CT216: Introduction to Communication Systems

Topic: Polar Codes

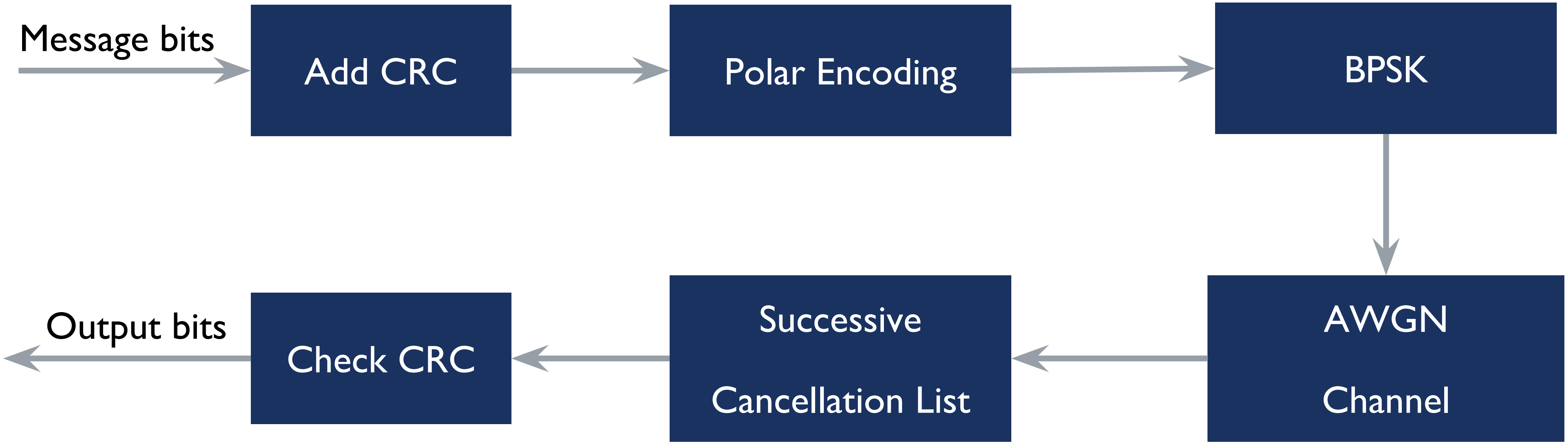
CONTENTS COVERED:

- Introduction
- Block Diagram
- Cyclic Redundancy Check(CRC)
- Polarization
- Polar Encoding
- BPSK Modulation in AWGN channel
- Successive Cancellation Decoder
- Successive Cancellation List Decoder
- References

INTRODUCTION:

- Polar codes are the most recent, proposed by Erdal Arikan in 2008.
- They are linear block error-correcting codes.
- Polar codes have been chosen as the channel coding scheme for control and data channels in the 5G wireless system in third generation partnership project New Radio - (3GPPNR) standard, ensuring high-speed and reliable communication in next-gen wireless networks.
- Polar codes are based on the concept of channel polarization, where noisy channels can transform into extremely noisy(least reliable) and ideal noiseless(highly reliable) channels.

BLOCK DIAGRAM



CYCLIC REDUNDANCY CHECK (CRC)

- A Cyclic Redundancy Check(CRC) is used to select a code word from the list of likely codewords.
- During Polar Encoding,

Message Bits → Adds L length parity check Bits → Message Bits

Length = A bits

Message bits get divided by standard polynomial of L degree

Length = K bits
= A+L Bits

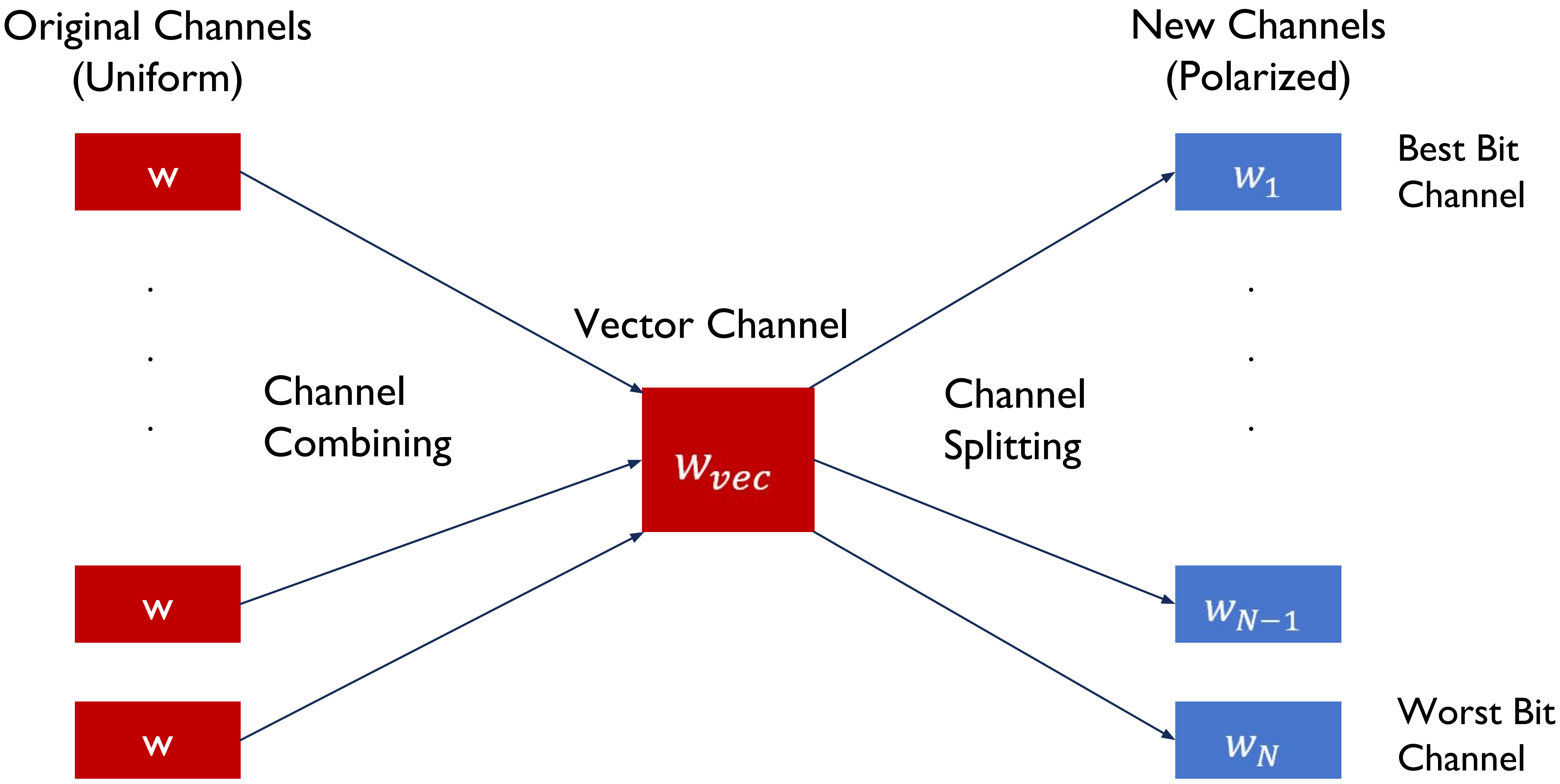
- After Successive Cancellation List Decoding, when all the parallel decoders have each selected their distinct prospective codewords, these candidate codewords are then checked by CRC, and the most likely candidate that passes the check is selected.
- If no candidate passes the CRC, then a decoding failure is reported.

