

Homework 1

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Problem 1: We have to make sure the matrix $I - \alpha A$ is invertible, otherwise the linear system has no solution.

$$\det(I - \alpha A) = 0$$

is equivalent to

$$\det(A - I/\alpha) = 0$$

We see that the largest value of $1/\alpha$ for which the determinant is zero is the the largest eigenvalue of matrix A . In practice α is often set close to the threshold $1/k_1$.

Problem 2: The number of walks of length 1 between V_i and V_j is denoted by A_{ij}

Number of walks of size 2 from V_i to V_j that go through V_k is equivalent to

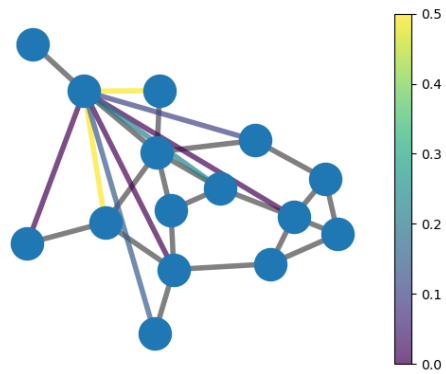
$$N_{ij}^{(2)} = \sum_{k=1}^n A_{ik} A_{kj} = [A^2]_{ij}$$

Problem 3: The idea is to define a function which calculate the Jaccard matrix by the definition, which is the common neighbors divided by their union neighbors.

Thus, we need to convert the graph into its adjacency matrix. From question 2, we can get the common neighbors by multiplying the adjacency matrix.

The union is their total neighbors minus the intersection. The code is shown below.

```
1  """
2  AOMS 88778-83: Deep Learning with Graphs
3  Instructor: Navid Shervani-Tabar
4  Fall 2022
5  University of Notre Dame
6
7  Homework 1: Programming assignment
8  """
9
10 import networkx as nx
11 import matplotlib.pyplot as plt
12 import numpy as np
13 from networkx.algorithms import bipartite
14 from networkx.generators.random_graphs import erdos_renyi_graph
15 import copy
16
17 # --- Initialize graphs
18 seed = 30
19 G = nx.florentine_families_graph()
20 nodes = G.nodes()
21
22 layout = nx.spring_layout(G, seed=seed)
23
24 # --- compute jaccard's similarity
25 """
26 This example is using NetworkX's native implementation to compute similarities.
27 Write a code to compute Jaccard's similarity and replace with this function.
28 """
29
30 def get_jaccard(G):
31     node_list = list(nodes)
32     A = nx.adjacency_matrix(G)
33     A = A.to_numpy_array()
34     A_numer = np.matmul(A,A)
35     A_deno = np.ones_like(A_numer)
36     for i in range(len(A)):
37         for j in range(len(A)):
38             A_deno[i,j] = np.sum(A[i]) + np.sum(A[j]) - A_numer[i,j]
39
40     Jaccard_matrix = A_numer/A_deno
41     item = (node_list[i],node_list[j],Jaccard_matrix[i][j] for i in range(len(A)) for j in range(len(A)))
42     return item
43
44 jred = get_jaccard(G)
45
46
47 # --- keep a copy of edges in the graph
48 old_edges = copy.deepcopy(G.edges())
49
50 # --- add new edges representing similarities.
51 new_edges, metric = [], []
52 for u, v, p in jred:
53     G.add_edge(u, v)
54     print(f"({u}, {v}) -> {p:.8f}")
55     new_edges.append((u, v))
56     metric.append(p)
57
58 # --- plot Florentine Families graph
59 nx.draw_networkx_nodes(G, nodelist=nodes, label=nodes, pos=layout, node_size=600)
60 nx.draw_networkx_edges(G, edgelist=old_edges, pos=layout, edge_color='gray', width=4)
61
62 # --- plot edges representing similarity
63 """
64 This example is randomly plotting similarities between 8 pairs of nodes in the graph.
65 Identify the "Ginori"
66 """
67 ne = nx.draw_networkx_edges(G, edgelist=new_edges[:8], pos=layout, edge_color=np.asarray(metric[:8]), width=4, alpha=0.7)
68 plt.colorbar(ne)
69 plt.axis('off')
70 plt.show()
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The code to access the Ginori family

```

66 # --- plot edges representing similarity
67 ---
68 This example is randomly plotting similarities between 8 pairs of nodes in the graph.
69 Identify the "Ginori"
70 ---
71 for i in range(len(A)):
72     if new_edges[i][0] == "Ginori":
73         break
74
75
76 ne = nx.draw_networkx_edges(G, edgelist=new_edges[i:len(A)], pos=layout, edge_color=mg.asarray(metric[:8]), width=4, alpha=0.7)
77 plt.colorbar(ne)
78 plt.axis('off')
79 plt.show()
80

```

