Camera Model

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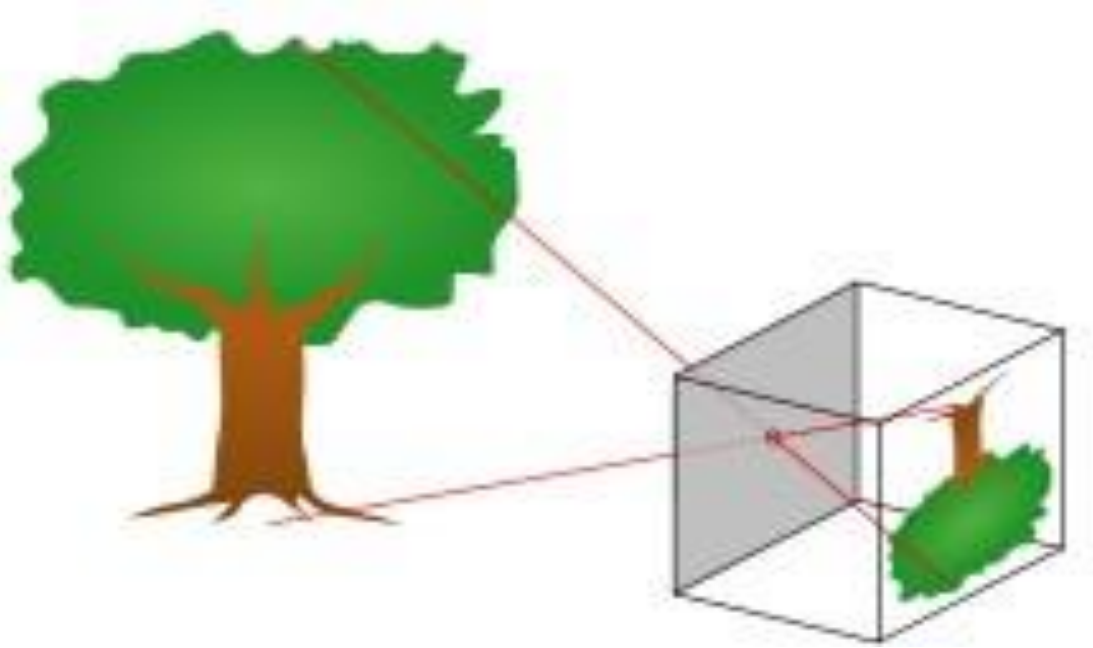
Introduction

- Camera is an equipment for mapping between the 3D scene space and a 2D image plane
- In image processing field, most geometric interpretations of images are based on the pinhole camera model.

