From Excell data

(i) $\hat{\mu} = 0.0516$, $\hat{\sigma} = 0.1988$, $C_0 = 2.3464$, $P_0 = 2.8716$

$r = 0.03$, $K = 64$, $T = 14/52$

At $t = 0$

$$V_0 = 1000 * S_0 + 2000 * C_0 + 1000 * P_0 = 100{,}164.4$$

At $t = 1/52$,

$$V_t = 1000 * S_t + 2000 * C_t(S_{1/52}, t, \hat{\sigma}, r, K, T) + 1000 * P_t(S_{1/52}, t, \hat{\sigma}, r, K, T)$$

here $S_t = S_0 * e^{(\hat{\mu} - \hat{\sigma}^2/2)t + \hat{\sigma}\sqrt{t}\, Z}$ where $Z \sim N(0,1)$

$$C_t = S_0 \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$$

$$P_t = K e^{-r(T-t)} \Phi(-d_2) - S_0 \Phi(-d_1)$$

$$\therefore \quad L = -(V_t - V_0)$$

$$L = 100{,}164.4 - 1000\, S_t - 2000\, C_t - 1000\, P_t$$

(ii) $L = 100{,}164.4 - 1000 S_t - 2000 C_t - 1000 P_t$

Now for linearized loss,

Risk measures are S_t & t

$$\therefore L^{\Delta} = L(S_0, t_0) + L_{S_t}(S_0, t_0)(S_t - S_0) + L_t(S_0, t_0)\, t$$

Now, $L(S_0, t_0) = 0$, $L_{S_t}(S_0, t_0) = -1000 - 2000\Delta_A^C - 1000\Delta_B^P$

& $L_t(S_0, t_0) = -2000\,\Theta_A^C - 1000\,\Theta_B^P$

$$\Delta_A^C = \frac{\partial C}{\partial S} = N(d_1) = 0.4885\,\&\qquad \Theta_A^C = \frac{\partial C}{\partial t} = -\frac{S_0\hat{\sigma}\, N'(d_1)}{2\sqrt{T}} - rKe^{-r(T)}N(d_2) = -5.6619$$

$$\Delta_B^P = \frac{\partial P}{\partial S} = -N(-d_1) = -0.5115\,\&\qquad \Theta_B^P = \frac{\partial P}{\partial t} = -\frac{S_0\hat{\sigma}}{2\sqrt{T}}N'(d_1) + rKe^{-r(T)}N(-d_2) = -3.7573$$

$\boxed{\because \text{ See Matlab Code}}$

$$\therefore L^{\Delta} = (-1000 - 2000\Delta_A^C - 1000\Delta_B^P)\, S_t - S_0(-1000 - 2000\Delta_A^C - 1000\Delta_B^P) + (-2000\,\Theta_A^C - 1000\,\Theta_B^P)\, t$$

$$\Rightarrow L^{\Delta} = -1465.4\, S_t + 92259 + 15081\, t$$

(iii) $$F_{L^{\Delta}}(l) = P(L^{\Delta} \le l) = P\left(-1465.4\, S_t + 92259 + 15081t \le l\right)$$

$$\Rightarrow F_{L^{\Delta}}(l) = P\left(S_t \ge \frac{92259 + 15081t - l}{1465.4}\right)$$

$$= P\left(e^{X_t} \ge \frac{92259 + 15081t - l}{1465.4 * S_0}\right)$$

$$= P\left(X_t \ge \ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right)\right)$$

$$\Rightarrow F_{L^{\Delta}}(l) = P\left((\mu - \sigma^2/2)t + \sigma\sqrt{t}\, Z \ge \ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right)\right)$$

$$= P\left(Z \ge \frac{1}{\sigma\sqrt{t}}\left[\ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right) - (\mu - \sigma^2/2)t\right]\right)$$

$$\therefore F_{L^{\Delta}}(l) = P\left(Z \le \frac{1}{\sigma\sqrt{t}}\left[(\mu - \sigma^2/2)t - \ln\left(\frac{92259 + 15081t - l}{1468.4 * S_0}\right)\right]\right)$$

$\because$ By symmetry.

$$\therefore F_{L^{\Delta}}(l) = \Phi\left(\frac{1}{\sigma\sqrt{t}}\left[(\mu - \sigma^2/2)t - \ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right)\right]\right)$$

