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Spatial Filtering

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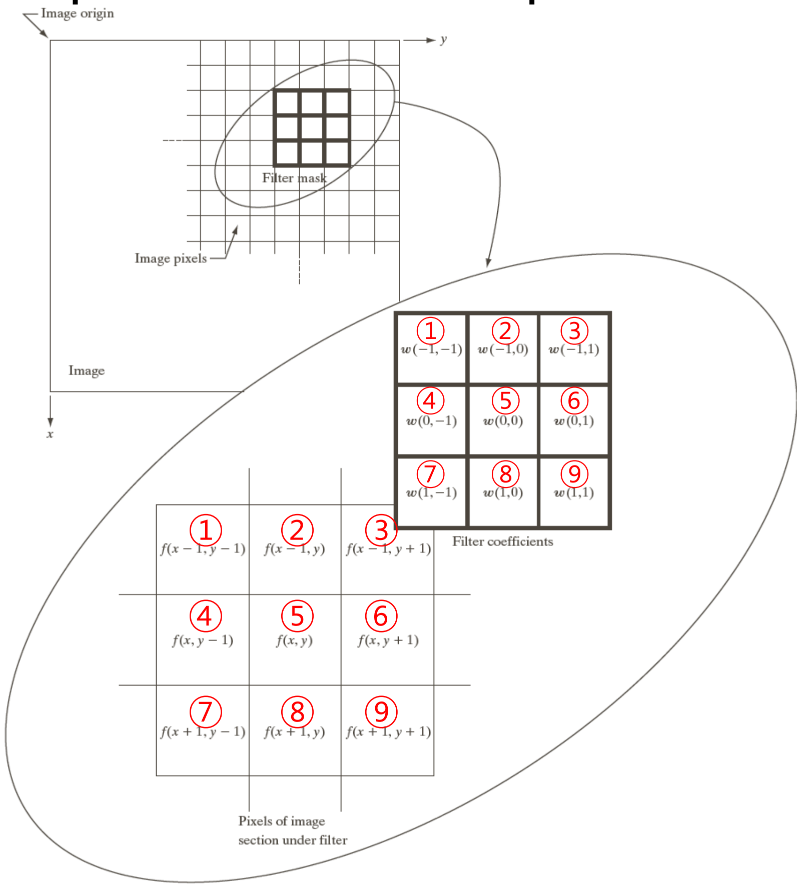
Introduction

- Spatial filtering
  - Spatial filters = spatial masks, kernels, templates, windows



When using 3X3 spatial filters,

$$
\begin{aligned}
g(x,y) = {} & w(-1,-1)f(x-1,y-1) \text{ ①} \\
& +w(-1,0)f(x-1,y) \text{ ②} \\
& \\
& +w(0,0)f(x,y) \text{ ⑤} \\
& +\cdots \\
& +w(1,1)f(x+1,y+1) \text{ ⑨}
\end{aligned}
$$

Introduction

- Spatial filtering
  - Example

5	5	5	3	3
5	5	5	3	5
5	5	5	3	3
5	5	3	3	3
5	5	3	3	3
5	5	3	3	3

0	1/5	0
1/5	1/5	1/5
0	1/5	0

- Result of spatial filtering on red pixel:

5*(0)+5*(1/5)+5*(0)+5*(1/5)+5*(1/5)+5*(1/5)+5*(0)+5*(1/5)+5*(0)=5

- Result of spatial filtering on blue pixel:

3*(0)+3*(1/5)+3*(0)+3*(1/5)+3*(1/5)+3(1/5)+3*(0)+3*(1/5)+3*(0)=3

- Result of spatial filtering on green pixel:

