Final Assignment

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1 Laplace's Equation

To solve Laplace's equation computationally in two dimensions, we will use the method of finite difference relaxation. This method involves updating elements of an array based on adjacent values as seen in Equation 1.

$$A_{ij} = \frac{1}{4}\left[A_{i+1,j} + A_{i-1,j} + A_{i,j+1} + A_{i,j-1}\right] \tag{1}$$

This method will propagate information from a boundary to the rest of the system as it iterates. One limitation of this method is that the indices $i+1, i-1, j+1, j-1$ must exist, so accuracy near the borders is decreased. This can be mitigated by increase the spatial resolution to minimize the discontinuity.

1.1 Single Constant Boundary

As a basis for more complex modeling, I first solved Laplace's equation, $\nabla^2 V = 0$, for a rectangular box with dimensions $a \times b$ with $a = 10$ and $b = 10$. Three sides were held at $V = 0$ and one side was held at $V = 1$ volt. The resulting plot can be seen in Figure 1.

1.2 Two Boundary Functions

After verifying our solver for a single simple boundary condition, I then changed the boundaries such that $V(x,0) = \cos^2(\frac{\pi x}{a})$ (top edge) and $V(x,b) = -\sin^2(\frac{\pi x}{a})$ (bottom edge).

1.3 Rectangular Capacitor Model

Now, we can move the source of the voltage to be on two plates in the center of the insulated rectangle. Figures 3 and 4 show the plot of a simulated rectangular square plate capacitor with the left plate held at $V = 1$ volt and the right plate held at $V = -1$ volt.

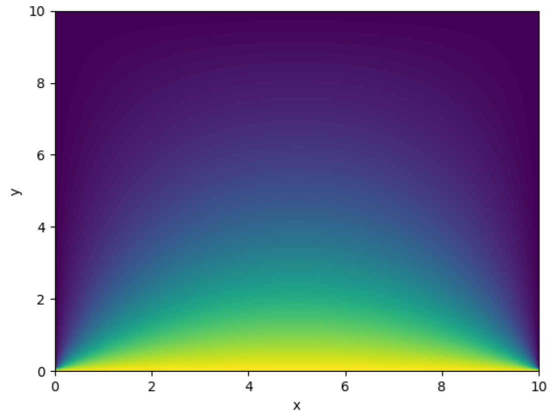

Figure 1: Contour plot of a rectangular region with the bottom side held at 1 V and the other three sides held at 0 V.

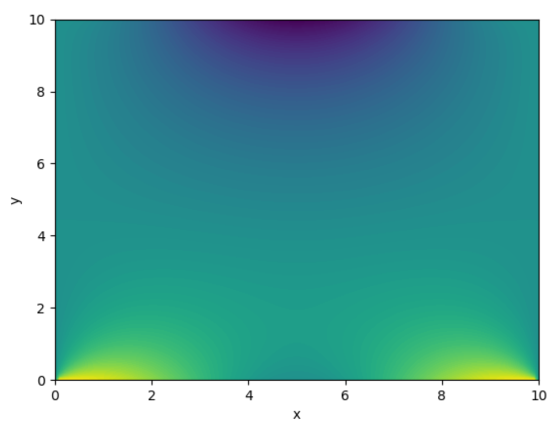

Figure 2: Contour plot of a rectangular region with $V(x,0) = \cos^2(\frac{\pi x}{a})$ (top edge) and $V(x,b) = -\sin^2(\frac{\pi x}{a})$ (bottom edge) where $a = 10$ and $b = 10$, and the side edges held at 0 V.

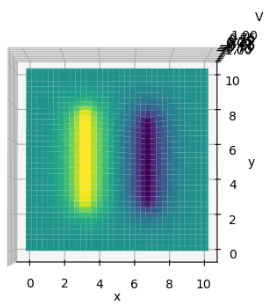

Figure 3: Top-down view of a surface plot depicting the voltage distribution of a rectangular plate capacitor with the left plate held at 1 V and the right plate held at -1 V.

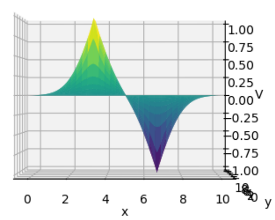

Figure 4: Side view of a surface plot depicting the voltage distribution of a rectangular plate capacitor with the left plate held at 1 V and the right plate held at -1 V.