

A Method of Enhancing the Image Edges in Super Resolution Reconstruction Based on Gradient

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Abstract—This paper theoretically analyzes the reasons that caused blurred edges in the traditional POCS (projection onto the convex sets) super-resolution reconstruction. In order to improve the quality of the reconstructed image, this paper analyzes and improves the algorithm at different process. First, the paper proposes to obtain the initial reference frame with a gradient interpolation algorithm in replacement of the traditional linear interpolation. Secondly, the paper uses a motion estimation method called pre-filter iterative gradient to achieve image registration. Finally, the paper selects the POCS algorithm for reconstruction. In order to get better edge information, we adaptively modify the PSF (point spread function) by combination of the edge neighborhood pixel gray information. The experimental results show that the proposed algorithm can significantly improve image quality, maintains the rich edge information and has a higher PSNR and better visual effect.

Keywords- *super resolution; POCS; gradient interpolation; pre-filter iterative registration; edge enhancement*

I. INTRODUCTION

Currently, super-resolution image has been more and more widely applied. Super-resolution reconstruction has been proposed due to the physical equipment and cost constraints. Its purpose is to compensate for the lack of hardware and to improve image resolution and enhance the image availability from the perspective of the software [1]. Super-resolution image reconstruction technique is to restore a high-resolution and high-quality image from multitudinous low-resolution observations degraded by warping, blurring, noise. Currently this technology has been widely used in astronomy, remote

sensing, military surveillance and medical diagnosis.

Stark and Oskoui [2] first applied the POCS (Project onto Convex Sets) to super resolution image reconstruction. Patti [3] proposed the image acquisition model considering to a variety of degraded factors. Eren [4] extended Patti method to multiple moving targets scene. In recent years, Papa [5] proposed the POCS algorithm based on particle swarm optimization. Ogawa [6] proposed the POCS algorithm based on principal component analysis (PCA). Gho [7] proposed applied POCS algorithm for the reconstruction of magnetic resonance images through low-pass and high pass filter.

POCS [8] super resolution reconstruction is proposed based on set theory. In this method, because the image correction process is based on PSF (point spread function), the reconstructed image has jagged edges and ringing and not high image quality. So this article proposes some improvements. First, we use gradient interpolation algorithm to obtain the initial value of super-resolution image, then adopt a four-parameter pre-filtering iterative registration for motion estimation to improve registration accuracy, finally modify the PSF to better enhance the reconstructed image edges.

This paper is structured as follows. It introduces POCS method in Section II and proposes three improvements in Section III. The effectiveness of the proposed method is proved by the experiment results in Section IV. Finally, the conclusion is presented in Section V.

II. THE TRADITIONAL POCS ALGORITHM

A. The Acquisition Model of Low Resolution Images

In the process of digital image acquisition, the images are deteriorated due to the presence of noise which is generally considered as additive noise. Therefore, the image observation model [9] is as follows

$$g_k = D_k M_k W_k f + n_k \quad 1 \leq k \leq p. \quad (1)$$

Where, f is of size for $M \times N$ HR (high resolution) image, g_k is the k th frame LR (low resolution) image of size for $M/q \times N/q$ (q is the down sampling parameter). W_k is the geometric transformation matrix. M_k is the optical fuzzy matrix. D_k is the decimation matrix. n_k is the noise vector. p is the number of LR observation frames.

Image super-resolution reconstruction is a reverse process based on the model which rebuilds HR f by g_1, g_2, \dots, g_p .

B. The Process of POCS Algorithm Reconstruction

POCS algorithm first selects one of the LR images as the reference image, then enlarge it by bilinear interpolation to as the initial estimate of POCS method. Other LR images are based on the reference frame for image registration and use the motion vector to compensate reference frame. Finally, the image is projected onto the convex and continuously iterates until producing the ideal HR image.