POLARIZATION

- The principle of polar code is based on polarization. The channel polarization takes place in two ways:

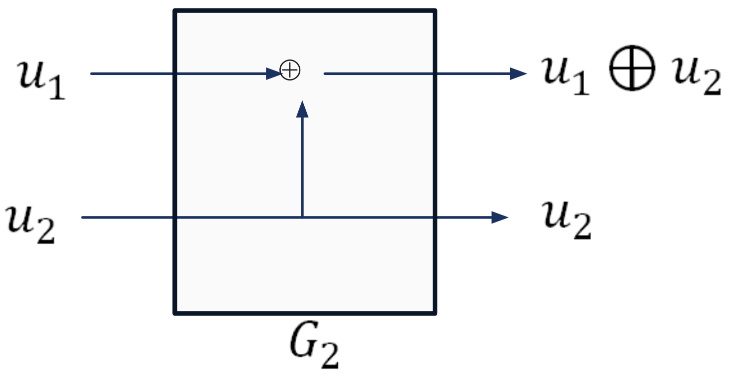
1. Channel Combining : In channel combining, the polarized channel are combined to create new set of channels with improved reliability. This is typically achieved through mathematical operations that merge the information from several channels into a single, more reliable channel. Channel combining is particularly useful when dealing with channels that exhibit different error characteristics.
2. Channel Splitting : In channel splitting, channel splitting involves breaking down a single polarized channel into multiple sub-channels. This can be advantageous when the original channel suffers from high error rates or variations in error probabilities across its length. By splitting the channel into smaller, more manageable sub-channels, it becomes easier to encode and decode information with greater accuracy. Channel splitting allows for finer control over the transmission process, enabling more efficient error correction.

- With the help of polarization we can differentiate between best bit channel (with low error probability) and worst bit channel (with high error probability).


Original Channels
(Uniform)
W
W
W
Channel
Combining
Vector Channel
W_{vec}
Channel
Splitting
New Channels
(Polarized)
W_1
Best Bit
Channel
W_{N-1}
W_N
Worst Bit
Channel

POLAR TRANSFORM:

- Generator matrix $G_N = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{\otimes n}$, where N = 2^n , $n = 1,2,3, \ldots$
- Here $\otimes$ represents tensor product, also known as kronector product.
-
- For e.g., $G_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $[u_1\ u_2]\ G_2 = [u_1 \oplus u_2\ u_2]$



BINARY TREE REPRESENTATION OF POLAR TRANSFORM (for N=8)

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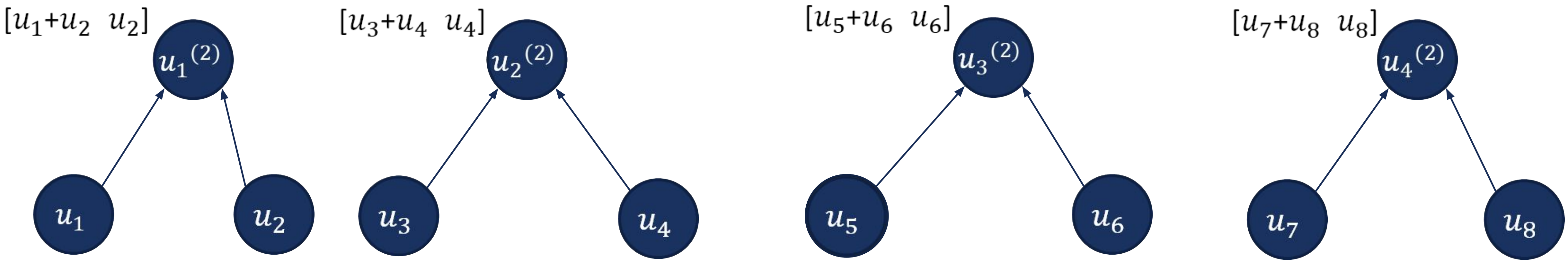

