Radon Optical Flow

***Abstract*—Optical flow estimation is one of the front-end tasks in most of the computer vision applications such as motion detection, object segmentation, time-to-collision and focus of expansion calculations, motion compensated encoding, and stereo disparity measurement. In this paper, we propose a novel but effective method based on Radon transform to estimate optical flow fields across frames. One of the most fundamental properties of the radon transform is that it is invariant to translation that means object movement results in shifted projections. We use this property as it relates translational motion in the image to simple displacement in the projections. We formulate a Radon based optical flow method using shift and derivative properties of Radon transform. The advantage of the proposed method over other methods is it's capability of solving aperture equation. We study the affect of this Radon Optical Flow on different previous popular optical flow methods such as Lukas-Kanade and Horn-Schunk methods and Phase correlation based methods and test the robustness of the proposed method towards different types of image degradations such as noise, illumination changes and blur etc.**

I. Introduction

Without doubt, the motion analysis is one of the important tasks in computer vision. The estimation of optical flow is a front-end task in motion image analysis. It is the process of approximating the movement of brightness patterns in an image sequence and, thus, provides useful information for the determination of the 3D structure of the environment and the relative motion between the camera and the objects in the image [2]. There are many techniques for computing optical flow that have been proposed including Gradient based techniques [10], [13], [7] that are based on spatio-temporal derivatives of image intensity; Variational techniques [3] that extend the gradient-based techniques by using different data or smoothness constraints [15]; Block matching based techniques [12], [2] that assume all pixels in a block undergo the same motion; Energy-based techniques [8] that are based on the output energy of velocity-tuned filters [5]; Bayesian techniques [11] that utilize probability smoothness constraints in the form of a Gibbs random field; and Phase correlation techniques [9] that compute the flow by applying the phase correlation to the images locally.

In all these methods, they assume some additional constraints such as smootheness or gradient constancy etc. in addition to aperture problem of optical flow as only aperture problem is not sufficient to solve the problems of finding the motion flow of pixels. Here, we propose a novel but effective method of solving the optical flow problems mainly aperture problem without the need of any additional constraints by using the projective information from the radon domain. In case of uniform motion, that means whole scene is moving with same velocity, we can estimate motion flow from aperture equation only very easily and accurately. In case of affine or non-uniform motion, we propose a novel method of computing the optical flow in Radon domain. Flow fields for each frame in a sequence is estimated by , first, dividing frame into a regular grid of patches and then, transforming the each pair of co-sited patches using the Radon transform (RT). The proposed method can estimate not only the pure translation, but also the scale and rotation motion of image patches (full similarity transforms) where as the basic phase correlation techniques [9] estimate flow by assuming pure translational motion between image patches. Hence, Radon transform based method can estimate flow fields for a full non-uniform motion model that ordinary phase correlation methods could not. As a result, a more accurate estimation of the optical flow can be achieved. This method should be robust towards the image distortions that are related to brightness changes because of the robustness of RT. The results are compared with the ground truth as well as results by Lucas-Kanades method [13] and Horn-Schunck's method [10], due to their popularity.

The remainder of the paper is organized as follows. Section2 describes the Radon transform and how it can handle the translation as well scale and rotation motion of images or image patches. The application of RT in calculating the optical flow of an image sequence is described in Section 3. In Section 4, we discuss the analysis of affine or non-uniform motion using proposed method. Next, in Section 5, we show a number of experimental results to demonstrate the effectiveness of our approach. Finally, the conclusions and the future directions are given in Section 6.

II. Radon transform and it's properties

The shift property of the Radon transform shows that 2-D translational motion in the image domain results in 1-D translational motion along projection axis. More specifically, if $R(\rho, \theta)$ isthe projection of $f(x, y)$ at angle θ defined by

$$R(\rho,\theta) = \int\int f(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy \qquad (1)$$

A. Shift Property

we have

$$f(x,y) \rightarrow R(\rho,\theta) \qquad (2)$$

$$f(x-x_0, y-y_0) \rightarrow R(\rho - x_0\cos\theta, \theta - y_0\sin\theta) \qquad (3)$$

where x_0 and y_0 represent the amounts of shift in X-direction and Y-direction respectively.