5*(0)+5*(1/5)+3*(0)+5*(1/5)+3*(1/5)+3*(1/5)+5*(0)+3*(1/5)+3*(0)=19/5

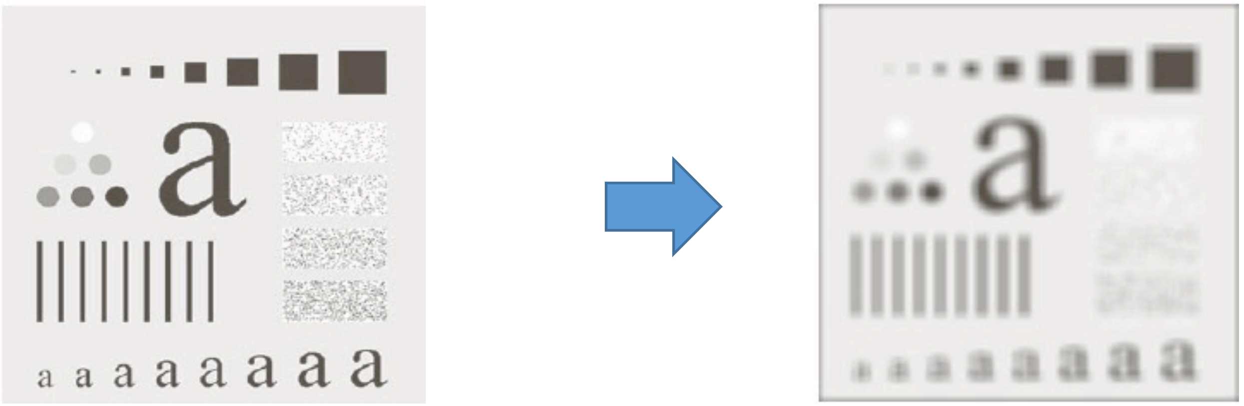
Spatial filtering

- Averaging filter
  - The average of the pixels contained in the neighborhood of the filter mask
  - Sometimes called low pass filters
  - For every pixel, replace the value of the pixel by the average of the intensity levels in the neighborhood
  - Advantage and disadvantage
    - It reduces random noises
    - It blurs an image

Spatial filtering

- Averaging filter

$$\frac{1}{9}\times\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}\qquad \frac{1}{16}\times\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$



Spatial filtering

- Gaussian filter

$$G(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Gaussian Function

1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16

3x3 filter mask

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Spatial filtering

- Floating-point Gaussian kernel ($\sigma = 1$)

0.075	0.124	0.075
0.124	0.204	0.124
0.075	0.124	0.075

3x3

0.003	0.013	0.022	0.013	0.003
0.013	0.060	0.098	0.060	0.013
0.022	0.098	0.162	0.098	0.022
0.013	0.060	0.098	0.060	0.013
0.003	0.013	0.022	0.013	0.003

5x5

0.000	0.000	0.001	0.002	0.001	0.000	0.000
0.000	0.003	0.013	0.022	0.013	0.003	0.000
0.001	0.013	0.059	0.097	0.059	0.013	0.001
0.002	0.022	0.097	0.159	0.097	0.022	0.002
0.001	0.013	0.059	0.097	0.059	0.013	0.001
0.000	0.003	0.013	0.022	0.013	0.003	0.000
0.000	0.000	0.001	0.002	0.001	0.000	0.000

7x7

Spatial filtering

- Discretized Gaussian kernel ($\sigma = 1$)

1/16 x

1	2	1
2	4	2
1	2	1

3x3

1/273 x

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

5x5

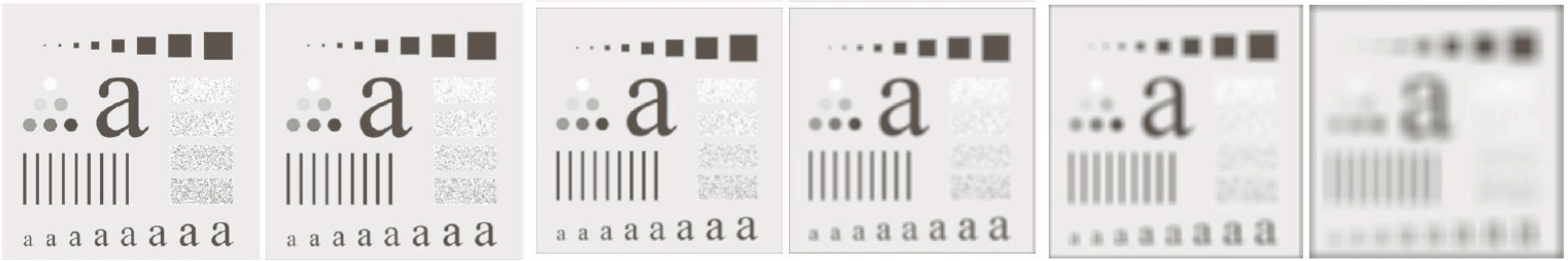
1/1003 x

0	0	1	2	1	0	0
0	3	13	22	13	3	0
1	13	59	97	59	13	1
2	22	97	159	97	22	2
1	13	59	97	59	13	1
0	3	13	22	13	3	0
0	0	1	2	1	0	0