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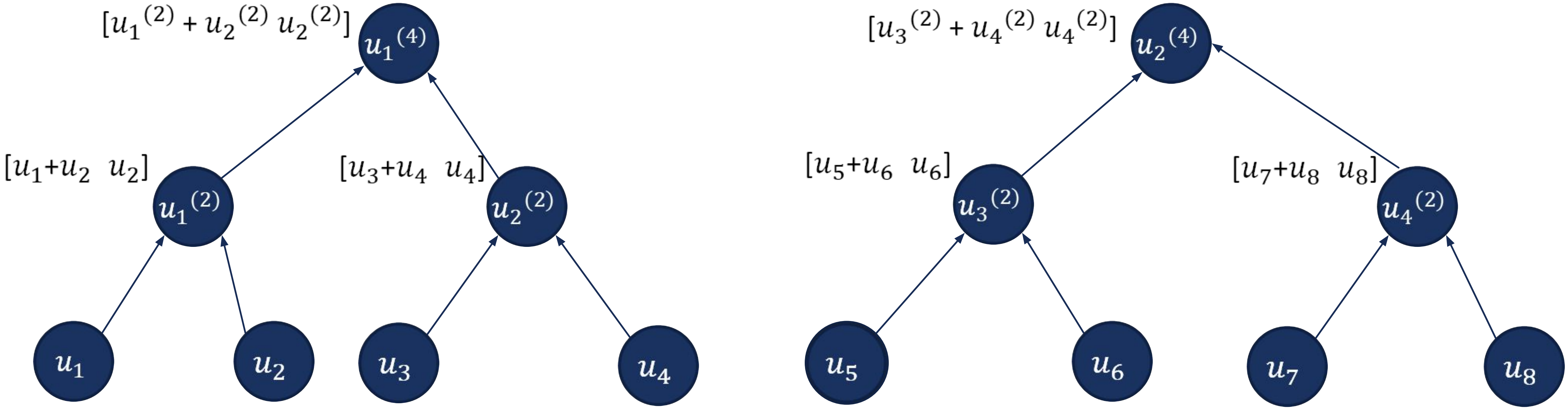

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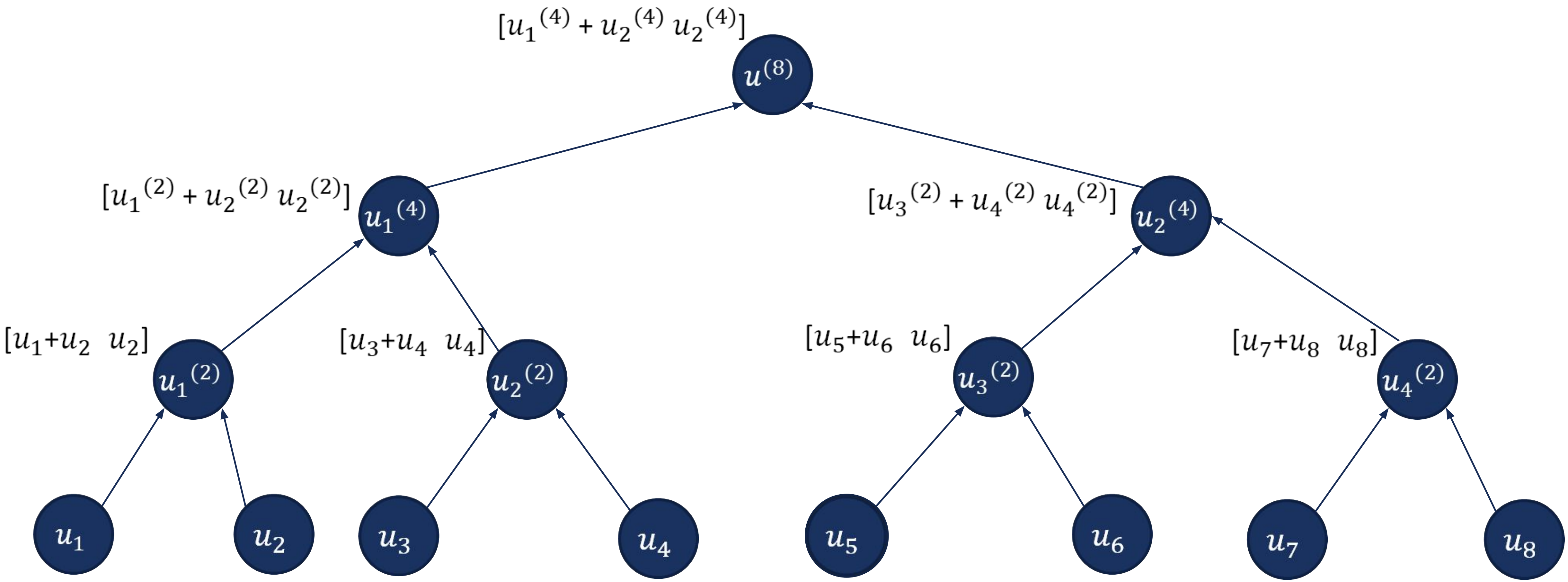


RELIABILITY SEQUENCE

- Reliability sequence tells the ordering of bit channels from worst to best.
- For N=8,

  [1 2 3 5 4 6 7 8]

