Probability machines

Risk estimation in a post-parameter world

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Effect size

Let's go back to your first regression course

> ***"How much does the outcome change, on average, when a predictor changes by one unit, all other predictors remaining the same?"***

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> ***"How much does the outcome change, on average, when a predictor changes by one unit, all other predictors remaining the same?"***

This is based on the concept of *counterfactuals*

- What would happen **if** a predictor changed by 1 unit keeping everything else the same?
- This is not something we can observe, but only something we can conceptualize
- Using multiple machines, we can actually estimate these counterfactuals directly **for each observation!!!**
- We don't need parameters in the model; we can do this nonparametrically
- Requires us to think about what exactly are the effects we want to interrogate

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Counterfactuals

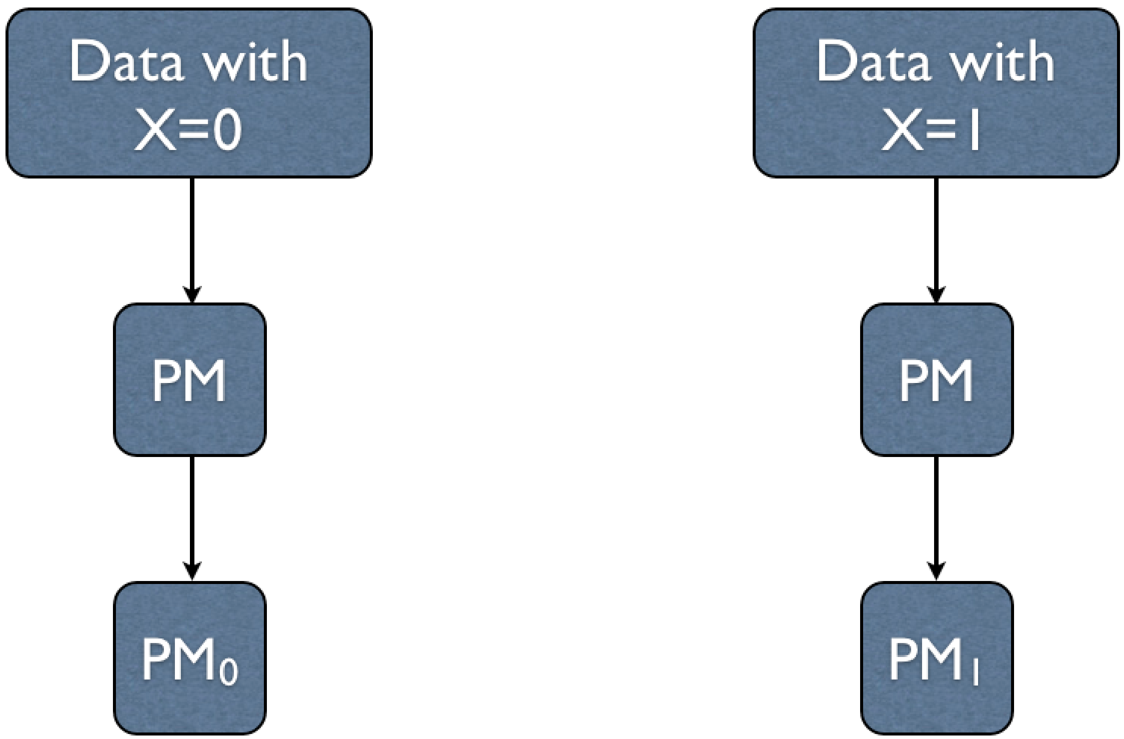
X = 1	X = 0
	

The counterfactual argument is, in essence

If we put an observation in the other landscape, what would it do?