7x7

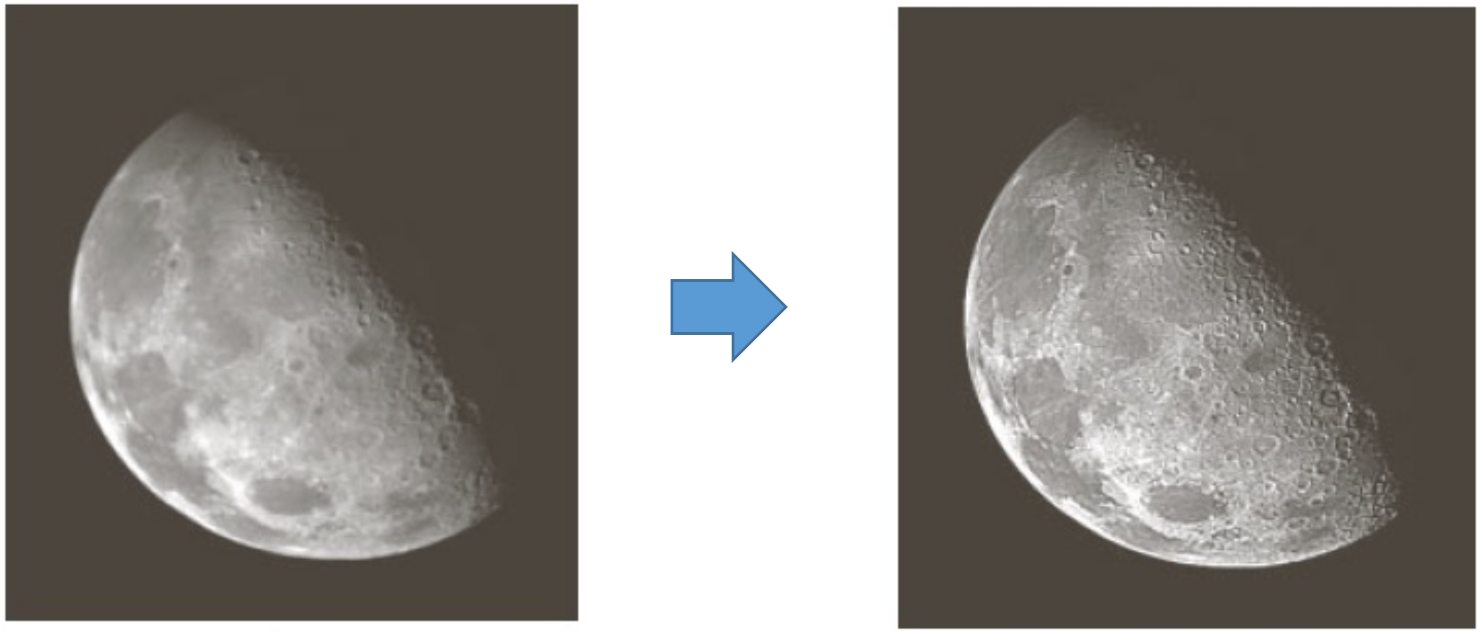
Spatial filtering

- Mask size
  - Mask size matters
  - If you want to blur small objects, use a small size mask
  - Using a large mask is computationally expensive



Spatial filtering

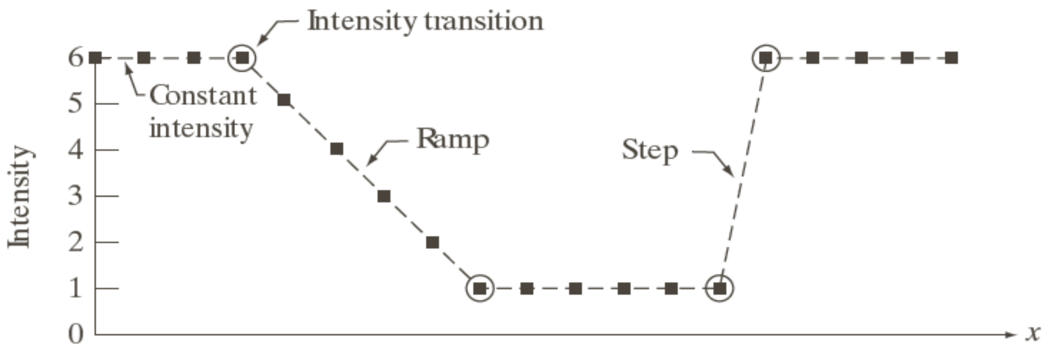
- Sharpening
  - The principal objective of sharpening is to highlight transitions in intensity



