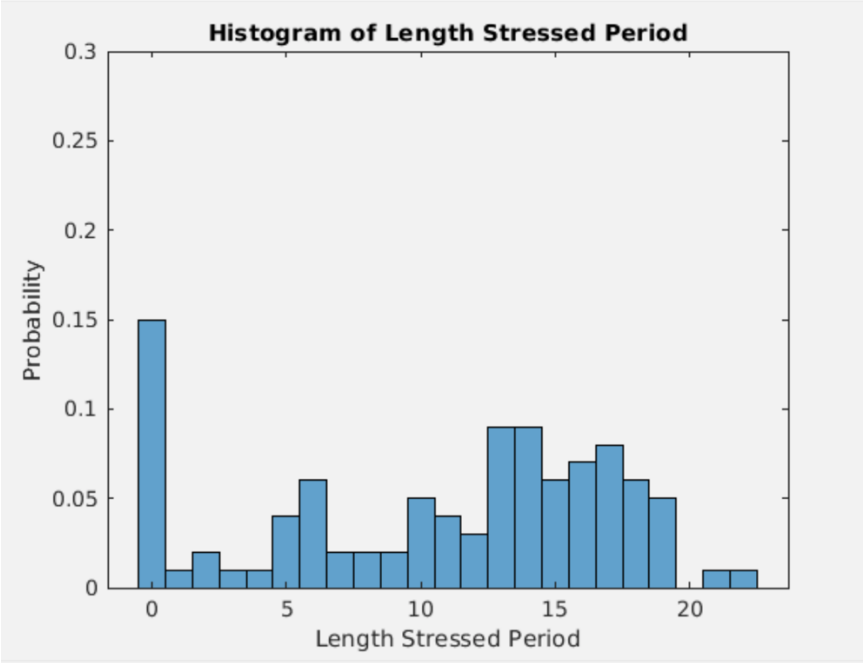

Figure: 7

In figure 5 and 6, we can see that the mean of τ_0 and τ_1 is nearby 40 and 50 respectively which is close to deterministic model. We can also observe some of the values that are near to 120 which are the values where I never exceeded threshold value.

While it is difficult to say same about the mean of the length of stressed period from the figure 7.

h).

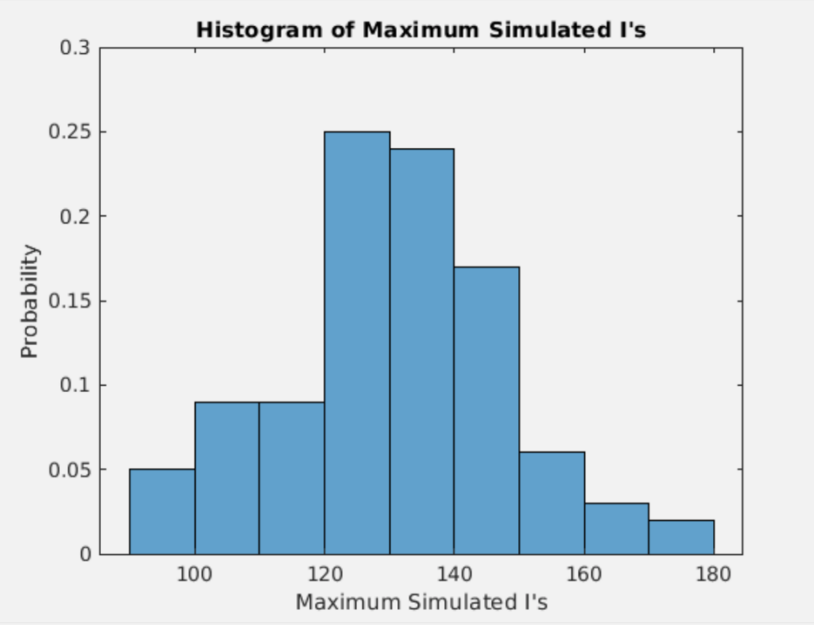

Figure: 8

The mean of Simulated Maximum $I's = 129.73$ which is close to the mean of the deterministic model which is equal to 122.0434

From the figure 8, it can be seen that it follows normal distribution and the mean is around 130 and the simulated maximum $I's$ range from 90 to 180.

i).

Probability that healthcare system will be overwhelmed in deterministic model will be zero because maximum I is I_{max}. Therefore, I will never be more than I_{max}. Hence probability will be always zero.

For stochastic model,

$$P\{\text{system will be overwhelmed}\} = 0.54$$

Which can also be observed from the sample paths plotted in the figure 2, we can observe that almost half of the sample paths cross the maximum value of infected(deterministic).

j).

Stochastic models provide a better understanding of the effects over health care system because they can present the best as well as the worst case scenarios of the epidemic. While deterministic models are good for approximating the stochastic model. By looking at stochastic models, policies and preparations can be done in advance for the effects of the epidemic.