B. Rotation property

$$f(x,y)/\alpha \rightarrow R(\frac{\rho}{\cos(\theta-\alpha)}, \theta-\alpha) \tag{4}$$

where left side term represents the image obtained by rotating the original image f with an angle of α in anti-clockwise direction.

C. Scaling property

$$f(\frac{x}{a}, \frac{y}{b}) \rightarrow \frac{1}{|a|} R(\rho\frac{a}{b}, \arctan(\frac{\tan\theta}{b})) \tag{5}$$

where a and b represent the scaling factors in X-direction and Y-direction respectively.

Before beginning our development of a model for Radon Optical Flow (ROF), we first introduce two useful differentiation properties of the Radon transform which will be used later in the paper.

D. Transform of Derivatives

Let $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ denote linear differential operators and we have

$$\frac{\partial f}{\partial x} \rightarrow \cos\theta \frac{\partial R}{\partial \rho} \tag{6}$$

$$\frac{\partial f}{\partial y} \rightarrow \sin\theta \frac{\partial R}{\partial \rho} \tag{7}$$

For instance, a useful corollary is

$$\nabla f \rightarrow [\cos\theta \sin\theta]^T \frac{\partial R}{\partial \rho} \tag{8}$$

III. Radon Optical flow

Now, let us consider an image sequence $f(x,y,t)$, which evolves in time according to the spatially varying motion vector field $v(x,y) = [v_x(x,y)v_y(x,y)]^T$. Generally, optical flow methods estimate motion fields between two image frames which are captured at times t and $t+\delta t$ at every pixel position. They use local Taylor series approximations of the image signal, which in turns partial derivatives with respect to the spatial and temporal coordinates. Thus, these methods are called differential methods.

For a $2D+t$ dimensional case (3D or n-D cases are similar) a pixel at location (x,y,t) with intensity $f(x,y,t)$ will have moved by δx, δy and δt between the two image frames, and the following image constraint equation can be given:

$$f(x,y,t) = f(x+\delta x, y+\delta y, t+\delta t) \tag{9}$$

Assuming the movement to be small, the image constraint at f(x,y,t) with Taylor series can be developed to get:

$$\frac{\partial f}{\partial x}v_x + \frac{\partial f}{\partial y}v_y + \frac{\partial f}{\partial t} = 0 \tag{10}$$

where v_x and v_y are the x and y components of the velocity or optical flow of $f(x,y,t)$ and $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial x}$ are the derivatives of the image at (x,y,t) in the corresponding directions. By applying Radon transform on both sides of Eq(10), we get

$$v_x \cos\theta \frac{\partial R}{\partial \rho} + v_y \sin\theta \frac{\partial R}{\partial \rho} + \frac{\partial R}{\partial t} = 0 \tag{11}$$

R_ρ and R_t can be written for the derivatives in the following. Thus:

$$(v_x \cos\theta + v_y \sin\theta) R_\rho = -R_t \tag{12}$$

or

$$v.[\cos\theta \sin\theta] R_\rho = -R_t \tag{13}$$

We term this relationship the aperture problem of Radon Optical flow. In general optical flow, Eq.(10) is an equation which contains two unknowns and cannot be solved as such. This is called as the aperture problem of the optical flow algorithms. To find the optical flow another set of equations is needed, which is given by some additional constraints in general. All optical flow methods introduce additional conditions or assumptions for estimating the actual flow.

But in Radon Optical flow, only the aperture equation Eq.(12) is enough to solve all the problems that related to Optical flow estimation. We can compute v_x by applying aperture equation at a pixel on $\theta = 0^0$ line and v_y by applying aperture equation at a pixel on $\theta = 90^0$ line.

At $\theta = 0^0$ Eq.(12) becomes $v_x = -\frac{R_t}{R_\rho}$;

At $\theta = 90^0$ Eq.(12) becomes $v_y = -\frac{R_t}{R_\rho}$;

Hence , without using any additional constraints, we found motion flow of $f(x,y,t)$. This is the advantage of this method.