According to prior information (such as positive definite, the energy-bounded, data consistency, smooth, etc), we can define multiple convex sets. Convex set is defined as follows

$$C_k(x, y) = \{\hat{f}(x, y) : |r_k(x_1, y_1) \leq \delta_k|\}. \quad (2)$$

Here, $\hat{f}(x, y)$ is the estimated high resolution image. $h_k(x_1, y_1; x, y)$ is the PSF of the observe frame at the pixel (m, n) .

$$r_k(x_1, y_1) = g_k(x_1, y_1) - \sum_{x=0}^M \sum_{y=0}^N \hat{f}(x, y) h_k(x_1, y_1; x, y) \quad (3)$$

$$k = 1, 2, 3, \dots, p$$

In the POCS method, there are m kinds of prior information, There are correspondingly m closed convex sets for C_i ,

$$i = 1, 2, \dots, m. \text{ The HR image } f \in C_0 = \bigcap_{i=1}^{i=m} C_i P_i.$$

Here, C_0 is a non-empty closed convex set. For the given constraint set, the corresponding projection operator is P_i , then the iterative sequence is

$$f_{n+1} = T_m T_{m-1} \dots T_1 f_n. \quad n = 1, 2, \dots, p \quad (4)$$

Where $T = (1 - \lambda)I + \lambda P$, $0 < \lambda < 2$ which is relaxation projection parameters. We can adjust the convergence rate of POCS method by adjusting the relaxation parameter λ . In this paper, we set λ for 0.1. As a result of relaxed projection operator, the convergence stability of POCS method is improved.

III. THE IMPROVED POCS ALGORITHM

This paper firstly uses gradient interpolation algorithm to obtain super-resolution the initial value of super-resolution image, then adopts a four-parameter pre-filtering iterative registration for motion estimation to improve registration accuracy. Finally in order to enhance the edge of the reconstructed image the paper uses the Canny operator to detect edge. When the template center is on the edge, we combine with neighboring pixel gray information to change PSF so to improve image quality.

A. Gradient Interpolation

In the POCS algorithm, the initial estimate for SR image has a great influence on the feasibility of the algorithm. Each SR image estimate is projected to the nearest point of the next set from it which means that the final solution is similar to the initial estimate in a way [10]. The traditional POCS method uses bilinear interpolation and does not consider the edge and smooth region that is the main reason causing blurred edge. In this paper, we adopt gradient interpolation algorithm to obtain the initial estimate and enhance the edges. Interpolation algorithm based on gradient [11] analyzes gray value changes of the edge of the pixel gray region and uses the gradient information of adjacent pixels so that the interpolated points can minimize the impact of the edge gray value and make edge more obvious.

The interpolation thought is as follows: Assuming that $g_{11}, g_{12}, g_{21}, g_{22}$ is corresponding to the four adjacent pixels in the LR image (Fig. 1), $f_{ij} (1 \leq i \leq 4)$ corresponds to the pixel blocks in the interpolated SR image. In gradient interpolation algorithm, first each point in LR image corresponds to each point in SR image such as $f_{11} = g_{11}, f_{13} = g_{12}, f_{31} = g_{21}, f_{33} = g_{22}$ (Fig. 2), then we use interpolation method to calculate f_{12}, f_{21}, f_{22} .

g_{11}	g_{12}	
g_{21}	g_{22}	

Fig. 1 The sub-image of the LR image

f_{11}	f_{12}	f_{13}
f_{21}	f_{22}	
f_{31}		f_{33}

Fig. 2 The sub-image of interpolated HR image

Let we first calculate the values of f_{12} , f_{21} . Because the edge reflects the relatively large areas of the gradient, we require to calculate the absolute value of the gradient for g_{11}, g_{12} and g_{21} , g_{22} . As follows

$$\begin{cases} grad_1 = |g_{12} - g_{11}| \\ grad_2 = |g_{21} - g_{11}| \end{cases} \quad (5)$$

To ensure the image edge, the interpolated points are as close as possible to small gray value pixels. The horizontal interpolation mainly uses the horizontal gradient information while the vertical interpolation mainly uses the vertical gradient information.

$$\begin{cases} f_{12} = \min(g_{11}, g_{12}) + grad_1 \times r \\ f_{21} = \min(g_{11}, g_{21}) + grad_2 \times r \end{cases} \quad (6)$$

Where, r is the interpolation parameter. Similarly, we calculate the gradient in the diagonal direction of 45° and 135° for the pixel f_{22} . Experimental results show that this method has significant improvement on maintaining more clear edges than the bilinear interpolation.