(iv) For VaR_{α} we need to solve,

$$F_{L^{\Delta}}(l) = u \Rightarrow \Phi\left(\frac{1}{\sigma\sqrt{t}}\left[(\mu - \sigma^2/2)t - \ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right)\right]\right) = u$$

$$\Rightarrow \sigma\sqrt{t}\, \Phi^{-1}(u) - (\mu - \sigma^2/2)t = -\ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right)$$

$$\Rightarrow \ln\left(\frac{92259 + 15081t - l}{1465.4 * S_0}\right) = (\mu - \sigma^2/2)t - \sigma\sqrt{t}\, \Phi^{-1}(u)$$

$$\Rightarrow 92259 + 15081t - l = 1465.4 * S_0 * \exp\{(\mu - \sigma^2/2)t - \sigma\sqrt{t}\, \Phi^{-1}(u)\}$$

$$\Rightarrow \quad l = 92259 + 15081t - 1465.4 * S_0 * \exp\left\{(\mu - \sigma^2/2)t - \sigma\sqrt{t}\,\Phi^{-1}(u)\right\}$$

$$\therefore \quad \boxed{VaR_\alpha = 92259 + 15081t - 1465.4 * S_0 * \exp\left\{(\mu - \sigma^2/2)t - \sigma\sqrt{t}\,\Phi^{-1}(\alpha)\right\}}$$

$$\Rightarrow \quad VaR_\alpha = 92549.02 - 135{,}696.04 * \exp\left\{0.00023228 - 0.02756\,\Phi^{-1}(\alpha)\right\}$$

$\therefore$ VaR at 95% = 4323.7

&

VaR at 99% = 5965.8

$\boxed{\because \text{See Matlab Code}}$

$$ES_\alpha(L^\Delta) = E\left[92549.02 - 1465.4 S_t \mid 92549.02 - 1465.4 S_t > VaR_\alpha(L^\Delta)\right]$$

$$\Rightarrow E\left[99549.02 - 1465.4 S_t \mid S < \frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4}\right]$$

$$\Rightarrow 99549.02 - 1465.4\, E\left[S_t \mid S < \frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4}\right]$$

$$\Rightarrow 99549.02 - 1465.4 \times S_0\, E\left[e^X \mid X < \ln\left(\frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4 \times S_0}\right)\right]$$

$$X \sim N\left((\mu - \sigma^2/2)t, \sigma^2 t\right)$$

$$\therefore \quad ES_\alpha(L^\Delta) = \frac{-1465.4 S_0}{1-\alpha} E\left[e^X I_{X < \ln\left(\frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4 \times S_0}\right)}\right] + 99549.02$$

Using the identity,

$$E\left[e^{Bx}\, \mathbb{I}_{[x<\Delta]}\right] = e^{\mu B + 1/2\, \sigma^2 B^2} \cdot \Phi\left(-\sigma B + \frac{\Delta-\mu}{\sigma}\right)$$

where $X \sim N(\mu, \sigma^2)$

$\therefore$ We get $\frac{-1465.4 S_0}{1-\alpha} E\left[e^{X} I_{x < \ln\left(\frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4 \times S_0}\right)}\right] + 99549.02$

$$ES_\alpha = \frac{-1465.4 \cdot S_0}{1-\alpha} \cdot \exp\left\{(\mu - \sigma^2/2)t + \frac{1}{2}\sigma^2 t\right\}$$

$$* \Phi\left(-\sigma\sqrt{t} + \frac{1}{\sigma}\left[\ln\left(\frac{92549.02 - VaR_\alpha(L^\Delta)}{1465.4 \times S_0}\right) - (\mu - \sigma^2/2)t\right]\right) + 99549.02$$

Putting in the values,

$$\therefore ES_\alpha = -\frac{135662.7}{1-\alpha} \cdot \exp\left\{0.000\,9923\right\}$$

$$* \Phi\left(-0.0275 + 5.03\left[\ln\left(\frac{92549.02 - VaR_\alpha(L^\Delta)}{135662.7}\right) - 0.000612\right]\right) + 99549.02$$

(V) $\Delta - \Gamma$ approximation

$$L = 100{,}164.4 - 1000 S_t - 2000 C_t - 1000 P_t$$

Risk Factor $- S_t$

$$\Rightarrow L^{\Delta-\Gamma} = L(S_0) + L^S(S_t - S_0) + \frac{1}{2} L^{SS}(S_t - S_0)^2$$

