# An Approach to Reduce the Blurred Edges for Super-Resolution Reconstruction

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*Abstract*—**In this paper, we theoretically analyze the reasons that caused blurred edges in the traditional POCS (projection onto the convex sets) super-resolution reconstruction. In order to improve the quality of the reconstructed image, this paper analyzes and improves the algorithm at different points. First, the paper proposes to obtain the initial reference frame with a gradient interpolation algorithm in replacement of the traditional bilinear interpolation. Secondly, the paper uses a motion estimation method called pre-filter iterative gradient to achieve image registration. Finally, the paper selects the POCS algorithm for reconstruction. In order to get better edge information, we adaptively modify the PSF (point spread function) by combination with the edge neighborhood pixel gray information. Experimental results show the proposed algorithm can significantly improve image quality, maintains the rich edge information and has a higher PSNR and better visual effect.**

***Keywords- super resolution; POCS; gradient interpolation; pre-filter iterative registration; edge enhancement***

## I. INTRODUCTION

Currently, super-resolution image has been more and more widely applied. Super-resolution reconstruction has been proposed due to the physical equipment and cost constraints. Its purpose is to compensate for the lack of hardware and to improve image resolution and enhance the image availability from the perspective of the software [1].Super-resolution image reconstruction technique is to restore a high-resolution and high-quality image from multitudinous low-resolution observations degraded by warping, blurring, noise. Currently this technology has been widely used in astronomy, remote sensing, military surveillance and medical diagnosis.

Stark and Oskoui [2] first applied the POCS (Project onto Convex Sets) to super resolution image reconstruction. Patti [3] proposed the image acquisition model considering a variety of degraded factors. Eren [4] extended Patti method to multiple moving targets scene. In recent years, Papa [5] put forward the POCS algorithm based on particle swarm optimization. Ogawa [6] proposed the POCS algorithm based on principal component analysis (PCA). Gho [7] proposed applied POCS algorithm for the reconstruction of magnetic resonance images through low-pass and high-pass filter.

POCS [8] super resolution reconstruction is proposed based on set theory. In this method, because the image correction process is based on PSF (point spread function), the reconstructed image has jagged edges and ringing. So this article proposes some improvements. First, we use the gradient interpolation algorithm to obtain the initial value of super-resolution image, then adopt a four-parameter pre-filtering iterative registration for motion estimation to improve registration accuracy, finally modify the PSF to better enhance the reconstructed image edges.

This paper is structured as follows. It introduces POCS method in Section II and proposes three improvements in Section III. The effectiveness of the proposed method is proved by the experiment results in Section IV. Finally, the conclusion is presented in Section V.

## II.THE TRADITIONAL POCS ALGORITHM

### *A. The Acquisition Model of Low Resolution Images*

In the process of digital image acquisition, the images are deteriorated due to the presence of noise which is generally

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considered as additive noise. Therefore, the image observation model [9] is as follows

$$g_k = D_k M_k W_k f + n_k \qquad 1 \le k \le p. \tag{1}$$

Where, $f$ is of size for $M \times N$ HR (high resolution) image, $g_k$ is the Kth frame LR (low resolution) image of size for $M/q \times N/q$($q$ is the down sampling parameter). $W_k$ is the geometric transformation matrix. $M_k$ is the optical fuzzy matrix. $D_k$ is the decimation matrix. $n_k$ is the noise vector . $p$ is the number of LR observation frames.

Image super-resolution reconstruction is a reverse process based on the model which rebuilds the HR$f$ by $g_1$, $g_2$, $\cdots g_k$.

### *B. The Process of POCS Algorithm Reconstruction*

POCS algorithm first selects one of the LR images as the reference image, then enlarge it by bilinear interpolation as the initial estimate of POCS method. Other LR images are based on the reference frame for the image registration. Finally, the image is projected onto the convex sets and continuously iterates until producing the ideal HR image.