IV. Analysis of Affine or Non-uniform motion

Any motion field can be locally approximated (to first order) by affine motion. Hence, it is important to consider the class of motions given by

$$v = v_0 + M \begin{pmatrix} x \\ y \end{pmatrix} \tag{14}$$

where $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the operator corresponds to rotation and scaling. v_0 is a fixed vector denoting translational motion. Affine motion model is six parameter system as it needs 6 equations to solve. Here, we can apply Eq. 13 on six different locations to estimate affine parameters.

If it is pure uniform motion, then estimation of motion flow becomes very easy problem. We can easily find flow parameters by transforming the images into Radon domain and applying aperture equation Eq.(12) at the vertical lines $\theta = 0^0$ for v_x and $\theta = 90^0$ for v_y. In case of affine or non-uniform motion, we apply same technique for the image blocks of size MxN by assuming the flow to be constant in a local neighbourhood around the central pixel under consideration at any given time. To find sub-pixel accurate flow vectors, we follow [14] and simply upscale the input images using bicubic interpolation. To prevent the sudden junks in final flow, we have used weighted averaging step [9].

V. Experiments and results

The performance of the proposed robust OF (ROF) estimation was evaluated using the benchmark Middlebury dataset [1], which includes both synthetic and real scenes. These sequences contain all the components that make the OF ambiguous and difficult including the aperture problem, textureless regions, motion discontinuities, occlusions, large motions, small objects, non-rigid motion, mixed pixels, changes in illumination, motion blur, non-Lambertian reflectance, and camera noise [4]. Thus, these sequences provide meaningful comparisons between different OF algorithms. Figure 1a shows frame 10 of the real Rubber Whale sequence. The ground truth flow and obtained result are shown in Figures 1b and 1c, respectively. Figure 2a shows frame 14 of the synthetic Street sequence which is a synthetic image sequence. The ground truth and our result are shown in Figures 2b and 2c, respectively. Figure 3a shows frame 10 of the Venus sequence which is a stereo image sequence. The ground truth and our result are shown in Figures 3b and 3c, respectively. In these figures, the flow field has been scaled and resized for closer examination. It can be seen clearly from the figures that our results coincides well with the ground truth flow field. The computed flow field was also assessed quantitatively using different error metrics. These are presented next.

A. Error metrics

Two error metrics were considered: Average Angular Error (AAE) and the Average End point Error (AEE) [4]. Let $V_0 = (u_0, v_0)$ be the ground truth flow and $V_1 = (u_1, v_1)$ be the estimated flow. The angular error (AE) is found from the dot product of these two vectors as

$$\phi_{AE} = \arccos(\vec{V_0}.\vec{V_1}) \tag{15}$$

where

$$\vec{V_0} = \frac{1}{\sqrt{u_0^2+v_0^2+1}}(u_0, v_0, 1)$$
$$\vec{V_1} = \frac{1}{\sqrt{u_1^2+v_1^2+1}}(u_1, v_1, 1)$$

The AAE is then obtained by calculating the average of all (entire field) the angular errors between ground truth and estimated velocities in the optical flow.

A second error metric, is the average end point error (AEE) [4] which is defined as the average of the absolute difference of the groundtruth and the estimated flow vectors.

Experiments were done to test both the key issues of interest: accuracy and robustness of proposed method. The accuracy of the proposed method was compared with the results of well-known optical flow algorithms such as Lucas-Kanade method [13], a modern Horn-Schunck method [10] and a Phase-correlation method which uses Fourier-Mellin Transform (FMT) [9]. The robustness was tested under some extreme cases of distortions. Here the comparison is done with methods which are blockbased and that use robust estimators such as kNNME [6] and FMT [9] in addition to Lucas-Kanade method.

B. Accuracy

The computed accuracy measures are summarized in Table 1 for six different data sequences. Lukas-Kanade and Horn-Schunck criteria appear to fare relatively poorly on the Venus and Dimetrodon sequences while the proposed method (listed as ROF) outperforms them. ROF performance is good as it ranks first in 6 of 6 sequences in terms of AAE and 5 of 6 sequences in terms of AEE. Across all sequences and both errors, it outperforms the other methods thereby confirming that ROF criterion does not affect the accuracy of OF estimation.

