Compressed sensing for magnetic resonance imaging

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***Abstract*—The application of compressed sensing (CS) in accelerating magnetic resonance imaging (MRI) is discussed. The sparsity inherent in typical MR images can be exploited in order to replace regular full sampling of measurement space with sparse sampling; CS reconstruction methods have proved to be useful in experiments by a number of groups. A recently proposed method to enhance image sparsity by data ordering is described and results for two new algorithms, PECS and SENSECS, which exploit the data ordering enhancement, are presented.**

***Index Terms*—magnetic resonance imaging; compressed sensing;**

I. Introduction

The significant time necessary to record each resonance echo from the volume being imaged in magnetic resonance imaging (MRI) has lead to much effort to develop methods which take fewer measurements. Faster methods mean less time for the patient in the scanner, increased efficiency in the use of expensive scanning facilities, improved temporal resolution in studies involving moving organs or flows, and less artifacts from patient motion. Images like those of the human body possess the property of *sparsity*, i.e., the property that in some transform space they can be represented much more compactly than in image space. The technique of compressed sensing, which aims to exploit sparsity, has therefore been adapted for use in MRI. This, coupled with the use of multiple receiving coils (parallel MRI) and the use of various forms of prior knowledge (e.g., support constraints in space and time), has resulted in significantly faster image acquisitions with only a modest penalty in the computational effort required for reconstruction. We briefly introduce the technique of compressed sensing and its use in MRI. We then present the use of data ordering as a method of further exploiting sparsity and show some preliminary results.

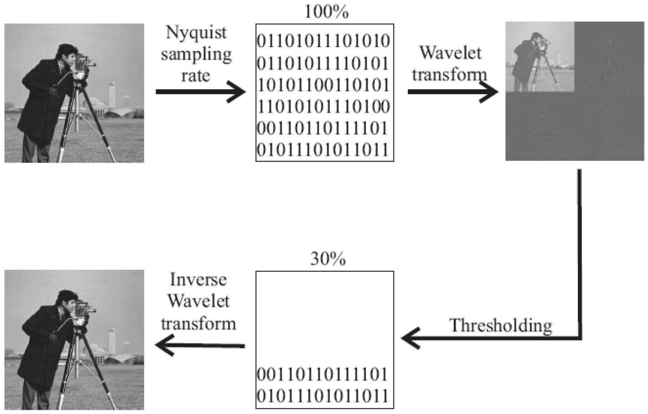
II. Compressed sensing

The conventional wisdom in signal processing is that the sampling rate for any signal must be at twice the maximum frequency present in the signal. However in the work performed in recent years related to signal compression, it has become obvious that the total amount of information which is needed to represent a signal or image to high accuracy is in many cases much less than that implied by the sampling theorem. This is nowhere more apparent than in the modern digital camera where quite acceptable images can be stored and recreated from a small fraction of the data volume that was associated with the original image sampling, a direct consequence of the property of sparsity. The technique of compressed sensing (also known as 'compressive sensing' and henceforth abbreviated to 'CS') was introduced to exploit image sparsity [1, 2]. Consider a 2D image with N pixels represented by the vector $\mathbf{x}$ and suppose that it can be accurately represented by $K << N$ data values under the linear transformation $\mathbf{y} = \Phi\mathbf{x}$. Rather than to measure the N pixel values and then to perform the transformation, we seek to make just M measurements $\mathbf{m}$, where $K \leq M << N$, and estimate the transformed version of the image. Thus $\mathbf{m} = \Psi\mathbf{y}$, where Ψ is a measurement matrix of dimension $M \times K$. While this might be of little direct benefit in the case cited above of a modern digital camera, for which the design of the sensor is most straightforwardly implemented as a regular 2D array of individual pixel detectors, there are many other applications, notably including MRI, for which making fewer measurements does offer an advantage.