According to prior information (such as positive definite, the energy-bounded, data consistency, smooth, etc), we can define multiple convex sets. Convex set is defined as follows

$$C_k(x,y) = \{\hat{f}(x,y) : |r_k(x_1,y_1)| \le \delta_k\}. \tag{2}$$

Here, $\hat{f}(x,y)$ is the estimated high resolution image. $h_k(x_1,y_1;x,y)$ is the PSF of the observed frame at the pixel $(x,y)$.

$$r_k(x_1,y_1) = g_k(x_1,y_1) - \sum_{x=0}^{M}\sum_{y=0}^{N}\hat{f}(x,y)h_k(x_1,y_1;x,y) \tag{3}$$

$$k = 1,2,3,\ldots\ldots p$$

In the POCS method, suppose there are $m$ kinds of prior information, there are correspondingly $m$ closed convex sets for $C_i$, $i = 1,2,\ldots,m$. The HR image$f \in C_0 = \bigcap_{i=1}^{i=m} C_i P_i$. Here, $C_0$ is a non-empty closed convex set. For the given constraint set, the corresponding projection operator is $P_i$, then the iterative sequence is

$$f_{n+1} = T_m T_{m-1} \ldots T_1 f_n. \qquad n = 1,2,\ldots\ldots \tag{4}$$

$$T = (1-\lambda)I + \lambda P \qquad 0 < \lambda < 2 \tag{5}$$

Here, $\lambda$ is relaxation projection parameters. We can speed up the convergence rate of POCS method by adjusting the relaxation parameter$\lambda$. As a result of the relaxation projection operator, the convergence stability of POCS method is improved.

## III.THE IMPROVED POCS ALGORITHM

This paper firstly uses gradient interpolation algorithm to obtain the initial value of the super-resolution image, then adopts a four-parameter pre-filtering iterative registration for motion estimation to improve registration accuracy. Finally in order to enhance the edge of the reconstructed image, we utilize the Canny operator to detect edge. When the template center is on the edge, we combine with neighboring pixel gray information to modify PSF so to improve image quality.

### *A. Gradient Interpolation*

In the POCS algorithm, the initial estimate for SR image has a great influence on the feasibility of the algorithm. Each SR image estimate is projected to the nearest point of the next set from it which means that the final solution is similar to the initial estimate in a way [10]. The traditional POCS method uses bilinear interpolation and does not consider the edge and smooth region that is the main reason causing blurred edges. In this paper, we adopt gradient interpolation algorithm to obtain the initial estimate and enhance the edges. The interpolation algorithm based on gradient [11] analyzes the gray value change of the edges of the pixel gray region and uses the gradient information of adjacent pixels so that the interpolated points can minimize the impact of the edge gray value and make edge more obvious.

The interpolation thought is as follows: Assuming that $g_{11}$,$g_{12}$, $g_{21}$, $g_{22}$ is corresponding to the four adjacent pixels in the LR image (Fig. 1), $f_{ij}(1 \le i \le 4)$ corresponds to the pixel blocks in the interpolated SR image.

| | | |
|---|---|---|
| $g_{11}$ | $g_{12}$ | |
| $g_{21}$ | $g_{22}$ | |
| | | |

Fig. 1 The sub-image of the LR image

In gradient interpolation algorithm, first each point in LR image corresponds to each point in SR image such as $f_{11}=g_{11}$, $f_{13}=g_{12}$, $f_{31}=g_{21}$, $f_{33}=g_{22}$ (Fig. 2), then we use the interpolation method to calculate $f_{12}$, $f_{21}$, $f_{22}$.

| | | |
|---|---|---|
| $f_{11}$ | $f_{12}$ | $f_{13}$ |
| $f_{21}$ | $f_{22}$ | |
| $f_{31}$ | | $f_{33}$ |

Fig. 2 The sub-image of the interpolated HR image

Let us first calculate the values of $f_{12}$, $f_{21}$. Because the edge reflects the relatively large areas of the gradient, we need to calculate the absolute value of the gradient for $g_{11}$, $g_{12}$ and $g_{21}$, $g_{22}$. As follows

$$\begin{cases} \frac{dg}{dx}=|g_{12}-g_{11}| \\ \frac{dg}{dy}=|g_{21}-g_{11}| \end{cases} . \tag{6}$$

To ensure the image edge, the interpolated points are as close as possible to small gray value pixels. The horizontal interpolation mainly uses the horizontal gradient information while the vertical interpolation mainly uses the vertical gradient information.

$$\begin{cases} f_{12}=\min(g_{11},g_{12})+\frac{dg}{dx}\times r \\ f_{21}=\min(g_{11},g_{21})+\frac{dg}{dy}\times r \end{cases} \tag{7}$$

Where, $r$ is the interpolation parameter. Similarly, we calculate the gradient in the diagonal direction of $45°$ and $135°$ for the pixel $f_{22}$. Experimental results show that this method has significant improvement and maintains clearer edges than the bilinear interpolation.