B. The Pre-filtering Iterative Registration of the Sub-pixels

After obtaining the initial estimate by the above gradient interpolation, we need to find the motion vector between the observation frame and the reference frame and add it to the current estimate of the HR image. The accuracy of motion vector estimation seriously affects the quality of reconstructed image. Firstly, the image preprocessing through low-pass filter

can effectively inhibit the effect of noise, then use sub-pixel iterative method for image registration. The traditional Keren [12] registration adopts three-parameter method for registration based on small-angle Taylor series expansion. This paper uses four-parameter method to improve it which can avoid the caused angle error by the Taylor series expansion.

Define two images for $g(x', y')$ and $f(x, y)$, there are the following relations in mathematics.

$$g(x', y') = T(f(x, y)). \quad (7)$$

T represents the geometric transformation. This paper adopts a simplified four parameters affine model.

$$x' = x + k_1x + k_2y + k_3 \quad (8)$$

$$y' = y + k_1y - k_2x + k_4 \quad (9)$$

Turn it into matrix form

$$M = C^{-1}V. \quad (10)$$

Here, M is the motion parameter matrix. C and V are the gradient matrixes.

$$M = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} V = \begin{bmatrix} \sum R_1(g-f) \\ \sum R(g-f) \\ \sum \frac{\partial f}{\partial x}(g-f) \\ \sum \frac{\partial f}{\partial y}(g-f) \end{bmatrix}$$

$$C = \begin{bmatrix} \sum R_1^2 & \sum RR_1 & \sum R_1 \frac{\partial f}{\partial x} & \sum R_1 \frac{\partial f}{\partial y} \\ \sum RR_1 & \sum R^2 & \sum R \frac{\partial f}{\partial x} & \sum R \frac{\partial f}{\partial y} \\ \sum R_1 \frac{\partial f}{\partial x} & \sum R \frac{\partial f}{\partial x} & \sum \left(\frac{\partial f}{\partial x}\right)^2 & \sum \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \\ \sum R_1 \frac{\partial f}{\partial y} & \sum R \frac{\partial f}{\partial y} & \sum \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} & \sum \left(\frac{\partial f}{\partial y}\right)^2 \end{bmatrix}$$

$$R = y \frac{\partial f}{\partial x} - x \frac{\partial f}{\partial y} \quad R_1 = x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$$

In order to improve registration accuracy and get good result no matter how much rotation angle is, this paper adopts the following formula to gradually approach the true value by iterating (K is the number of iteration).

$$M_{k+1} = C_k^{-1}V_k + M_k \quad (11)$$

In order to increase the computational efficiency and robustness to noise, this paper adopts the three-layer Gaussian pyramid with the resolution from coarse to fine.

C. Edge Adaptive Constraints

Using the above gradient interpolation improves the edge quality of the initial estimate of HR image to some extent, while it is not very good and there are still jagged edges and

ringing. This is because the PSF $h(x, y)$ in the projection just uses the neighboring pixel spatial information, while ignoring the neighboring pixel gray value. Therefore, this paper proposes the edge adaptive constraints to modify PSF based on the idea of bilateral filtering [13]. It simultaneously uses the neighboring pixel spatial information and gray information which effectively maintains the edge of the image when eliminating noise. The weighting coefficient $w(i, j)$ of the modified PSF consists of two parts as follows

$$w(i, j) = w_s(i, j) \cdot w_r(i, j). \quad (12)$$

Where, $w_s(i, j)$ is the space similarity factor, $w_r(i, j)$ is the gray similarity factor. Their definitions are

$$w_s(i, j) = e^{-\frac{|i-x|^2 + |j-y|^2}{2\sigma_s^2}}, \quad (13)$$

$$w_r(i, j) = e^{-\frac{|f(i, j) - f(x, y)|^2}{2\sigma_r^2}}. \quad (14)$$

Where, σ_s , σ_r are the standard deviations. Generally set $\sigma_r = 2\delta_n$. δ_n is the noise standard deviation. After the above changes, the fuzzy edge in the rebuilding process is ultimately inhibited and the quality of reconstructed SR image is also significantly improved.