- For N=16,

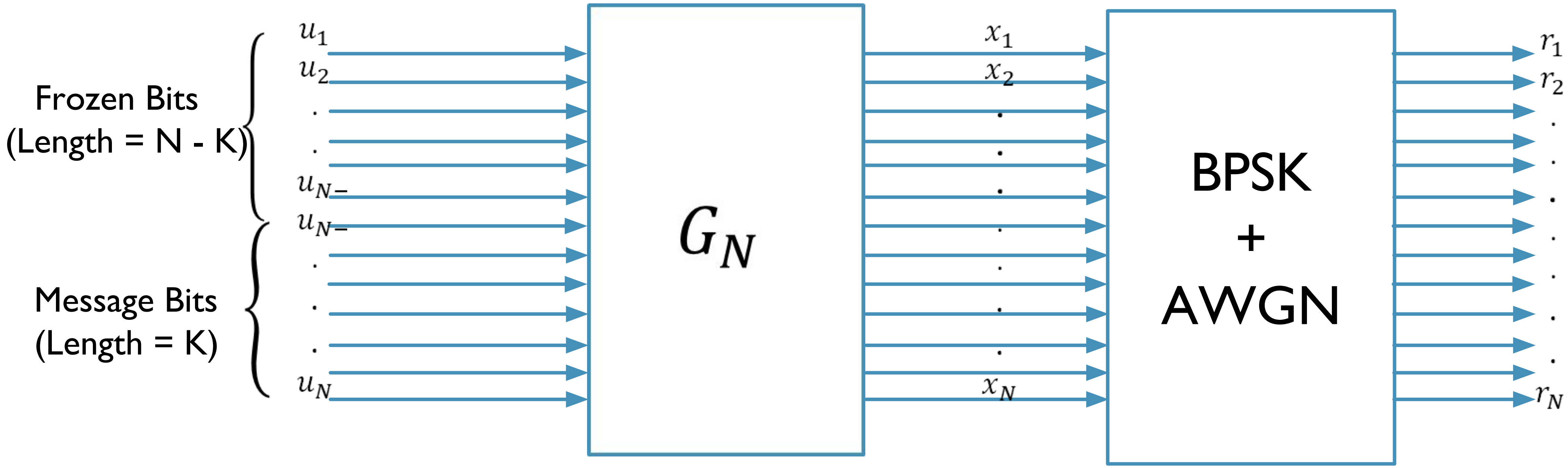
  [1 2 3 5 9 4 6 10 7 11 13 8 12 14 15 16]

- For N=32,

  [1 2 3 5 9 17 4 6 10 7 18 11 19 13 21 25 8 12 20 14 15 22 27 26 23 29 16 24 28 30 31 32]

POLAR ENCODING

- The Information bits, u_N of length N bits is passed to polar transformation. After getting encoded bits we transform it to BPSK modulation and further pass into AWGN channel resulting in received vector r_N.

- 

- Polar Encoding for $N = 8$, $K = 5$.

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  Frozen bits : [1 2 3]

  Msg bits : [5 4 6 7 8]

0 0 0 m_2 m_1 m_3 m_4 m_5

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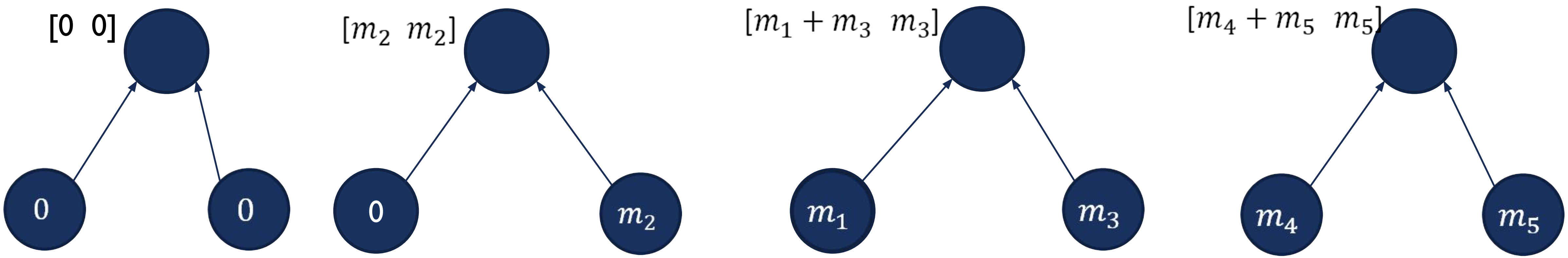

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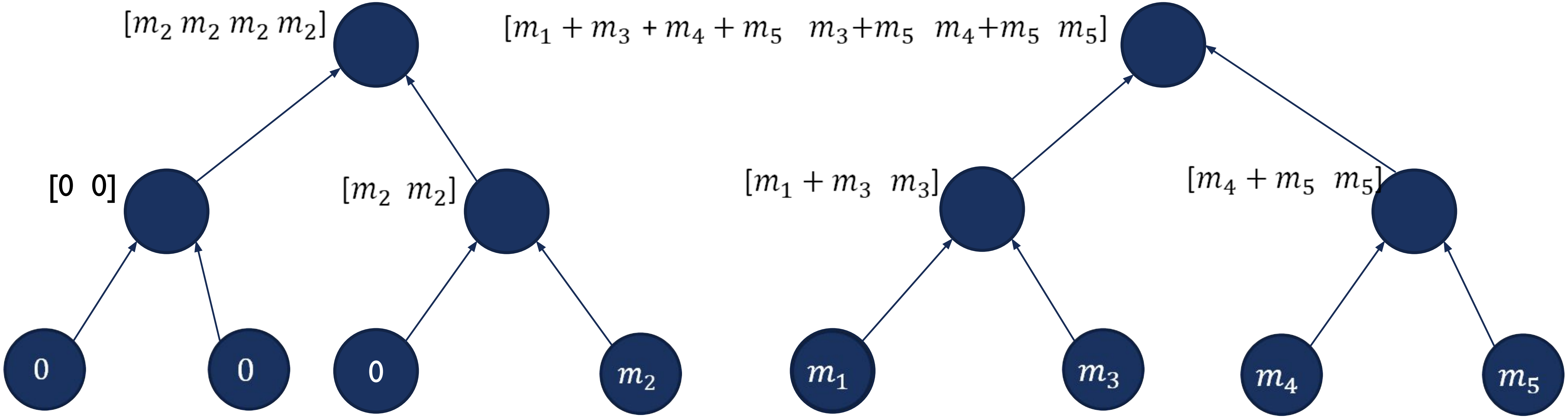