& $L(S_0) = 0$, $L^S(S_0, t_0) = -1000 - 2000\Delta_A^C - 1000\Delta_B^P$

& $L^{SS}(S_0, t_0) = -2000\Gamma_A^C - 1000\Gamma_A^P = -3000\Gamma$

here $\Delta_A^C = 0.4885$, $\Delta_B^P = -0.5115$

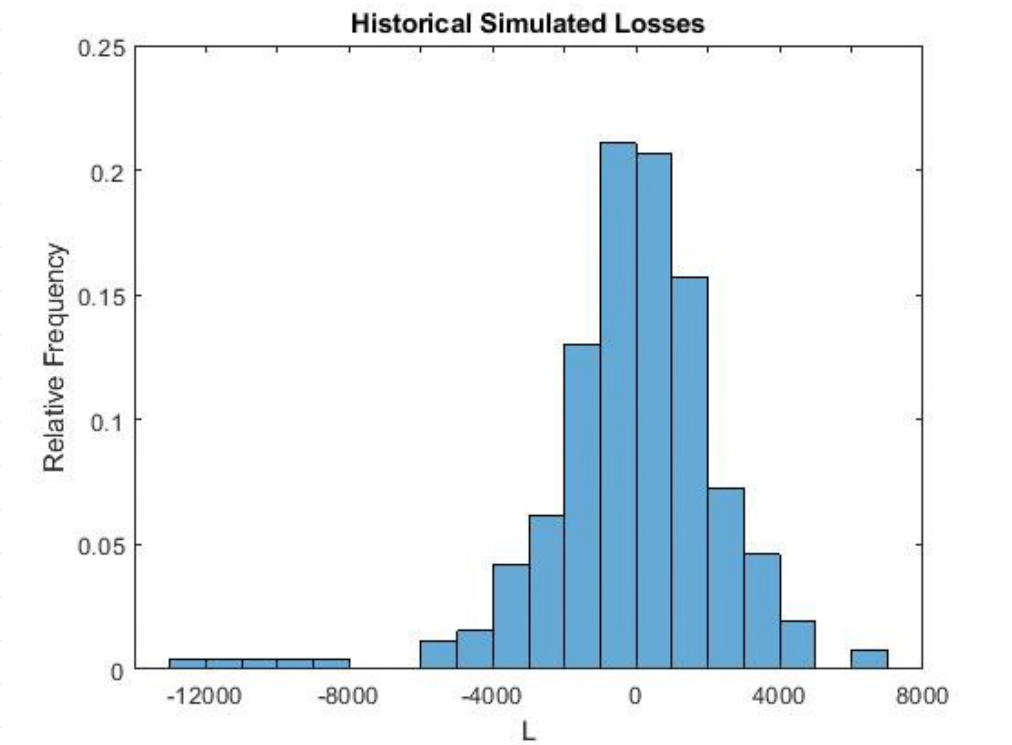
& $\Gamma_A^C = \Gamma_B^P = \dfrac{\varphi(d_1)}{S_0 \hat{\sigma} \sqrt{T}} = 0.0614$

$$\therefore L^{\Delta-\Gamma} = (-1000 - 2000\Delta_A^C - 1000\Delta_B^P)(S_t - S_0) - 1500\Gamma_A (S_t - S_0)^2$$

$$\Rightarrow L^{\Delta-\Gamma} = 1465.5(92.6 - S_t) - 92.1(S_t - 92.6)^2$$

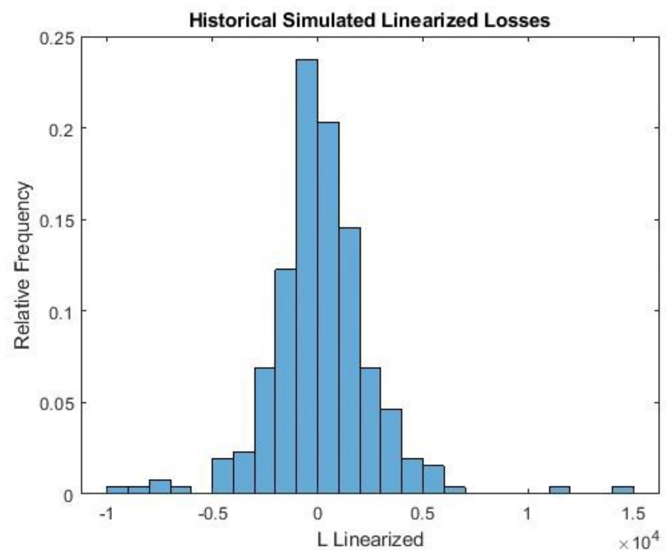
(b). Historical Simulations

(i)



(ii) $VaR_{0.95} = \$3280.5$ & $ES_{0.95} = \$4324.9$

(iii)