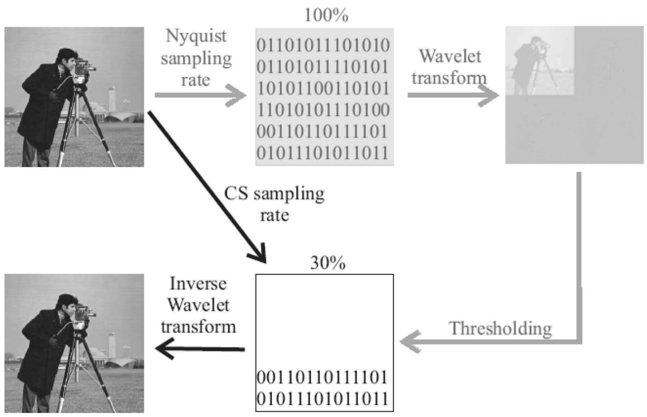
An illustration of how CS might work for the general optical imaging case is shown in Fig. 1. Suppose that the well known cameraman image is being acquired with a conventional digital camera in Fig. 1(a) and a full set of pixels are being recorded and stored. Applying a linear transform, such as the discrete wavelet transform (DWT), to the image allows many of the DWT coefficients to be set to zero, resulting in a compressed form of storage. The compressed data set can be used to reconstruct a good likeness of the original image. The alternative CS approach is illustrated in Fig. 1(b): by some process, many fewer samples are made of the original scene and the non-zero coefficients of the compressed image are directly estimated. Thus the waste associated with full data measurement followed by compression is avoided in CS.

III. Applying compressed sensing to MRI

MRI represents the entire set of methods which apply the principles first developed for chemistry as nuclear magnetic resonance in such a way that the spatial variation of a property within an object is observed. Because of the relationship between the resonance frequency of certain atoms and magnetic field strength, virtually all MRI method make measurements in spatial frequency space, or 'k-space' as the MRI community



(a)



(b)

Fig. 1. Comparing (a) conventional image sensing and compression to (b) compressed sensing. In (a) the image is sampled at the Nyquist sampling rate and stored. Then all of the smallest coefficients in the image's wavelet transform are discarded to reduce the storage volume. In (b) the significant coefficients of the wavelet transform are directly estimated from a lesser number of samples of the image.

refers to it. The use of MRI is restricted in one specific way: the physical processes involved in the excitation and reception of MR signals are inherently quite slow. Thus the time taken to invoke a specific sequence and to measure the signals that are generated is measured in a matter of milliseconds per k-space sample. Note that this has nothing to do with the electronics associated with the scanner - faster hardware does not solve the problem. To take a complete set of measurements for, say, a 3-D imaging study of the brain may take many minutes; some acquisition sequences require periods approaching one hour. A key method to speed up acquisition has been the introduction of multiple sensing coils and corresponding algorithms to exploit the extra information, such as SENSE, for performing what is now known as 'parallel MRI'[3]. Further strenuous efforts are being made to achieve further acceleration through signal processing, including with CS.

The formal introduction of CS into MRI methods was made by Lustig, Donoho and Pauly in 2007 [4]. Their key contribution is the explicit use of a different transform domain for appropriate application of the $\mathcal{L}_1$ norm. The authors identified the use of discrete wavelet transform (DWT) and discrete cosine transform (DCT) as suitable transform bases for application in MR images, as evidenced by their sparse representation under DWT and DCT. A reconstruction framework was given, which converts the CS formulation into a convex optimization problem and hence allows for computational efficiency [4]. The authors also spelt out that a key requirement in data measurement for successful compressed sensing recovery is to achieve incoherent aliasing. In MRI, such a requirement can be satisfied by employing a pseudo-random data acquisition pattern on Cartesian k-space grid, but it has also been extended to the use of non-Cartesian sampling trajectories [5, 6].