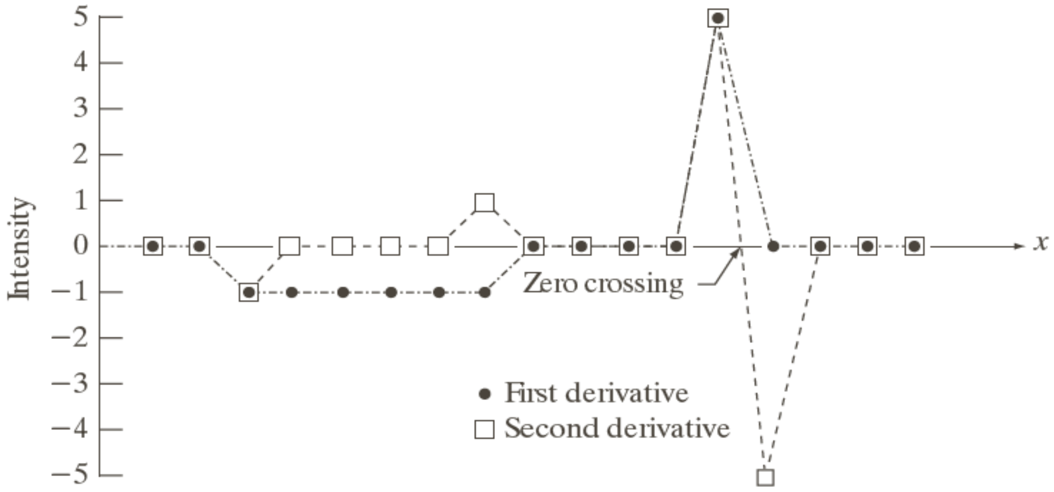
  - Sharpening can be accomplished by spatial differentiation

Spatial filtering

- Sharpening using second derivative



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



Spatial filtering

- Sharpening using second derivative
  - Mask for applying second derivative

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

  - Algorithm
    1. Obtain second derivative of the input image
    2. Add the second derivative with the input image

Spatial filtering

- First derivative and second derivative

$$One-Dimensional\ \ Signal\ \ f(x)$$

$1-Order:$

$$\frac{\partial f}{\partial x}=f(x+1)-f(x)$$

$2-Order:$

$$\frac{\partial^2 f}{\partial x^2}=f(x+1)+f(x-1)-2f(x)$$

2nd order derivative

$$\frac{\partial^2 f}{\partial x^2}=f(x+1)+f(x-1)-2f(x)$$

$$\frac{\partial^2 f}{\partial y^2}=f(y+1)+f(y-1)-2f(y)$$

Convolution Kernel

0	1	0
1	-4	1
0	1	0

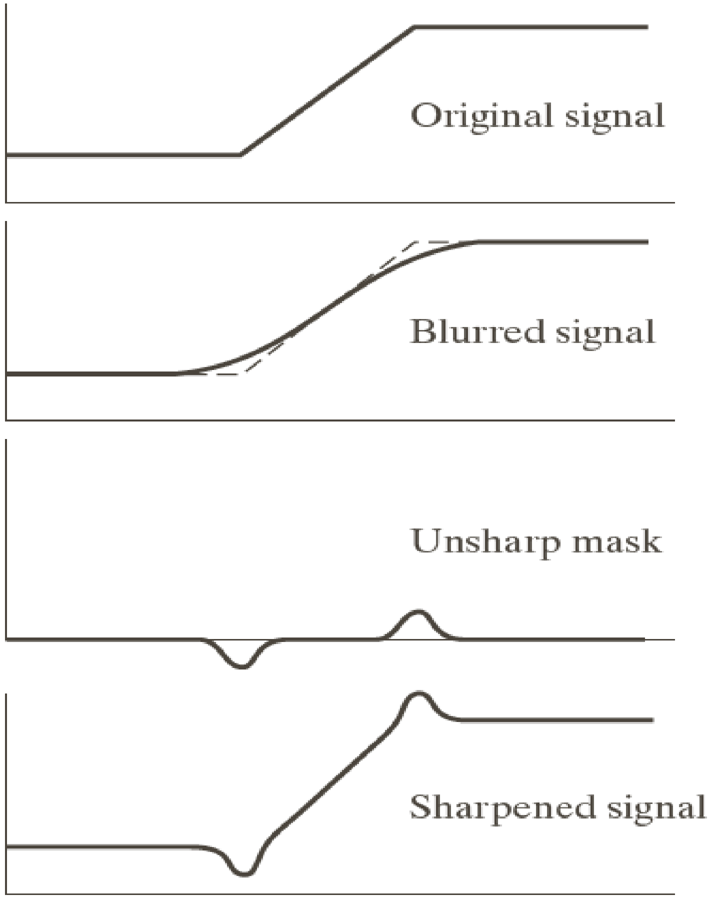
Laplacian

$$\nabla^2 f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial x^2}$$

$$=f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)-4f(x,y)$$

Spatial filtering

- Sharpening using unsharp masking
  - Unsharp masking



$$f(x,y)$$

$$\bar{f}(x,y)$$

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

Other filter - Median filter

- Median value
  - For 3X3 neighborhood, the median is the 5th largest
  - For 5X5 neighborhood, the median is the 13th largest
- Median filter
  - Find the median value of a mask, and replace the values of pixels in the mask with the median value
  - Isolated clusters of pixels that are light or dark with respect to their neighbors, and whose area is less than $m^2/2$ are eliminated by an mXm median filter
  - It is effective in the presence of impulse noise (or salt-and pepper noise)