Counterfactual machine

Counterfactual machine



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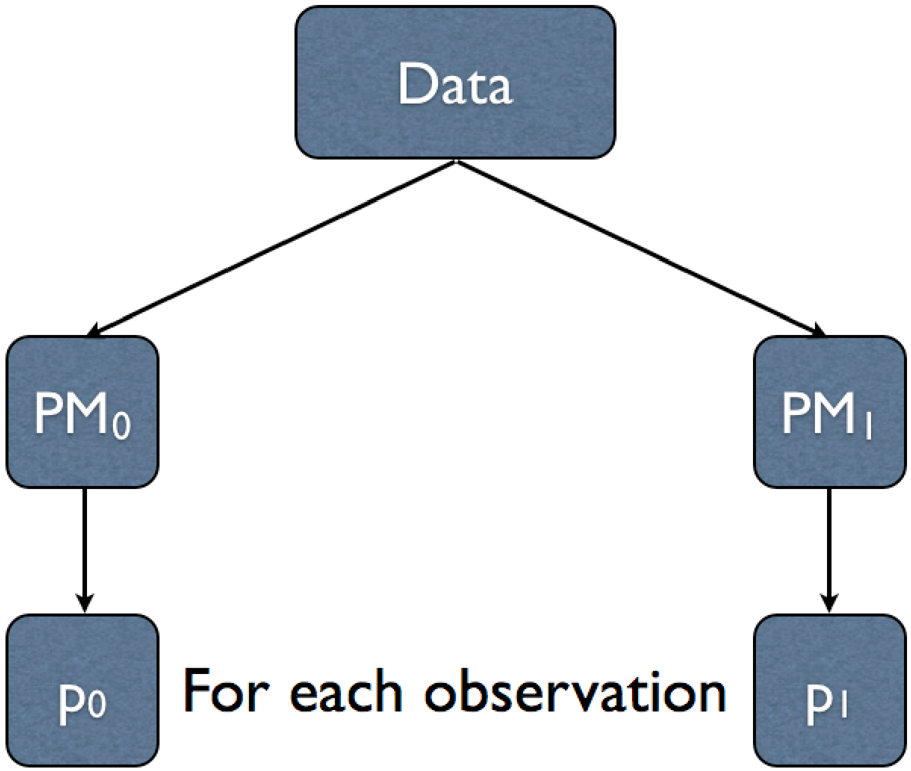
Counterfactual machine



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Risk machine

- PM_1 captures the landscape for X=1
- PM_0 captures the landscape for X=0

Now put each observation in each landscape and record its predicted outcome

Note, for each observation we now have a p_1 and a p_0

Now we can compute *conditional odds ratios* using

$$OR = \frac{p_1(1-p_0)}{(1-p_1)p_0}$$

for each observation, and look at group-specific odds ratios by averaging or taking medians

Simulations

We generate data from a logistic regression model with

- 10 independent binary features
- 3 features associated with outcome to various degrees
- 7 features not associated with outcome (to mimic sparseness)