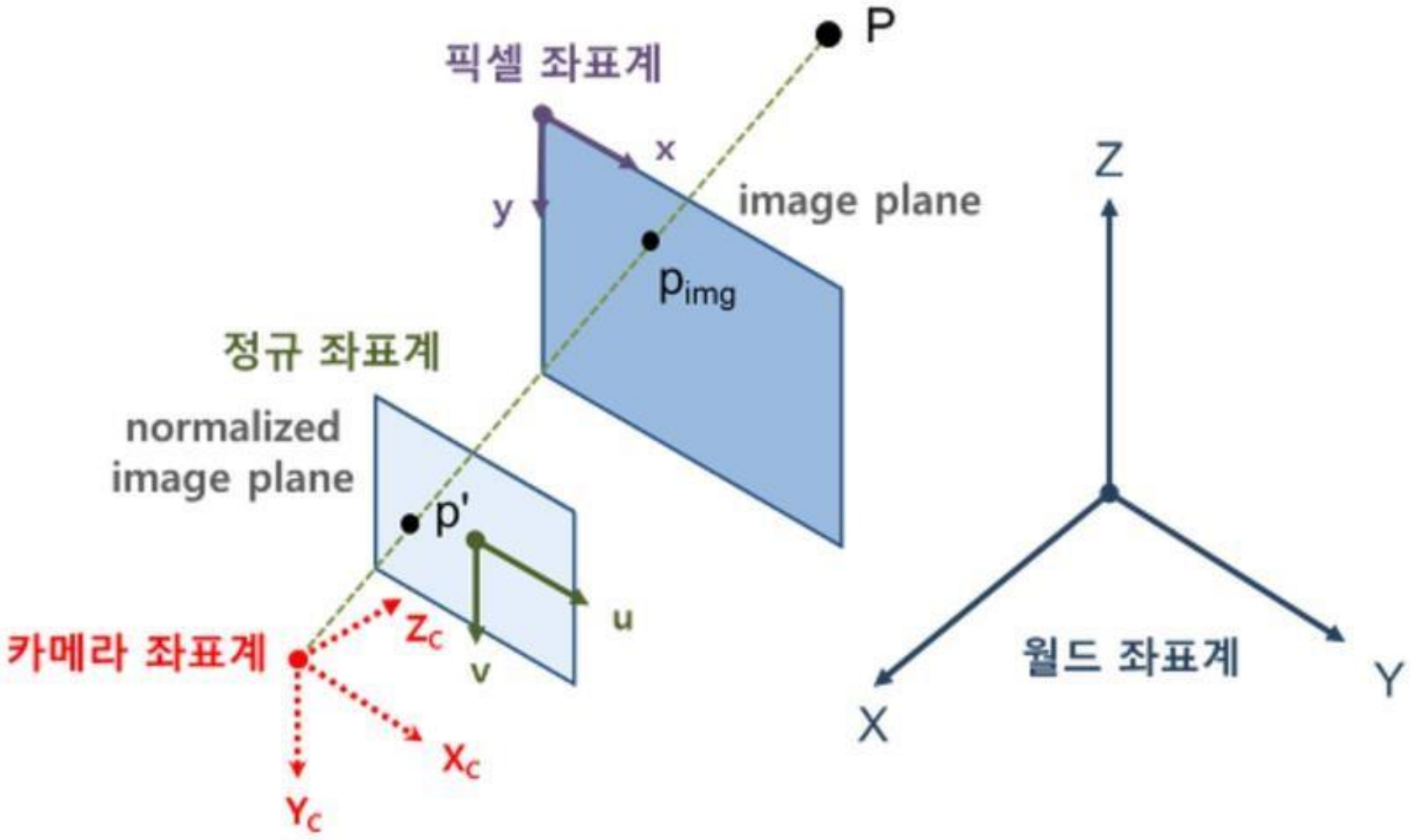


Introduction

- Preliminaries
  - World coordinate(3D)
    - Coordinate system used as a reference when expressing the position of an object. Designate the origin at (0,0,0)
  - Camera coordinate(3D)
    - Coordinate system relative to the camera origin
  - Pixel coordinate(=image plane, 2D)
    - Coordinate system of images
  - Normalized image plane
    - Coordinate system for images removing the effect of intrinsic parameters of a camera, defining virtual image plane whose focal length is 1

Introduction

- Preliminaries



Introduction

- Preliminaries
  - Inhomogeneous coordinates

    2D point➔ (x,y) 3D point➔ (x,y,z)