C. Some extreme cases of Robustness

Since a key motivation of the proposed formulation was the tendency of brightness consistency to be violated easily, we assessed the robustness of the proposed method by imposing different distortions on the input data sequences. The distorted images were derived from the original sequences. Four types of degradation that were considered are

- Noise :salt and pepper noise covering 10% of a frame,
- Overexposure : (to simulate a camera flash),
- Non-uniform blur : Spatially varying Gaussian blur,
- Combo : combination of the above 3 distortions.

Sample distorted images for RubberWhale and Street frames are shown in First row of Table2 and Table3 respectively. The robustness is assessed once again in terms of the AAE and AEE metrics. The quantitative results for a real (RubberWhale) and a synthetic (Street) image sequence are summarized in Tables 2 and 3 respectively. The results on the real image sequence indicate that overall, the block-based approaches (the bottom two rows) fare better under distortion compared to the Horn-Schunck method as the error increases rapidly for the latter. This is due to the fact that the block based approaches by design have attempted to increase robustness. However, the kNN-ME method is susceptible to mixed, noisy, and blurry environments while the FMT method is relatively robust to only brightness changes. The proposed ROF consistently performs well across sequences as the increase in error under distortions is quite small. This confirms that the Radon transform based criterion is indeed a good strategy for robust OF estimation. The results on the synthetic sequence exhibit a trend similar to that on real sequence.

VI. Conclusions and Future work

In this paper, a formulation for OF based on Radon transfrom rather than brightness pattern-based movement was proposed and a novel method for OF estimation for an image sequence was presented. The radon transform-based formulation was found to marginally boost the accuracy and greatly improve the robustness of the flow field estimation. The OF computation from a pair of frames took about 2 minutes on a single core processor without any code optimisation. Future investigation would consider a reduction in this computation time to enable real-time applications.

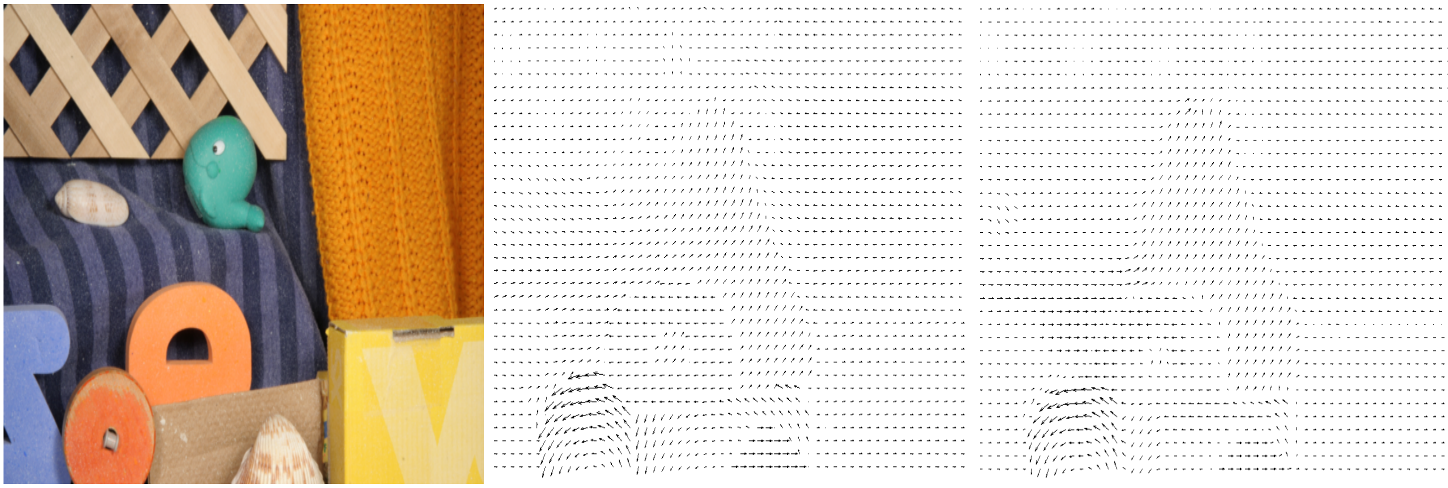

Fig. 1. Experimental result for RubberWhale image sequence (Real scene) a) Frame10, b) Ground truth and c) Our result