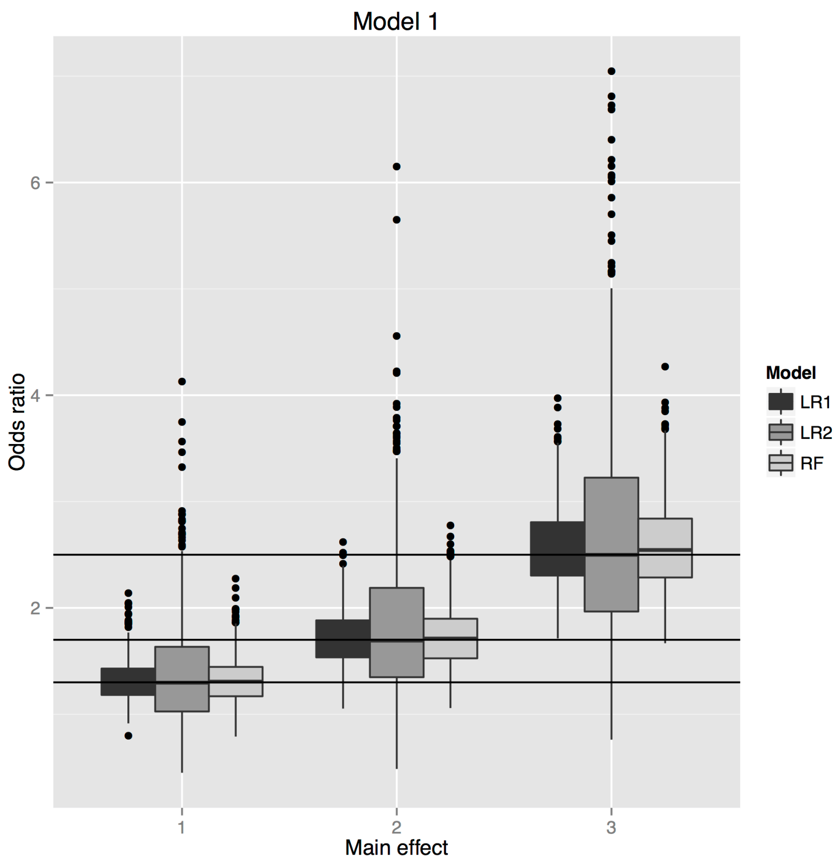
Simulations

We fit three models to the generated data

- Main effects logistic regression
  ```
  glm(y~x1+x2+x3+..., family=binomial)
  ```
- Main effects + two-way interactions logistic regression
  ```
  glm(y~(x1+x2+x3+...)^2, family=binomial)
  ```
- Random forest regression
  ```
  randomForest(y~x1+x2+x3+...)
  ```

For this entire exercise, **we do not change this code**

Simulations



Look at a main effects model

- All 3 are unbiased
- RF does as well as logistic regression for efficiency

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WARNING: Soapbox time

Logistic regression and odds ratios

Logistic regression (the industry standard) gives us odds ratios (the industry standard)

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Please try explaining what an odds ratio is

Logistic regression and odds ratios

Logistic regression (the industry standard) gives us odds ratios (the industry standard)

- The odds ratio is interpreted as a risk ratio
  - Can only do this when the outcome is rare
- The odds ratio might be reasonable for gamblers (Hello, Bernoullis), but probably not for clinicians
- If I had my way, we'd report either risk ratios or risk differences

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Now back to regular programming

Risk machine

We have individual p_1 and p_0, so we can directly compute

- risk differences

$$RD = p_1 - p_0$$

- risk ratios

$$RR = p_1 / p_0$$

Risk machine

What about interactions?

Since we have a way of estimating counterfactuals, estimating conditional interaction effects are straightforward

Make 4 machines to "capture landscapes" when

- X1 = 0, X2 = 0 ($\rightarrow p_{00}$)
- X1 = 1, X2 = 0 ($\rightarrow p_{10}$)
- X1 = 0, X2 = 1 ($\rightarrow p_{01}$)
- X1 = 1, X2 = 1 ($\rightarrow p_{11}$)

Now compute the appropriate contrast ($p_{11} - p_{10} - p_{01} + p_{00}$) or ratio

$$\frac{p_{11}(1-p_{10})}{p_{10}(1-p_{11})} / \frac{p_{01}(1-p_{00})}{p_{00}(1-p_{01})}$$