IV. EXPERIMENTAL ANALYSIS AND RESULT

Based on the above theoretical analysis, this paper uses two sets of simulation experiments to verify the improved POCS reconstruction algorithm in matlab7.0 platform. The reconstructed SR image is four times of the LR image and global motion among the LR images is translation and rotation. Set the standard deviation of the PSF in the experiment $\sigma_s = 1$, the size of support domain is 5×5 , $\delta_n = 0.01$, $r = 0.3$. The number of iterations is 10.

In Fig. 3, we use eight degraded LR images sequence (size for 249×239) called Table to obtain a reconstructed SR image (size for 498×478). The image in (a) is the HR image using bilinear interpolation. The image in (b) is the HR image using gradient interpolation. The images in (c) and (d) are respectively the HR image using the traditional POCS method and the improved POCS method. Obviously the image in (b) is much clearer than image in (a). Although image in (c) remains a lot of details, there are still some fuzzy edges. The image in (d) obtained by the improved algorithm in this paper is not only much more clear but also maintains many more

perfect edges. The edge ringing effect of the table items almost disappeared. So it greatly improves the quality of the reconstructed image.

In Fig. 4, we use eight degraded LR images sequence (size for 128×128) called Lena to obtain a reconstructed SR image (size for 256×256). The image in (a) is the HR image using bilinear interpolation. The image in (b) is the HR image using gradient interpolation. The images in (c) and (d) are respectively the HR image using the traditional POCS method and the improved POCS method. The figure shows that image in (b) is much clearer than image in (a). Image in (c) remains a lot of details while there are ringing and jagged edges. The image in (d) obtained by the improved algorithm in this paper is not only much more clear but also maintained more effective edge (especially the edge of the brim). The quality of reconstructed image is significantly improved.

This paper uses PSNR (peak signal to noise ratio) to quantitatively evaluate the quality of the obtained images in the experiments. The PSNR between the initial high resolution image $f(m, n)$ and the reconstructed high resolution image is defined as

$$\text{PSNR} = 10 \cdot \log_{10} \frac{255^2 \times M \times N}{\sum_{m=1}^M \sum_{n=1}^N \left(f(m, n) - \hat{f}(m, n) \right)^2}. \quad (15)$$

Here, M and N are respectively the length and width of the high resolution image.

The comparison result of PSNR is shown in Table I. The table shows the improved POCS super-resolution reconstruction algorithm in this paper is superior to the traditional POCS algorithm.

TABLE I. THE COMPARISON OF PSNR (db)

	Table	Lena
bilinear interpolation	24.880	21.649
gradient interpolation	25.433	22.757
the traditional POCS	25.781	22.788
the improved POCS	26.716	23.514

V. CONCLUSIONS

This paper proposes an effective method to comprehensively improve the image quality. First, this paper uses gradient interpolation to obtain the initial reference frame

closer to the original high resolution image. Then this paper adopts the improved four-parameter method based on Keren sub-pixel image registration which improves the registration accuracy. Finally, the paper adaptively modifies the PSF based on the idea of bilateral filter in the reconstruction process and completes the POCS image reconstruction. The experimental results proved that the improved method in this paper not only reduce jagged edges and ringing effect, but also effectively enhances the edge and greatly improves the quality of the reconstructed high resolution image.