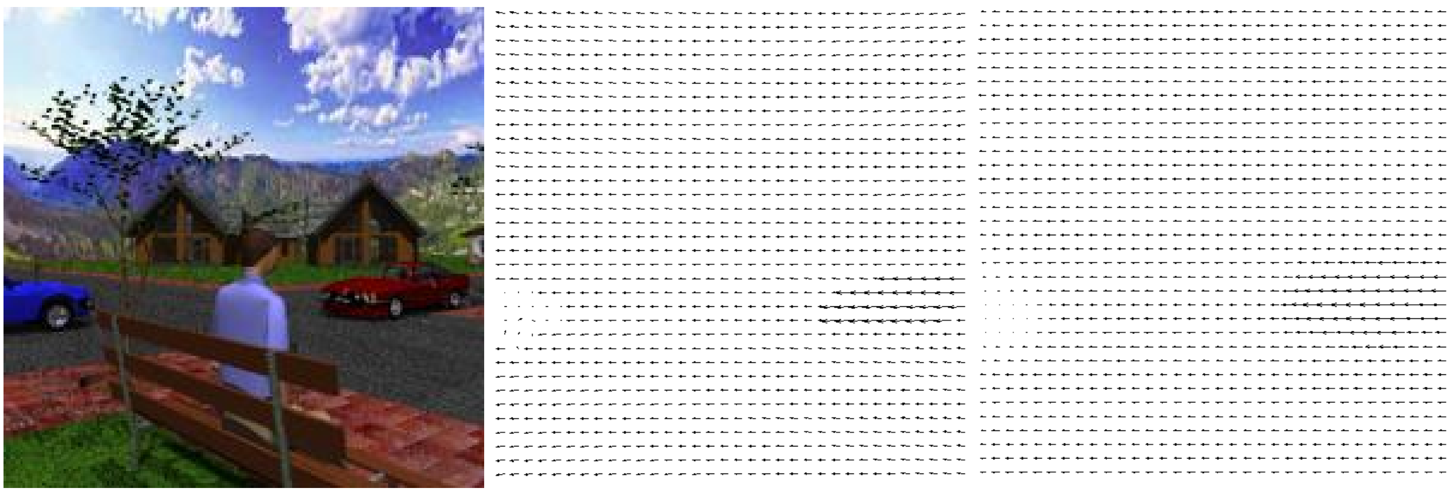

Fig. 2. Experimental result for Street image sequence (Synthetic scene) a) Frame14, b) Ground truth and c) Our result