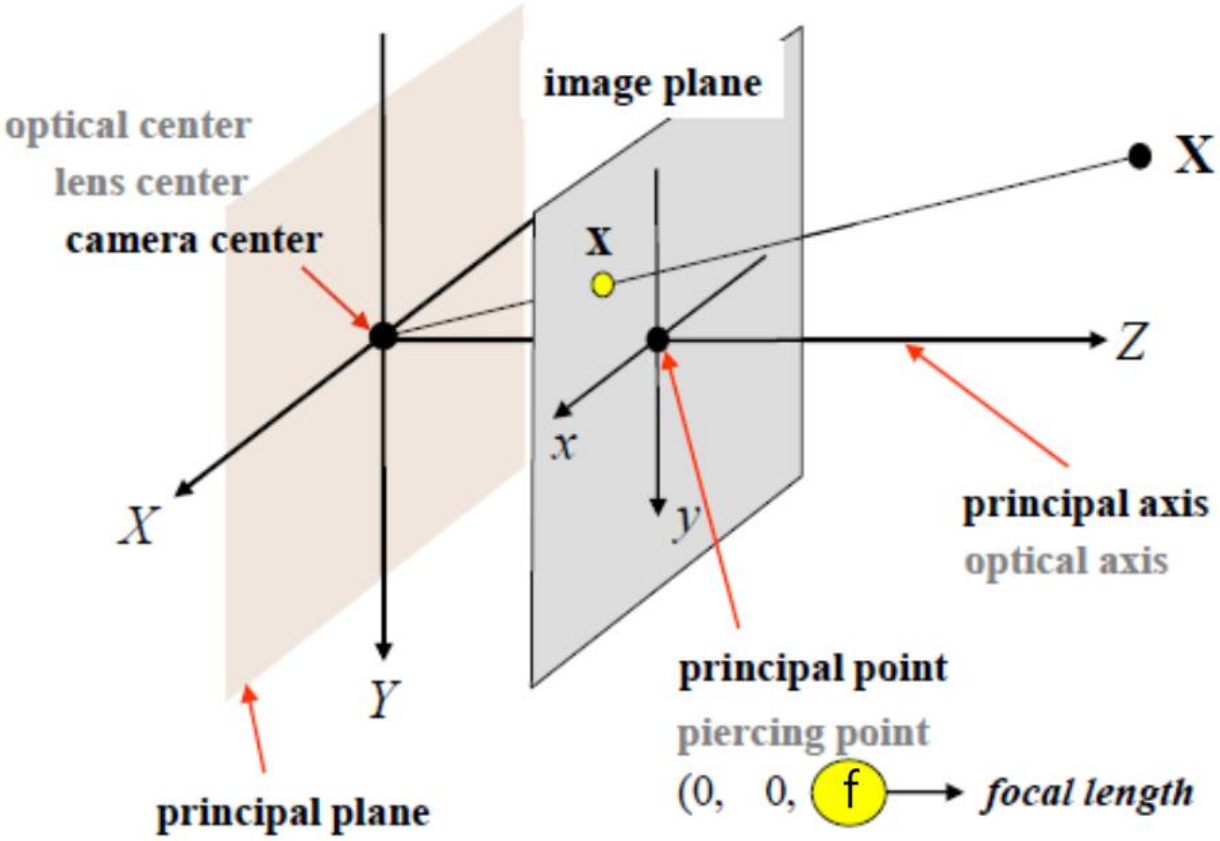
  - Homogeneous coordinates

    2D point➔ (x,y,1) 3D point➔ (x,y,z,1)