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REFERENCES

- [1] G. K. Moon and S. Chaudhuri, Super-resolution image reconstruction, IEEE Signal Processing Magazine, 3rd ed. vol. 20, 2003, pp.19-20.
- [2] S. Harris, P. Oskoui, High-resolution image recovery from image plane arrays using convex projections, Journal of the Optical Society of America A. 11th ed, vol. 6, 1989, pp. 1715-1726.
- [3] J. Patti, M. I. Sezan, A. M. Tekalp, Super-resolution Video Re-construction with Arbitrary Sampling Lattices and Nonzero Aperture Time, IEEE Transactions on Image Processing. 6th ed, vol. 8, 1997, pp. 1064-1076.
- [4] P. E. Eren, M. I. Sezan and A. M. Tekalp, Robust, Object-based High-resolution Image Reconstruction from Low-resolution Video, IEEE Transactions on Image Processing, 6th ed, vol.10, 1997, pp. 1446-1451.
- [5] J. P. Papa, L. M. G. Fonseca, L. A. S. de Carvalho. Projections Onto Convex Sets through Particle Swarm Optimization and its application for remote sensing image restoration. Pattern Recognition Letters. 13th ed, vol. 31, 2010, pp. 1876-1886.
- [6] T. Ogawa, M. Haseyama, Missing Intensity Interpolation Using a Kernel PCA-Based POCS Algorithm and its Applications, IEEE Transactions on Image Processing, 2nd ed, vol. 20, 2011, pp. 417-432.
- [7] S. Gho, C. Liu, D. Kim, "Application of Low-pass & High-pass reconstruction for improving the performance of the POCS based algorithm," Circuits and Systems (MWSCAS), 2011 IEEE 54th International Midwest Symposium on. Korea, pp. 1-3, August 2011.
- [8] P. Oskoui, H. Stark, Tomographic Image Reconstruction Using The Theory of Convex Projection, IEEE Trans. Medical Imaging Processing, 1st ed, vol. 7, 1998, pp. 45-48.
- [9] T. Akgun, Y. Altunbasak, R. M. Mersereau. Super resolution reconstruction of hyperspectral images, IEEE Translation on Image Processing, 11th ed, vol.14, 2005, pp. 1860-1875.
- [10] F. Qin, L. Zhong, Z. Lihong, "Research on the influence of initial value estimation on image super resolution reconstruction," 2010 2nd International Conference on Information Science and Engineering. China, pp. 1-4, Dec 2010.
- [11] J. X. Xiao, S. L. Wang, S. H. Li, The Edge-Preserving-based POCS Super Resolution Image Reconstruction, Communications Technologies, vol.1, 2009, pp.66-68.
- [12] F. Chong, Z. Jianjun, G. Jianya, "POCS Super Resolution Sequence Image Reconstruction Based on Improvement Approach of Keren Registration Method," Sixth International Conference on Intelligent Systems Design and Applications, ISDA '06. China, vol. 2, pp. 333-337, October 2006.
- [13] D. Li, Y. C. Lim, Y. Lian, A polynomial-time algorithm for designing FIR filters with power-of-two coefficients, IEEE Transactions on Signal Processing, 8th ed, vol.50, 2002, pp.1935-1941.



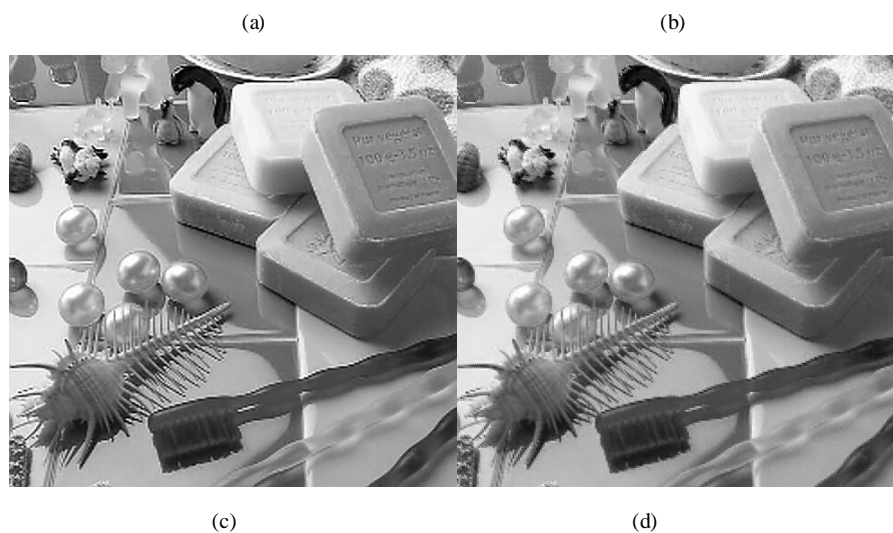


Fig. 3 Image called Table (a) HR image using bilinear interpolation (b) HR image using gradient interpolation (c) HR image using the traditional POCS method (d) HR image using the paper improved POCS method



Fig. 4 Image called Lena (a) HR image using bilinear interpolation (b) HR image using gradient interpolation (c) HR image using the traditional POCS method (d) HR image using the paper improved POCS method.