IV. Increased sparsity by data ordering

A quite distinctly different approach to increasing sparsity has recently been proposed. In 2008 Adluru and DiBella [7] and Wu et al [8] independently proposed performing a sorting operation on the signal or image as part of the reconstruction process. The principle is presented in Fig. 2 for a 2D axial brain image. In Fig. 2(a) the situation is shown whereby the image is transformed by the 2D DCT and then a compression occurs by setting all coefficients less than a given threshold to zero. The resulting reconstruction is similar to the original, but noticeably smoother due to the loss of some small amplitude high frequency components. In Fig. 2(b), the image pixels are sorted from largest amplitude in the lower right to highest amplitude in the upper left to make the resulting function monotonic and the mapping required to do this is retained (denoted 'R'). The same transformation and recovery operation after thresholding as in (a) is performed and a re-sorting (denoted 'R^{-1}') is performed. Because the compression retains much of the shape of the image after sorting, the result has much higher fidelity than in Fig. 2(a). We argue that many fewer coefficients need to be retained in the DCT of the sorted image than in the original, hence the more successful reconstruction.

Clearly the process depicted in Fig. 2 requires knowledge of the original signal in order to derive R. The practical utility of what has been demonstrated is likely therefore to be questioned. However we show in the next section that several methods to derive an approximate R are possible and that they lead to useful and practical algorithms for MR image recovery.

V. New algorithms

Our work in applying sparse sampling in MRI has lead to the development of two new algorithms: PECS (prior estimate-based compressed sensing) and SENSECS (sensitivity encoding with compressed sensing). PECS has been demonstrated in both brain imaging, i.e., imaging of a static structure, and in contrast-enhanced angiography, i.e., dynamic imaging as part of a pilot study on normal volunteers [9, 10]. SENSECS has been demonstrated in brain imaging [11]. We briefly summarize these methods and present preliminary results. The images presented are all for 2-D 'slices'; the accelerations achieved are in this case gained by under-sampling in one direction only (the phase encoding direction). The 'acceleration factor' is the inverse of the fraction of the total k-space samples which are

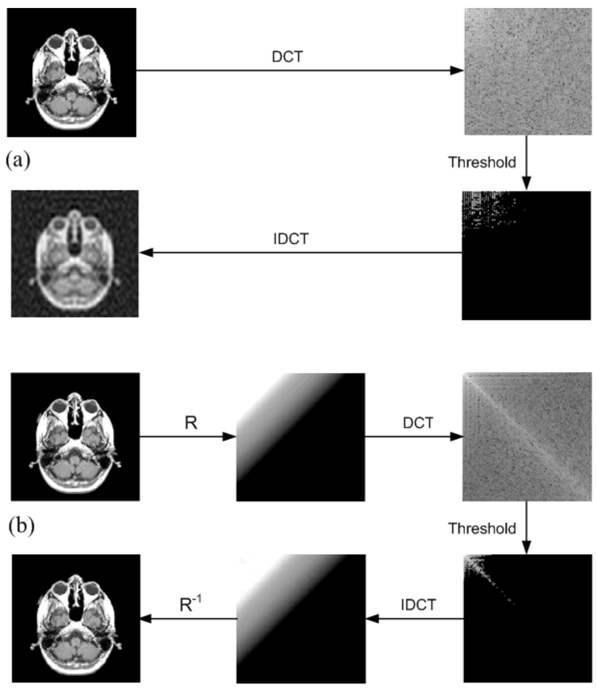


Fig. 2. Illustration of how a data ordering can achieve a higher sparsity for a 2D image. In (a) the signal is compressed by retaining only those DCT coefficients with amplitudes higher than a threshold. In (b) the image pixels are sorted to generate a monotonic function and then the same recovery operation is performed before a final resorting. Because the sorted data in (b) is more sparse, the recovery is of higher quality.