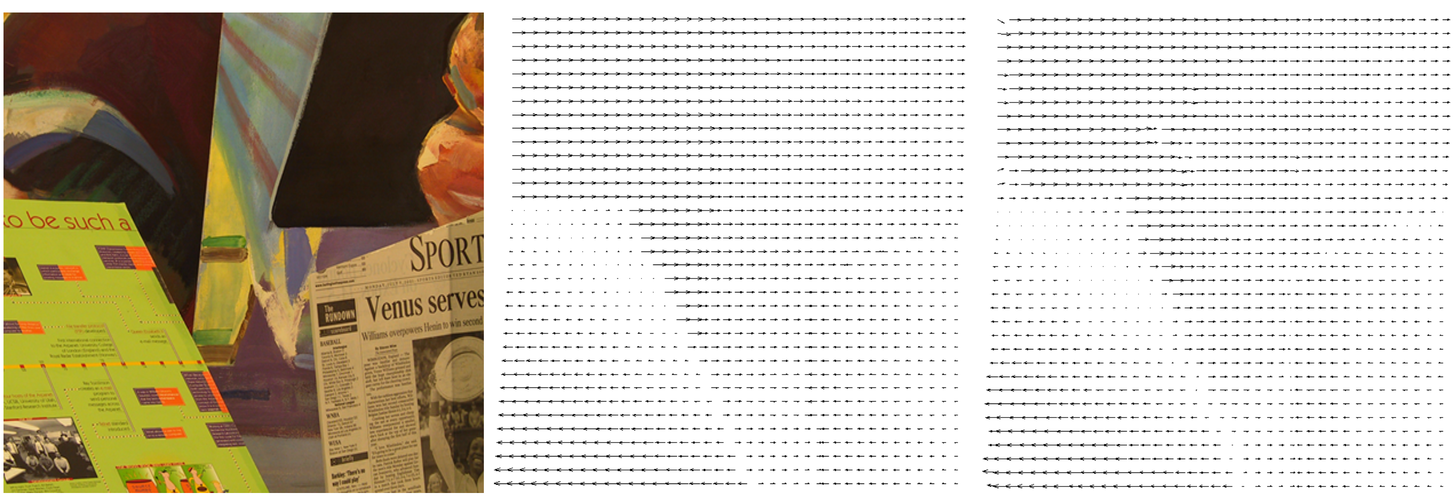

Fig. 3. Experimental result for Venus (Stereo) image sequence a) Frame10, b) Ground truth and c) Our result

										
	Street		Venus		Rubberwhale		Dimetrodon		Hydrangea	
Method	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE
Lucas-Kanade[13]	6.45	0.18	41.27	0.74	18.69	0.46	37.14	0.66	29.85	0.81
Horn-Schunck[10]	4.75	0.15	19.38	0.45	16.75	0.37	18.59	0.40	27.87	0.81
FMT[9]	4.66	0.14	5.51	0.14	10.07	0.26	7.33	0.18	11.83	0.56
ROF	**3.19**	**0.12**	**5.44**	**0.28**	**8.97**	**0.16**	**6.67**	**0.15**	**9.19**	**0.45**

TABLE I

COMPARISON OF AAE AND AEE ERROR METRICS FOR DIFFERENT METHODS ON SYNTHETIC AND REAL IMAGE SEQUENCES.

			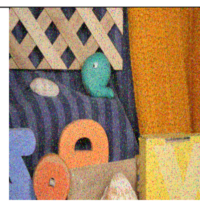							
	Original		Noise		Overexposure		Blur		Combo	
Method	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE
kNN-ME[6]	17.51	0.34	26.07	0.45	20.10	0.38	28.54	0.42	38.22	0.50
FMT[9]	10.07	0.26	19.43	0.54	12.08	0.32	18.31	0.30	28.15	0.61
Lucas-Kanade[13]	18.69	0.46	27.14	0.63	22.82	0.46	28.44	0.51	29.15	0.81
ROF	**8.97**	**0.16**	**9.87**	**0.18**	**9.15**	**0.16**	**9.18**	**0.16**	**11.46**	**0.19**

TABLE II

COMPARISON OF AAE AND AEE ERROR METRICS FOR DIFFERENT DISTORTIONS ON RUBBERWHALE IMAGE SEQUENCE FOR DIFFERENT METHODS.

									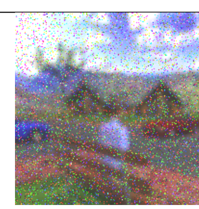	
	Original		Noisy		Overexposure		Blur		Combo	
Method	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE
kNN-ME[6]	6.44	0.11	8.48	0.41	7.95	0.13	9.54	0.43	10.44	0.46
FMT[9]	4.66	0.14	9.18	0.33	5.94	0.17	8.46	0.26	16.19	0.49
Lucas-Kanade[13]	6.45	0.18	19.61	0.47	11.71	0.35	14.79	0.42	26.67	0.78
ROF	**3.19**	**0.12**	**15.03**	**0.19**	**3.94**	**0.16**	**4.78**	**0.17**	**6.29**	**0.26**

TABLE III

COMPARISON OF AAE AND AEE ERROR METRICS FOR DIFFERENT DISTORTIONS ON STREET IMAGE SEQUENCE FOR DIFFERENT METHODS.

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