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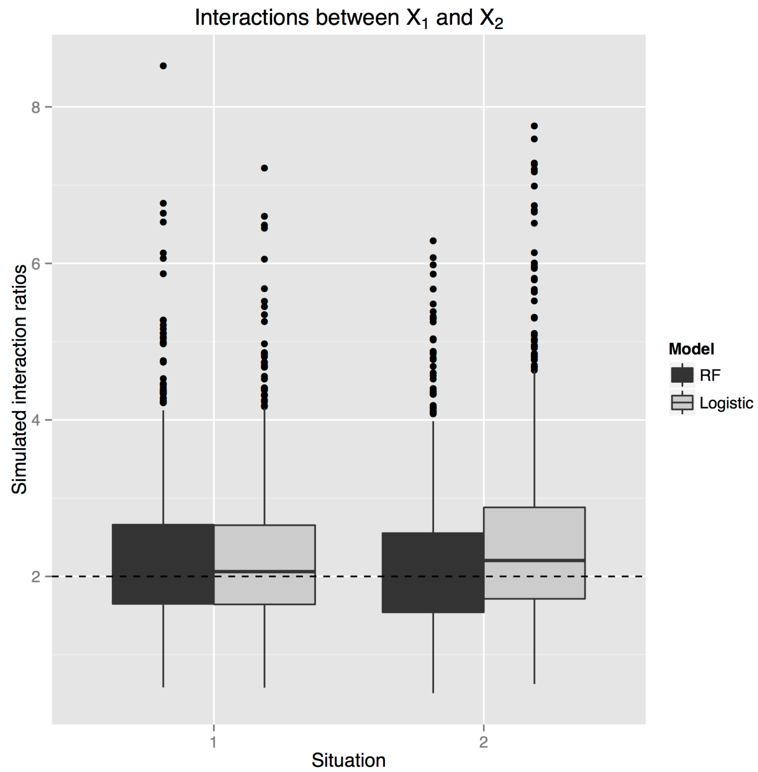
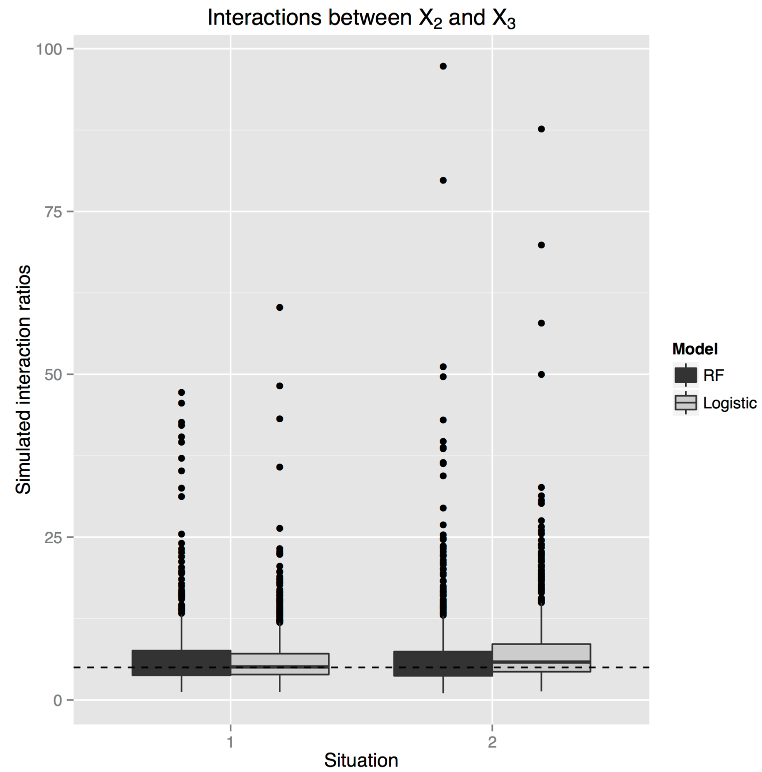
Risk machine





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More about interactions

The Risk Machine^{TM} is cumbersome when you have many features.