*B. The Pre-filtering Iterative Registration of the Sub-pixels*

After obtaining the initial estimate by the above gradient interpolation, we need to find the motion vector between the observed frames and the reference frame and add it to the current estimate of the HR image. The accuracy of motion vector estimation seriously affects the quality of reconstructed image. Firstly, the image preprocessing through low-pass filter can effectively inhibit the effect of noise, then use sub-pixel iterative method for image registration. The traditional Keren [12] registration adopts three-parameter method based on the small-angle Taylor series expansion. This paper utilizes four-parameter method to reduce the angle error caused by the Taylor series expansion.

Define two images for $g(x',y')$ and $f(x,y)$, there are the following relations in mathematics [13].

$$g(x',y')=T(f(x,y)). \tag{8}$$

Here, $T$ represents the geometric transformation. This paper adopts a simplified four parameters affine model.

$$x'=x+k_1x+k_2y+k_3 \tag{9}$$

$$y'=y+k_1y-k_2x+k_4 \tag{10}$$

Turn it into matrix form

$$M=C^{-1}V. \tag{11}$$

Here, $M$ is the motion parameter matrix. $C$ and $V$ are the gradient matrixes.

$$M=\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} V=\begin{bmatrix} \sum R_1(g-f) \\ \sum R(g-f) \\ \sum \frac{\partial f}{\partial x}(g-f) \\ \sum \frac{\partial f}{\partial y}(g-f) \end{bmatrix}$$

$$C=\begin{bmatrix} \sum R_1^2 & \sum RR_1 & \sum R_1\frac{\partial f}{\partial x} & \sum R_1\frac{\partial f}{\partial y} \\ \sum RR_1 & \sum R^2 & \sum R\frac{\partial f}{\partial x} & \sum R\frac{\partial f}{\partial y} \\ \sum R_1\frac{\partial f}{\partial x} & \sum R\frac{\partial f}{\partial x} & \sum\left(\frac{\partial f}{\partial x}\right)^2 & \sum\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} \\ \sum R_1\frac{\partial f}{\partial y} & \sum R\frac{\partial f}{\partial y} & \sum\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} & \sum\left(\frac{\partial f}{\partial y}\right)^2 \end{bmatrix}$$

$$R=y\frac{\partial f}{\partial x}-x\frac{\partial f}{\partial y} \qquad R_1=x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$$

In order to improve registration accuracy and get good result no matter how much rotation angle is, this paper adopts the following formula to gradually approach the true value by iterating (K is the number of iteration).

$$M_{k+1}=C_k{}^{-1}V_k+M_k \tag{12}$$

In order to increase the computational efficiency and robustness to noise, this paper adopts the three-layer Gaussian pyramid with the resolution from coarse to fine.

*C. Edge Adaptive Constraints*

Using the above gradient interpolation improves the edge quality of the initial estimate of HR image to some extent, while it is not very good and there are still jagged edges and ringing. This is because the PSF $h(x,y)$ in the projection just uses the neighboring pixel spatial information, while ignoring the neighboring pixel gray value. Therefore, this paper proposes the edge adaptive constraints to modify PSF based on the idea of bilateral filtering [14]. It simultaneously uses the neighboring pixel spatial information and gray information which effectively maintains the edges of the image when eliminating noise. The weighting coefficient $w(i,j)$ of the modified PSF consists of two parts as follows

$$w(i,j) = w_s(i,j) \cdot w_r(i,j). \quad (13)$$

Where, $w_s(i,j)$ is the space similarity factor, $w_r(i,j)$ is the gray similarity factor. Their definitions are

$$w_s(i,j) = e^{-\frac{|i-x|^2+|j-y|^2}{2\sigma_s^2}}, \quad (14)$$

$$w_r(i,j) = e^{-\frac{|f(i,j)-f(x,y)|^2}{2\sigma_r^2}}. \quad (15)$$

Where, $\sigma_s$, $\sigma_r$ are the standard deviations. After the above changes, the fuzzy edge in the rebuilding process is ultimately inhibited and the quality of the reconstructed SR image is also significantly improved.