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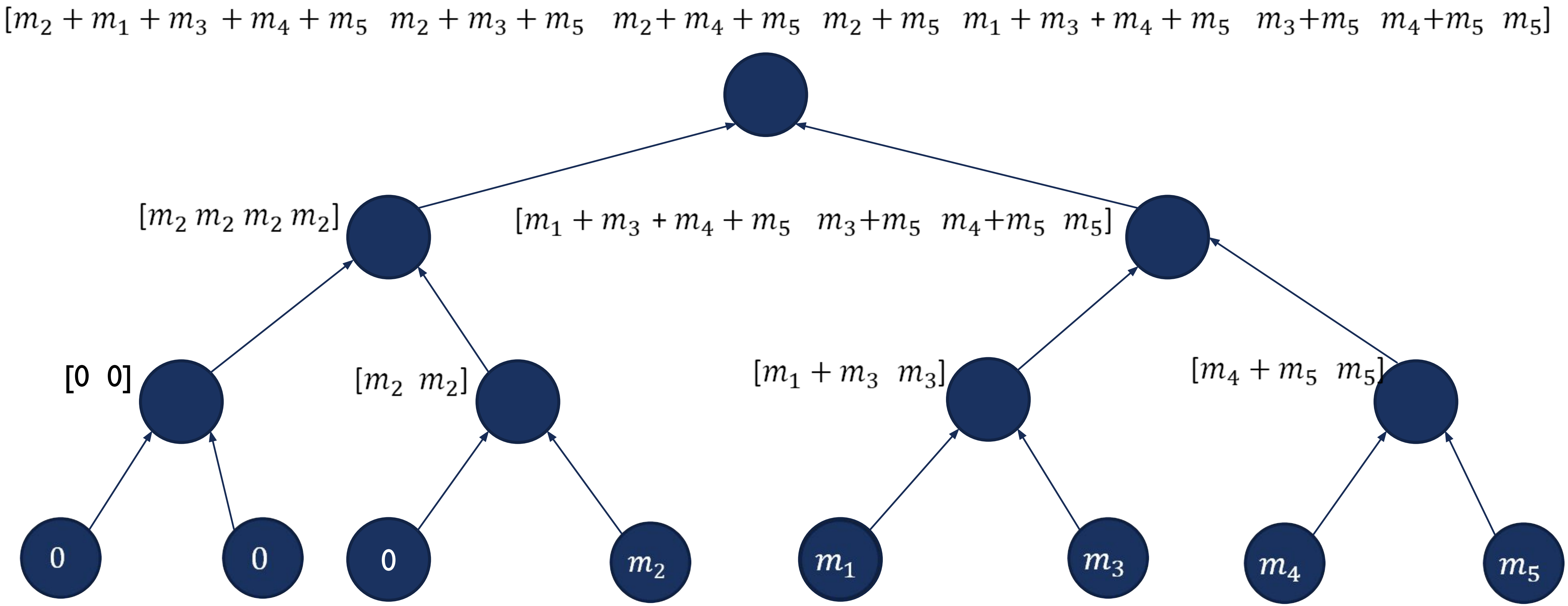


SUCCESSIVE CANCELLATION(SC) DECODER

Successive Cancellation(SC) Decoder is the basic decoding algorithm for polar code which work sequentially. Due to sequential nature of SC decoder, latency is high.

- Firstly, the received vector r_N is decoded with the help of soft-decision decoding algorithm. In soft-decision algorithm, the reliability of each received bits are identified with the help of Log-Likelihood Ratio(LLR). Further this soft bits help to compute the estimated hard bits using F-function, G-function and partial sum.

- The estimation of hard bits is performed with the help of F-function and G-function.

1. F–function : The F-function is applied to estimate the information bits in polar codes. It is designed by min-sum equation

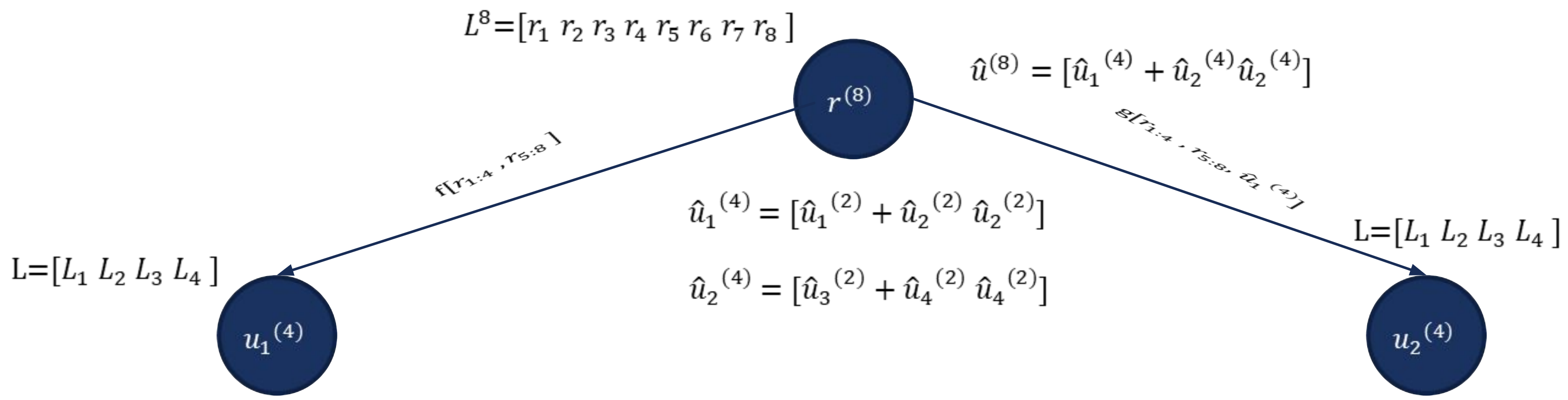
$L^8 = [r_1 \ r_2 \ r_3 \ r_4 \ r_5 \ r_6 \ r_7 \ r_8 \ ]$