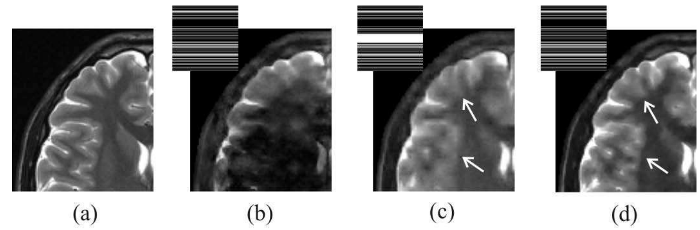


Fig. 3. Reconstructions comparing PECS with other methods: (a) top left quadrant of a reconstruction of an axial brain slice with full k-space sampling; (b) to (d) reconstructions at an acceleration factor of 4 (the sampling patterns used are shown in the insets). (b) CS with uniformly distributed randomly selected k-space samples; (c) CS with randomly selected k-space samples, but including the centre 32 lines; and (d) PECS with ordering derived from a low resolution approximation using just the centre 32 lines of k-space. The arrows indicate areas where (d) shows better recovery than (c).

measured. The methods have also been successfully applied in 3-D imaging.

A. PECS

The success of CS is determined by the sparsity of the underlying signal. In our experience to date, the sparsest representation of a typical MRI anatomical image is obtained by ordering the set of pixel (or voxel) amplitudes as described in Section IV. Assume that a set of under-sampled k-space data have been collected with a particular MRI sequence with the purpose of forming a high resolution image. In addition, a prior estimate is available of the image, for example a low resolution image. In PECS the prior estimate of the image is first used to derive a data ordering, R; CS is then used to recover an image from the measured k-space data, incorporating the approximate ordering (to promote sparsity) and a total variation (TV) minimization (to promote piecewise smoothness in the resulting image) [11, 12]. In this case the approximate ordering, R, is derived from a low resolution (low pass filtered) version of the signal. When R is applied to the high resolution data, the result is a highly noisy signal which only approximates in form to a monotonic function. Under the transform the largest coefficients retain the form, while the errors tend to generate a noise-like spectrum with low amplitudes spread across many coefficients. After thresholding, only the significant coefficients are non-zero and these retain the form of the true ordering. We argue that prior knowledge about the signal is thereby introduced by the application of the approximate data ordering [9].

A result for PECS is shown in Fig. 3. A 1.5T GE scanner equipped with an 8-channel head coil was used to obtain a 2D T2-weighted axial brain slice of a healthy adult volunteer. A fully sampled k-space data set (256×256) was obtained and then various forms of sampling patterns were applied in post processing to simulate the under-sampling required [11]. Note that in this case the under-sampling is applied in the single phase encoding direction (anterior-posterior). In Fig. 3(a) is shown the reconstruction for the slice utilizing the fully sampled k-space data; the top left quadrant is selected for more detailed study. To the right are three different reconstructions obtained from only one quarter of the k-space data, simulating an acceleration factor of 4 for the imaging process. In Fig. 3(b) the reconstruction is by CS with a uniform sampling density, while in Fig. 3(c) an otherwise similar sampling pattern is altered to make the centre 32 lines of k-space fully sampled. The improvement in the reconstruction in (c) compared to that in (b) is obvious. A PECS reconstruction is shown in Fig. 3(d), with the prior estimate used to generate R being a low resolution image formed from the centre 32 k-space lines. The arrows indicate particular areas where PECS in (d) has performed better than CS in (c).

B. SENSECS

As the name implies, SENSECS combines the well-known SENSE algorithm [3] with CS. A regular sampling pattern is employed which promotes the performance of SENSE, except that several additional lines are sampled at the centre of k-space. SENSE is first applied to achieve an intermediate reconstruction [3, 11]. Because of the high acceleration factor being used, the image is likely to be quite noisy. The noise is in part at least due to the imperfect knowledge available of the sensitivities of the individual receiver coils. An approximate sorting order R is therefore derived from the intermediate SENSE-derived image. Then PECS is performed with the same set of k-space data and using R.