    (x,y,z,1)= (2x,2y,2z,2) ➔ equal up to scale

    Point at infinity(2D) ?? $(x, y, 0), x, y \neq 0$

Camera Model



Central Projection

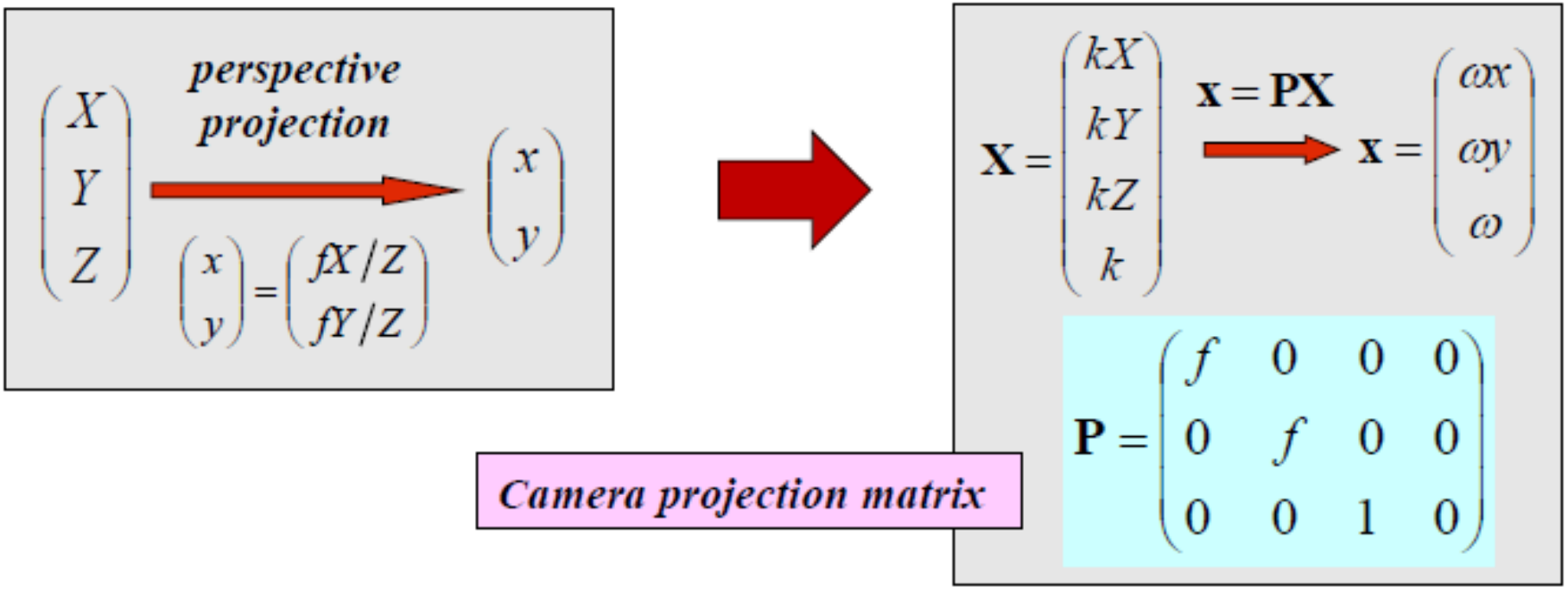
perspective projection

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix}$$

From the lecture "Multiple View geometry", by Prof. Kim, Seong Dae, KAIST

Camera Model

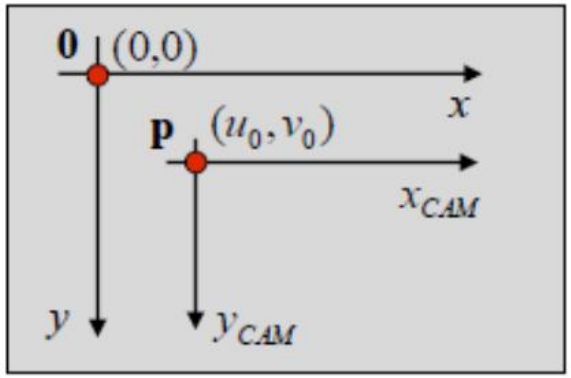


$$P = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \Rightarrow P = K[I|0]$$

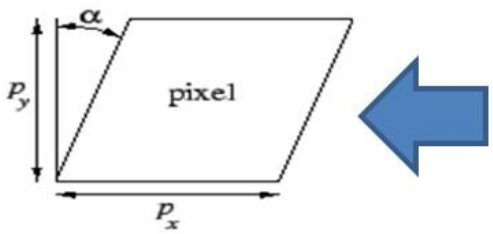
Camera calibration matrix

From the lecture "Multiple View geometry", by Prof. Kim, Seong Dae, KAIST

Camera Model



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} s_{xx} & 0 & u_0 \\ 0 & s_{yy} & v_0 \end{pmatrix} \begin{pmatrix} x_{CAM} \\ y_{CAM} \\ 1 \end{pmatrix} \Rightarrow \mathbf{x} = \begin{pmatrix} s_{xx} & 0 & u_0 \\ 0 & s_{yy} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{x}_{CAM}$$



skew

skew parameter

$$\mathbf{x} = \begin{pmatrix} s_{xx} & s_{xy} & u_0 \\ 0 & s_{yy} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{x}_{CAM}$$

$$\mathbf{P} = \begin{pmatrix} s_{xx} & s_{xy} & u_0 \\ 0 & s_{yy} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} fs_{xx} & fs_{xy} & u_0 \\ 0 & fs_{yy} & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

camera calibration matrix

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} \mathbf{I} \mid \mathbf{0} \end{bmatrix}$$