$r^{(8)}$

$$\mathrm{f}(a_{1:p}, b_{1:p}) = [\mathrm{f}(a_1, b_1) \ \mathrm{f}(a_2, b_2) \ \ldots \ \mathrm{f}(a_p, b_p)]$$

$$\mathrm{g}(r_1, r_2, \mathrm{b}) = r_2 + (1 - 2\mathrm{b}) \ r_1$$

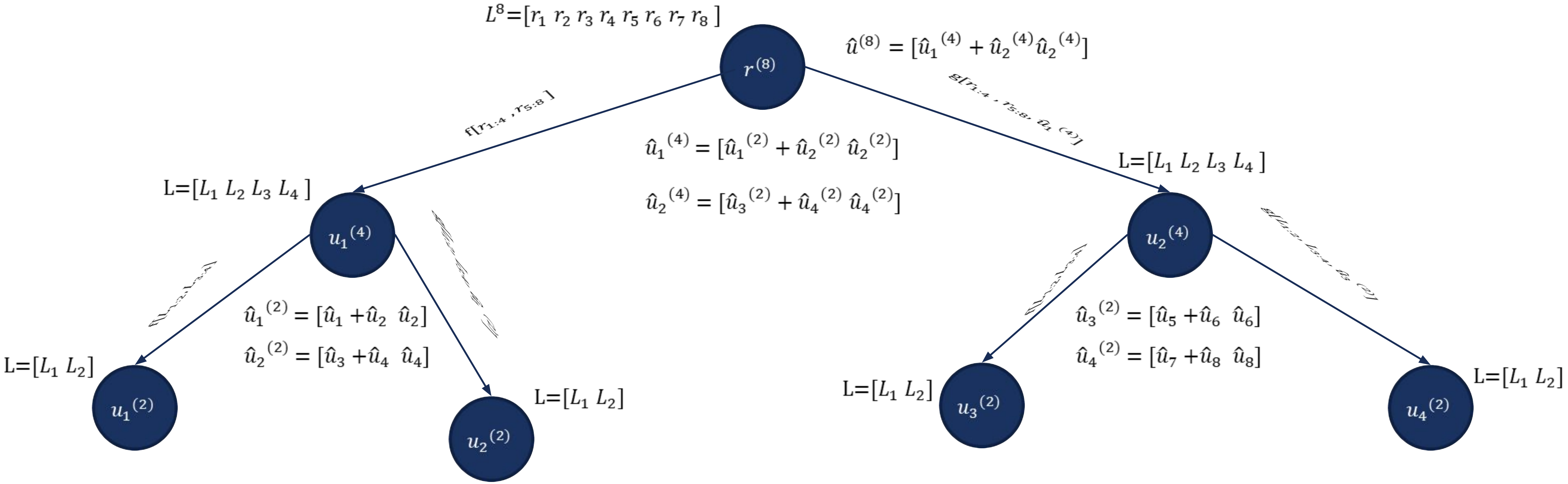
$$\mathrm{f}(r_1, r_2) = \mathrm{sgn}(r_1) \ \mathrm{sgn}(r_2) \ \min(| \ r_1 \ |, | \ r_2 \ |)$$



$$f(a_{1:p}, b_{1:p}) = [f(a_1, b_1)\ f(a_2, b_2) \ldots f(a_p, b_p)]$$

$$g(r_1, r_2, b) = r_2 + (1 - 2b)\, r_1$$

$$f(r_1, r_2) = \operatorname{sgn}(r_1)\ \operatorname{sgn}(r_2)\ \min(|\, r_1 \,|, |\, r_2 \,|)$$



$$f(a_{1:p}\ , b_{1:p}\ ) = [f(a_1, b_1)\ f(a_2, b_2)\ \dots\ f(a_p, b_p)]$$

$$g(r_1, r_2, b) = r_2 + (1 - 2b)\ r_1$$

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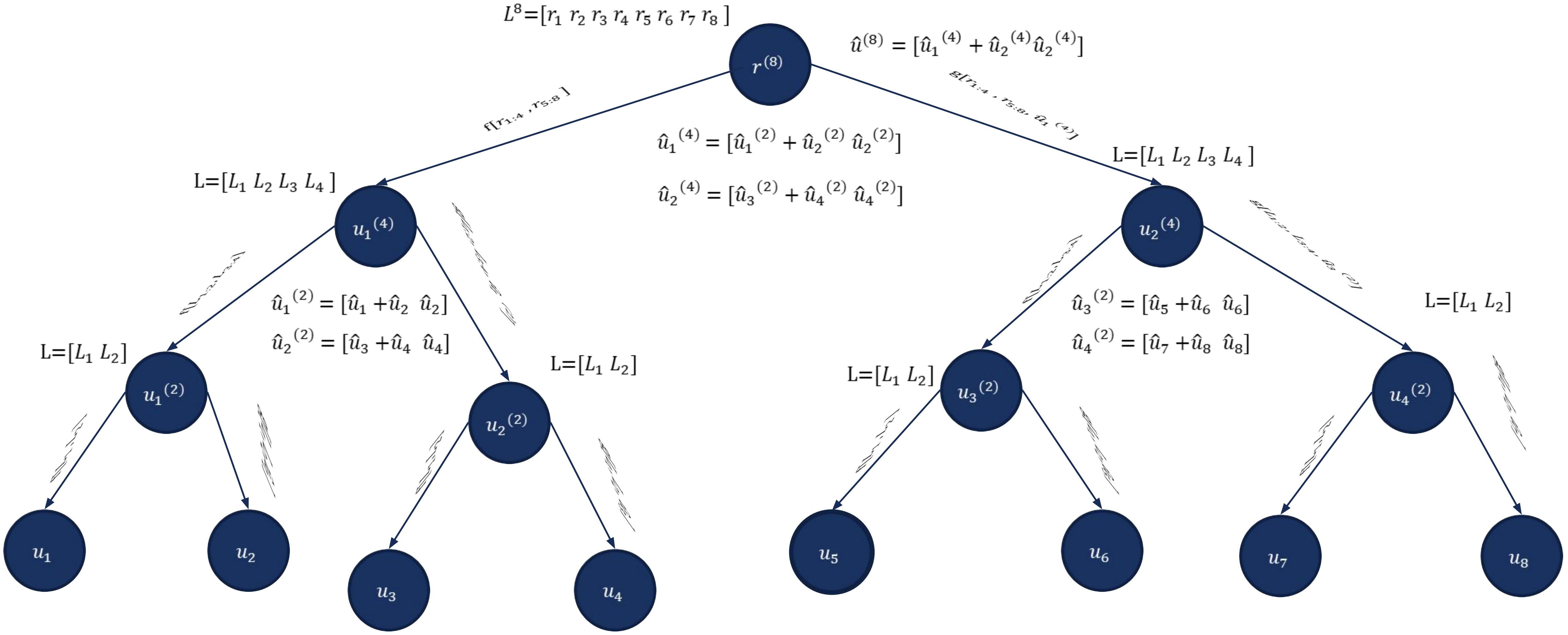