$VaR_{0.95} = \$3876.5$

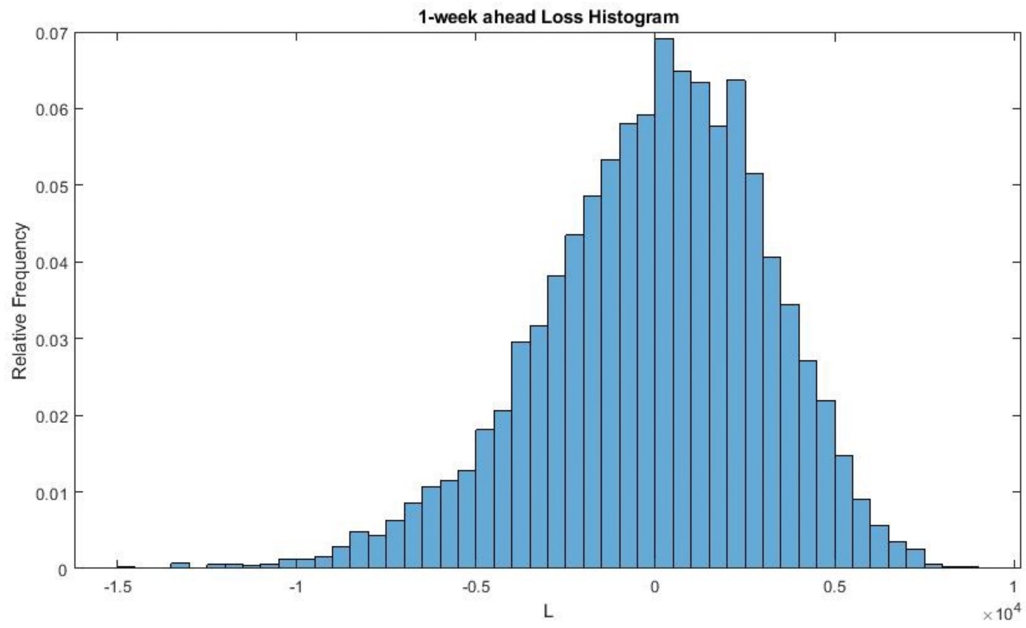
&

$ES_{0.95} = \$6154.4$

The Historical Linearized VaR is less than Linearized VaR this can be due to the fact we only have few observations for estimating Historical Linearized VaR.

(c)

(i) Monte - Carlo Simulation



For $\alpha = 0.95$

$VaR_{0.95} = \$ 4480.1$

$ES_{0.95} = \$ 5439.4$

For $\alpha = 0.99$

$VaR_{0.99} = \$ 6075.9$

$ES_{0.99} = \$ 6819.1$

(ii) Using $\Delta - \Gamma$ approximation, $\tilde{L}$,

For $\alpha = 0.95$

$VaR_{0.95} = \$ 3337$ & $ES_{0.95} = \$ 3915.3$

For $\alpha = 0.99$

$VaR_{0.99} = \$ 4278$ & $ES_{0.99} = \$ 4638.5$

(iii) We can see that $\Delta-\Gamma$ approximations, $\tilde{L}$, are different from the values of monte-carlo simulations of L.

It can be due to the fact that $\Delta-\Gamma$ approximations does not consider t as risk measure to approximate it.

(iv) Monte-Carlo simulations estimations are similar to L^{Δ} estimations of 95% & 99% VaR.

While historical estimators are different from the the monte-carlo simulations. It can be due to the fact that historical estimations rely on the data.

(a).

$$\text{Standard error} = \frac{1}{f(x)}\sqrt{\frac{(1-q)q}{n}}$$

(i) $n = 261$, $q = 0.95$, Standard Error $= 102.7706$

95% CI for 95% VaR using Historical simulation.

lower CI $= \$3079.1$

Upper CI $= \$3481.9$

[See Matlab]

(ii) $n = 10{,}000$, $q = 0.95$, Standard Error $= 20.5175$

95% CI for 95% VaR using Monte Carlo Simulation

$VaR_{0.95} = \$4480.1$

lower CI $= \$4439.8$

Upper CI $= \$4520.5$

[See Matlab]

(b)

Average $VaR_{0.95} = \$4597.7$

& Standard Deviation $VaR_{0.95} = \$51.8688$

&

lower CI $= \$4558.8$

Upper CI $= \$4593.5$

[See Matlab]

Average $ES_{0.95}$ = \$ 5556.6

& Standard Deviation $VaR_{0.95}$ = \$ 55.8642

&

lower CI = \$ 5536.6 [See Matlab]

Upper CI = \$ 5576.6