

1D DCT: 
$$F(u) = \sqrt{\frac{2}{N}} \cdot c(u) \sum_{i=0}^{N-1} \cos \frac{(2i+1) \cdot u \pi}{2N} \cdot f(i)$$

when  $N=4$ ,

$$F(u) = \sqrt{\frac{1}{2}} \cdot c(u) \sum_{i=0}^3 \cos \frac{(2i+1) \cdot u \pi}{8} \cdot f(i)$$

$$c(u) = \begin{cases} \frac{\sqrt{2}}{2} & u=0 \\ 1 & \text{otherwise} \end{cases}$$

If  $T$  is the DCT matrix

$$F(u, v) = T \cdot f(i, j) \cdot T^T$$

$$f(i, j) = T^T \cdot F(u, v) \cdot T$$

when  $N=4$ ,

$$T_4 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{3\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{5\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{7\pi}{8} \\ \sqrt{\frac{1}{2}} \cos \frac{\pi}{4} & \sqrt{\frac{1}{2}} \cos \frac{3\pi}{4} & \sqrt{\frac{1}{2}} \cos \frac{5\pi}{4} & \sqrt{\frac{1}{2}} \cos \frac{7\pi}{4} \\ \sqrt{\frac{1}{2}} \cos \frac{3\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{9\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{15\pi}{8} & \sqrt{\frac{1}{2}} \cos \frac{21\pi}{8} \end{bmatrix}$$

$$= \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix}$$

where  $a = \frac{1}{2}$ ,  
 $b = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8}$ ,  
 $c = \sqrt{\frac{1}{2}} \cos \frac{3\pi}{8}$ .