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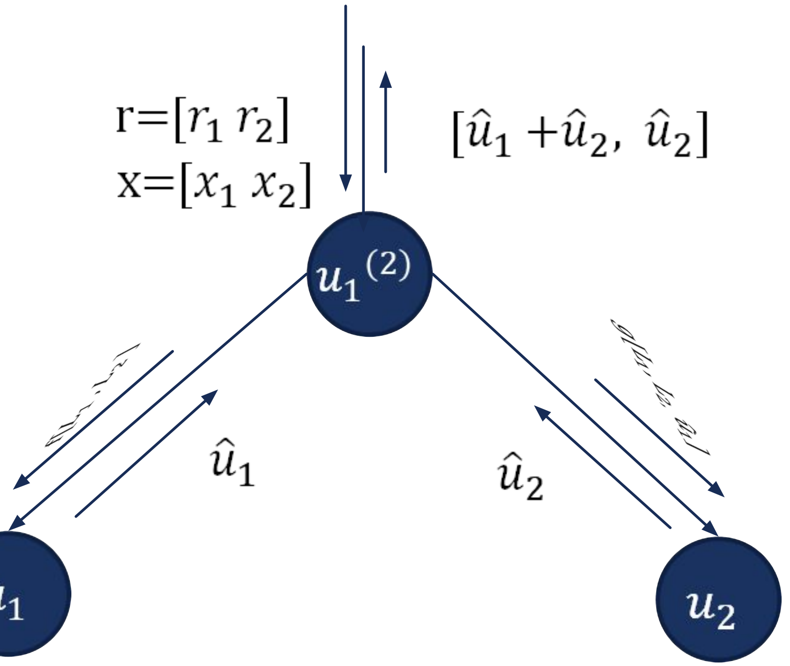


- First ,SPC decode msg u_1

$$\hat{u}_1 = 0, \ \ \text{if } L(u_1) \geq 0;$$
$$\hat{u}_1 = 1, \ \ \text{if } L(u_1) < 0$$

- Given $\hat{u}_1$, decode u_2 (Repetition)

If $\hat{u}_1 = 0, L(u_2) = r_2 + r_1$ ($x = [u2\ u2]$)

If $\hat{u}_1 = 1, L(u_2) = r_2 - r_1$ ($x = [\bar{u}2\ u2]$)

F-function node: It is a mathematical operation that estimates the likelihood of each information bit being 0 or 1, based on the received channel information and prior beliefs about the transmitted bits.

$$f(r_1, r_2) = 2\tan^{-1}\left(\tanh(\tfrac{r1}{2})\tanh(\tfrac{r2}{2})\right)$$
$$f(r_1, r_2) = \text{sgn}(r_1)\ \text{sgn}(r_2)\ \min(|\ r_1\ |, |\ r_2\ |)$$

G-function node: The G-function in polar code decoding is a mathematical operation applied to estimate the LLR (Log-Likelihood Ratio) of a frozen bit based on the LLRs of the received bits and their parent bits in the binary tree representation of the code.

SUCCESSIVE CANCELLATION LIST DECODING

if L$(u_i) \geq 0$: $\hat{u}_i$ = 0 has DM_i = 0,

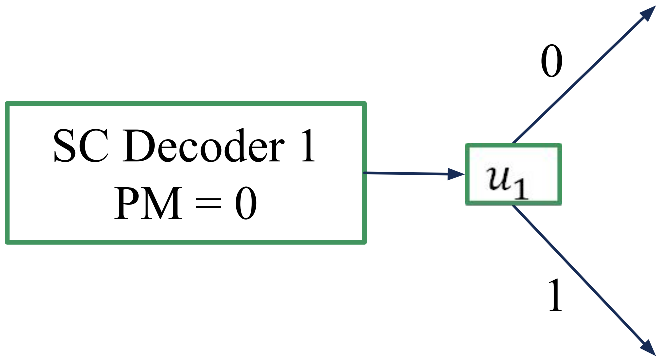
$\hat{u}_i$ = 1 has DM_i = |L(u_i)|;

if L$(u_i) < 0$: $\hat{u}_i$ = 1 has DM_i = 0,

- $\hat{u}_i$ = 0 has DM_i = |L(u_i)|;

Path Metric: Sum of decision metrics on a path of choices.

