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In[9]:= LazyQuantumRandomWalk [State0_, Steps0_] := Module[{State = State0, Steps = Steps0},
  BitOrder = 3;
  
$$\sigma = e^{\frac{\pi 2 i}{\text{BitOrder}}};$$

  
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = H = \frac{\begin{pmatrix} 1 & 1 & 1 \\ 1 & \sigma^{\text{BitOrder}-1} & \sigma \\ 1 & \sigma & \sigma^{\text{BitOrder}-1} \end{pmatrix}}{\sqrt{\text{BitOrder}}};$$

  
$$M = \begin{pmatrix} a & 0 & 0 & 0 & b & 0 & 0 & 0 & 0 & 0 & 0 & c \\ d & 0 & 0 & 0 & e & 0 & 0 & 0 & 0 & 0 & 0 & f \\ g & 0 & 0 & 0 & h & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & c & a & 0 & 0 & 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & f & d & 0 & 0 & 0 & e & 0 & 0 & 0 & 0 \\ 0 & 0 & i & g & 0 & 0 & 0 & h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c & a & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & f & d & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & i & g & 0 & 0 & 0 & h & 0 \\ 0 & b & 0 & 0 & 0 & 0 & 0 & 0 & c & a & 0 & 0 \\ 0 & e & 0 & 0 & 0 & 0 & 0 & 0 & f & d & 0 & 0 \\ 0 & h & 0 & 0 & 0 & 0 & 0 & 0 & i & g & 0 & 0 \end{pmatrix};$$

  For[j = 0, j < Steps, j++, State = Simplify[M.State]];
  Return[State];
]

In[10]:= FoutStepCircleStatesToPositionProbability [State0_] :=
Module[{State = State0},
  ProbabilityAll = Simplify[Conjugate[State] * State];
  ProbabilaityMixed = Transpose[{{
    ProbabilityAll[[1, 1]] + ProbabilityAll[[2, 1]] + ProbabilityAll[[3, 1]],
    ProbabilityAll[[4, 1]] + ProbabilityAll[[5, 1]] + ProbabilityAll[[6, 1]],
    ProbabilityAll[[7, 1]] + ProbabilityAll[[8, 1]] + ProbabilityAll[[9, 1]],
    ProbabilityAll[[10, 1]] + ProbabilityAll[[11, 1]] + ProbabilityAll[[12, 1]]
  }}];
  For[k = 1, k ≤  $\frac{\text{Dimensions}[ProbabilityAll][[1]]}{\text{BitOrder}}$ , k++,  $\sum_{j=(\text{BitOrder} (k-1))+1}^{\text{BitOrder} k} \text{ProbabilityAll}[[j, 1]]$ 
  ];
  Return[ProbabilaityMixed];
]

In[11]:= InputState = SparseArray[{
  {12, 1} → 0,
  {1, 1} → 1
}];

In[58]:= OutState = LazyQuantumRandomWalk [InputState, 1];

In[57]:= PositionProbability = FoutStepCircleStatesToPositionProbability [OutState];

In[51]:=
WalkResult = {};
For[LoopVar = 0, LoopVar < 100, LoopVar++,
  OutState = LazyQuantumRandomWalk [InputState, LoopVar];
  AppendTo[WalkResult, Flatten[FoutStepCircleStatesToPositionProbability [OutState]]]
]

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In[56]:= ListPointPlot3D[WalkResult, Filling -> Bottom, BoxRatios -> {2, 10, 1}]
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