We can actually do a faster scan of the data to find 2-way interactions

We call it the **Interactor**^{TM}

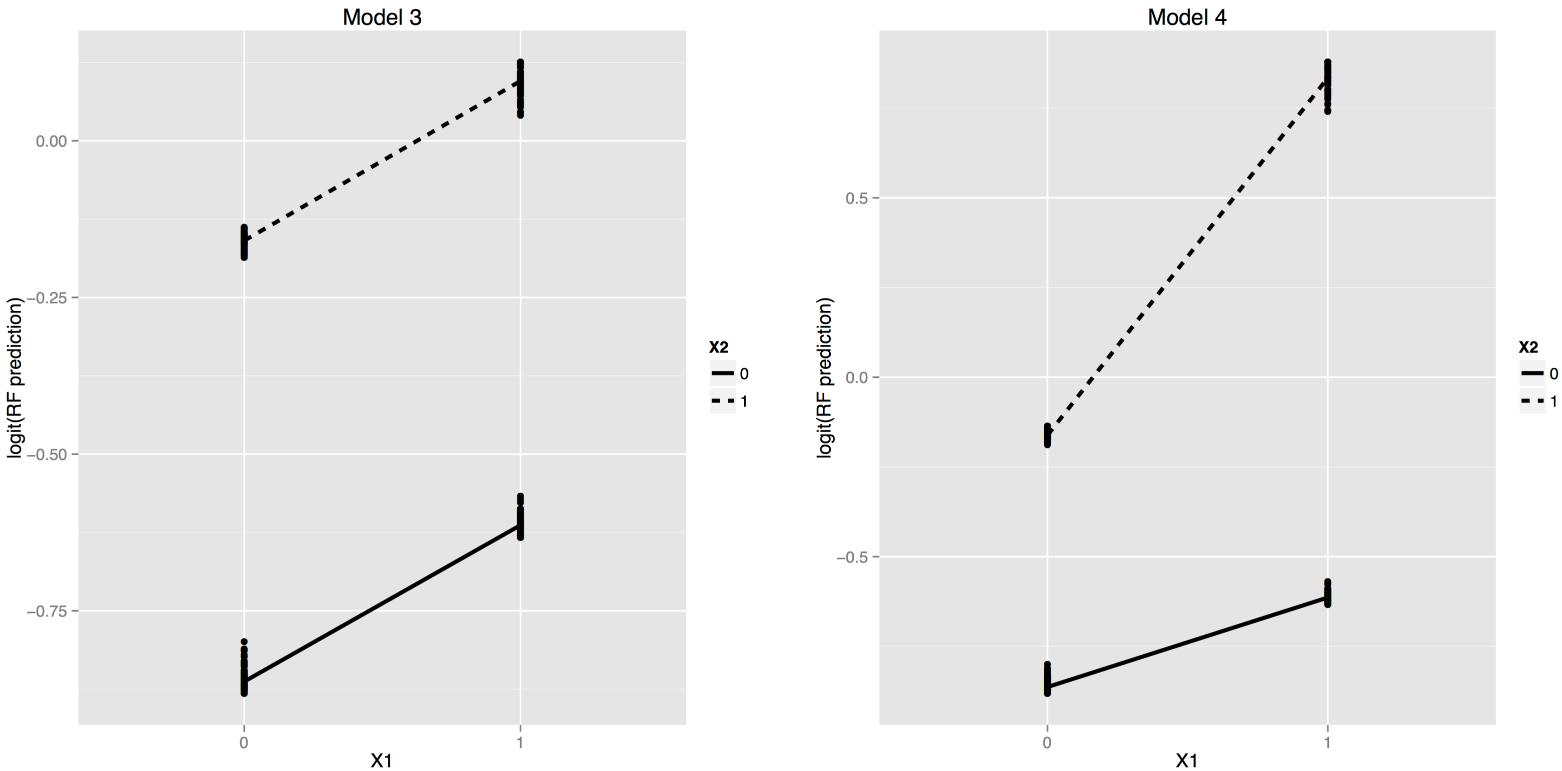
The Interactor

We fit one PM to the data and get predicted probabilities

- Average probabilities over X1=1, X2=1 → P_{11}
- Average probabilities over X1=0, X2=1 → P_{01}
- Average probabilities over X1=1, X2=0 → P_{10}
- Average probabilities over X1=0, X2=0 → P_{00}

We can now create classical interaction plots either on natural or logit scale

The Interactor



The Interactor

How about when we need to scan an entire genome?

1. Run 1 PM on the dataset
2. Compute interaction contrasts for each pair of features
3. Create an interaction heatmap to find interaction hotspots
4. Drill down (MDS, more analyses)

On to the post-parameter ecosystem