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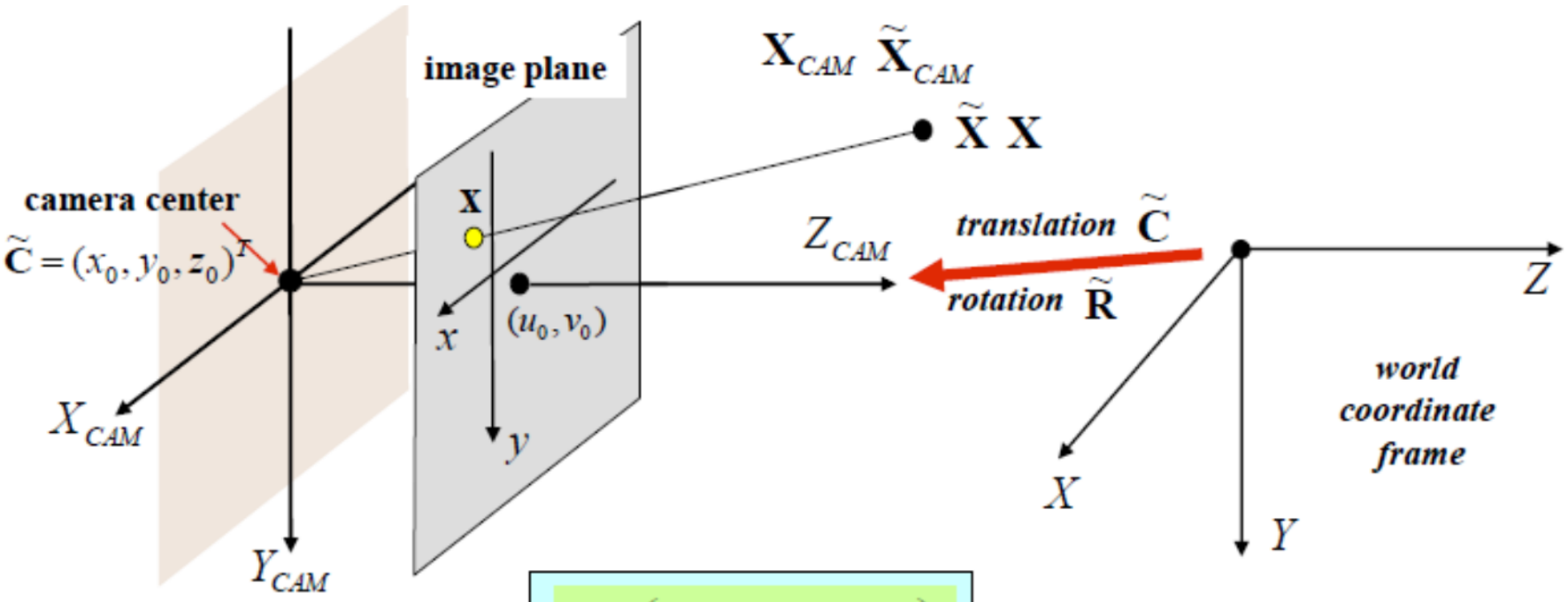
Camera Model



$$\tilde{\mathbf{X}}_{CAM} = \tilde{\mathbf{R}}(\tilde{\mathbf{X}} - \tilde{\mathbf{C}})$$

$$\tilde{\mathbf{X}}_{CAM} = \tilde{\mathbf{R}}[\mathbf{I} | -\tilde{\mathbf{C}}]\mathbf{X}$$

$$\mathbf{X}_{CAM} = \begin{pmatrix} \tilde{\mathbf{R}} & -\tilde{\mathbf{R}}\tilde{\mathbf{C}} \\ \mathbf{0}^T & 1 \end{pmatrix} \mathbf{X}$$

$$\mathbf{K} = \begin{pmatrix} fs_{xx} & fs_{xy} & u_0 \\ 0 & fs_{yy} & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{K}[\mathbf{I} | \mathbf{0}]\mathbf{X}_{CAM}$$

$$\mathbf{x} = \mathbf{K}\tilde{\mathbf{R}}[\mathbf{I} | -\tilde{\mathbf{C}}]\mathbf{X}$$

$$\mathbf{P} = \mathbf{K}\tilde{\mathbf{R}}[\mathbf{I} | -\tilde{\mathbf{C}}]$$

$$\mathbf{P} = \mathbf{K}[\tilde{\mathbf{R}} | \mathbf{t}] \qquad \mathbf{t} = -\tilde{\mathbf{R}}\tilde{\mathbf{C}}$$

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