## IV. EXPERIMENTAL ANALYSIS AND RESULT

Based on the above theoretical analysis, this paper uses two sets of simulation experiments to verify the improved POCS reconstruction algorithm in matlab7.0 platform. The reconstructed SR image is four times the size of the LR image and global motion among the LR images is translation and rotation. Set the standard deviation of the PSF in the experiment $\sigma_s = 1$, the size of support domain is $5 \times 5$, $\sigma_r = 0.02$, $r = 0.3$, $\lambda = 0.1$. The number of iterations is 10.

In Fig. 3, we use eight degraded LR images (size for $249 \times 239$) called Table to obtain a reconstructed SR image (size for $498 \times 478$). The image in (a) is the HR image using bilinear interpolation. The image in (b) is the HR image using gradient interpolation. The images in (c) and (d) are respectively the HR image using the traditional POCS method and the improved POCS method. Obviously the image in (b) is much clearer than image in (a). Although image in (c) remains a lot of details, there are still some fuzzy edges. The image in (d) obtained by the improved algorithm in this paper is not only much clearer but also maintains more perfect edges. The edge ringing effect of the table items almost disappeared. So it greatly improves the quality of the reconstructed image.

In Fig. 4, we use eight degraded LR images (size for $128 \times 128$) called Lena to obtain a reconstructed SR image (size for $256 \times 256$). The image in (a) is the HR image using bilinear interpolation. The image in (b) is the HR image using gradient interpolation. The images in (c) and (d) are respectively the HR image using the traditional POCS method and the improved POCS method. The figure shows that image in (b) is much clearer than image in (a). The image in (c) remains a lot of details while there are ringing and jagged edges. The image in (d) obtained by the improved algorithm in this paper is not only much clearer but also maintained more effective edge (especially the edge of the brim). The quality of the reconstructed image is significantly improved.

This paper uses PSNR (peak signal to noise ratio) and AG (average gradient) to quantitatively evaluate the quality of the HR images obtained by the experiments. AG reflects the contrast in the image details. The PSNR between the initial high resolution image $f(x,y)$ and the reconstructed high resolution image $\hat{f}(x,y)$ is defined as

$$\mathrm{PSNR} = 10 \cdot \log 10 \frac{255^2 \times M \times N}{\sum_{m=1}^{M} \sum_{n=1}^{N} \left( f(x,y) - \hat{f}(x,y) \right)^2}. \quad (16)$$

Here, $M$ and $N$ are respectively the length and width of the high resolution image.

The comparison results of PSNR and AG are respectively shown in Table I and Table II. The tables show the improved POCS super-resolution reconstruction algorithm in this paper is superior to the traditional POCS algorithm.

TABLE I. THE COMPARISON OF PSNR (db)

| **Method** | **Test Images** | |
|---|---|---|
| | ***Table*** | ***Lena*** |
| bilinear interpolation | 24.880 | 21.649 |
| gradient interpolation | 25.433 | 22.757 |
| the traditional POCS | 25.781 | 22.788 |
| the improved POCS | 26.716 | 23.514 |

TABLE II. THE COMPARISON OF AG (average gradient)

| **Method** | **Test Images** | |
|---|---|---|
| | ***Table*** | ***Lena*** |
| bilinear interpolation | 9.042 | 9.828 |
| gradient interpolation | 11.347 | 13.248 |
| the traditional POCS | 15.004 | 17.903 |
| the improved POCS | 16.324 | 19.431 |

(a) (b)

(c) (d)

Fig. 3 The image called Table (a) the HR image using bilinear interpolation (b) the HR image using gradient interpolation (c) the HR image using the traditional POCS method (d) the HR image using the improved POCS method

(a) (b) (c) (d)

Fig. 4 The image called Lena (a) the HR image using bilinear interpolation (b) the HR image using gradient interpolation (c) the HR image using the traditional POCS method (d) the HR image using the improved POCS method.

## V. CONCLUSIONS

This paper proposes an effective approach to comprehensively improve the image quality. First, this paper uses gradient interpolation to obtain the initial reference frame closer to the original high resolution image. Then this paper adopts the improved four-parameter method based on Keren sub-pixel image registration which improves the registration accuracy. Finally, the paper adaptively modifies the PSF based on the idea of bilateral filter in the reconstruction process and achieves the POCS image reconstruction. The experimental results proved that the improved method in this paper not only reduce jagged edges and ringing effect, but also effectively enhances the edge and greatly improves the quality of the reconstructed high resolution image.

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