Again a single set of results is shown for the new algorithm in Fig. 4. The same set of data as described above in this section was used and the fully sampled k-space data was again under-sampled in post processing [11]. High acceleration factors of 5.8 and 6.5 were simulated. In Fig. 4(a) is shown the

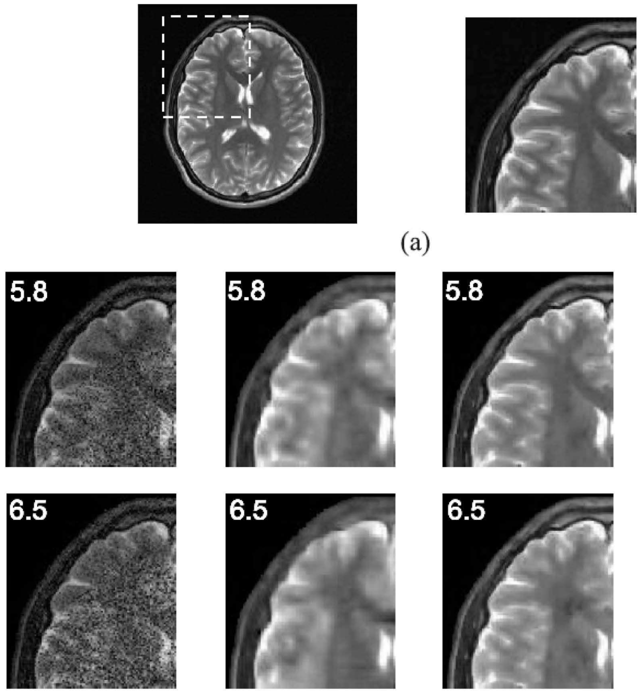


Fig. 4. Reconstructions comparing SENSECS with other methods at high acceleration factors (shown in the top left of the reconstructions): (a) reconstruction of an axial brain slice with full k-space sampling; in the lower section of the figure the left column is SENSE, the centre column is CS, and the right column is SENSECS. Note that the SENSECS reconstructions use the SENSE reconstructions to derive the data sorting order.

reconstruction for the slice utilizing the fully sampled k-space data; the top left quadrant is selected for more detailed study. In each of the two lower rows three different reconstructions are shown, obtained from a fraction of the k-space data, simulating the acceleration factors indicated. In the left column are shown the SENSE reconstructions, which are clearly noisy and unlikely to be diagnostically useful. CS reconstructions are shown in the centre column; in this case the sampling pattern employed was designed specifically for CS. The results are superior to the SENSE reconstructions, but somewhat blurred in appearance. SENSECS reconstructions are shown in the right column; they show better fidelity than the other reconstructions and diagnostically useful results up to at least an acceleration factor of 6.5. Note that the images obtained by SENSE were used to derive the sorting order here.

VI. Discussion

We have presented some preliminary and very encouraging results for incorporating a data ordering step in the recovery of MR images by compressed sensing. There remains considerable scope for putting this non-linear processing on a firm theoretical footing. Cands and others have provided such rigour to the basic CS recovery of certain classes of image [1, 2], but no such attention has to our knowledge been directed at the data ordering and its use in incorporating prior knowledge.

We have demonstrated the exploitation of several forms of sparsity. Briefly, this includes the sparsity achieved by ordering the image into a monotonic function and the use of a compressive transform such as DCT or DWT. Other forms of sparsity may also be exploited. Other authors have for example exploited piecewise homogeneity [12, 13]. The subtraction of the contribution to signal from static structures in dynamic contrast-enhanced MR angiography also shows considerable promise [11, 14, 15]. Given that CS is relatively new as a practical method in signal processing, it seems likely that other transforms may be available, or as yet undiscovered, which may allow more gains to be made. Our work and the work of many others in the area of applying sparse sampling in MRI suggests that it has a bright future and we should see the manufacturers of MRI scanning systems incorporating some of the algorithms based on sparse sampling soon.

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