SUCCESSIVE CANCELLATION LIST (for N=4)



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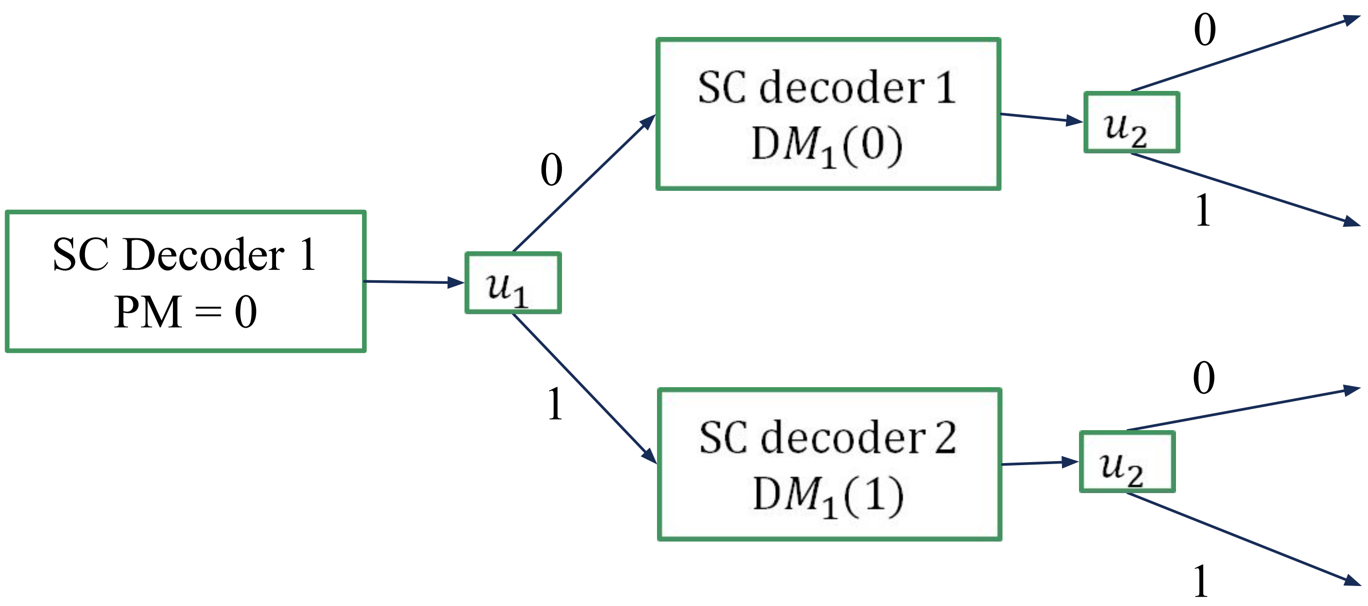

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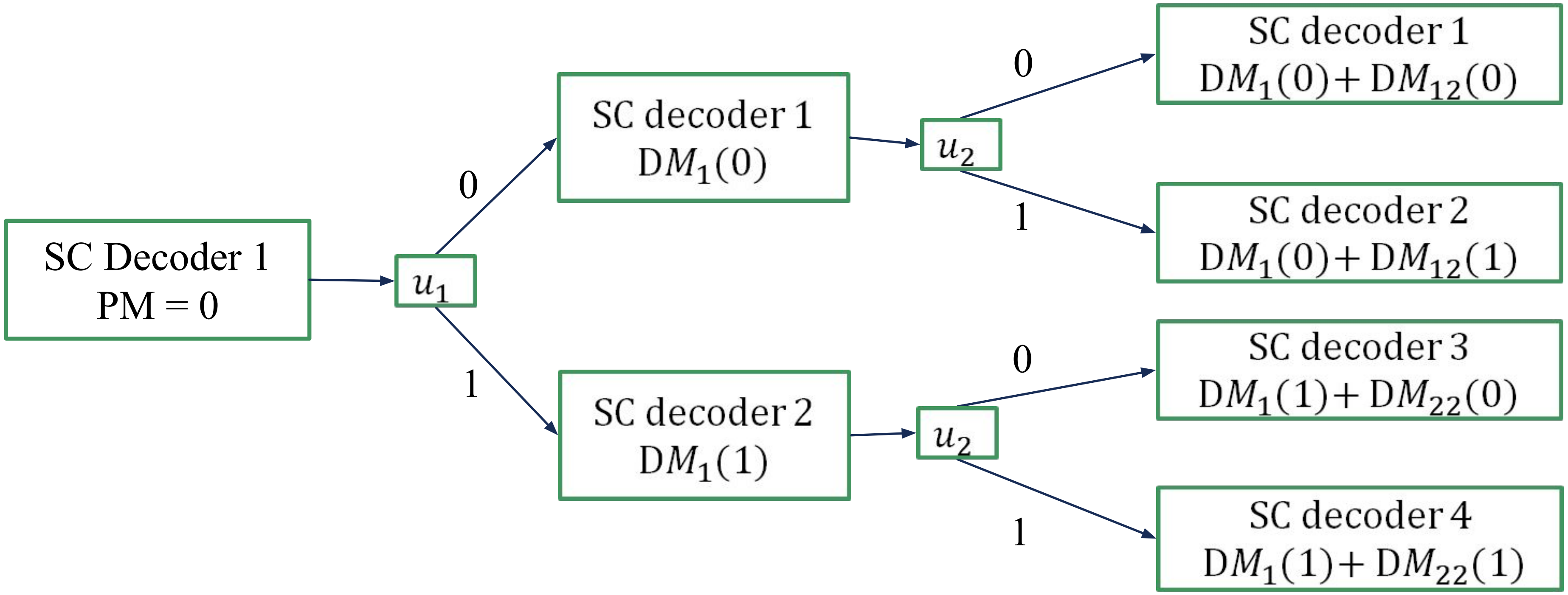
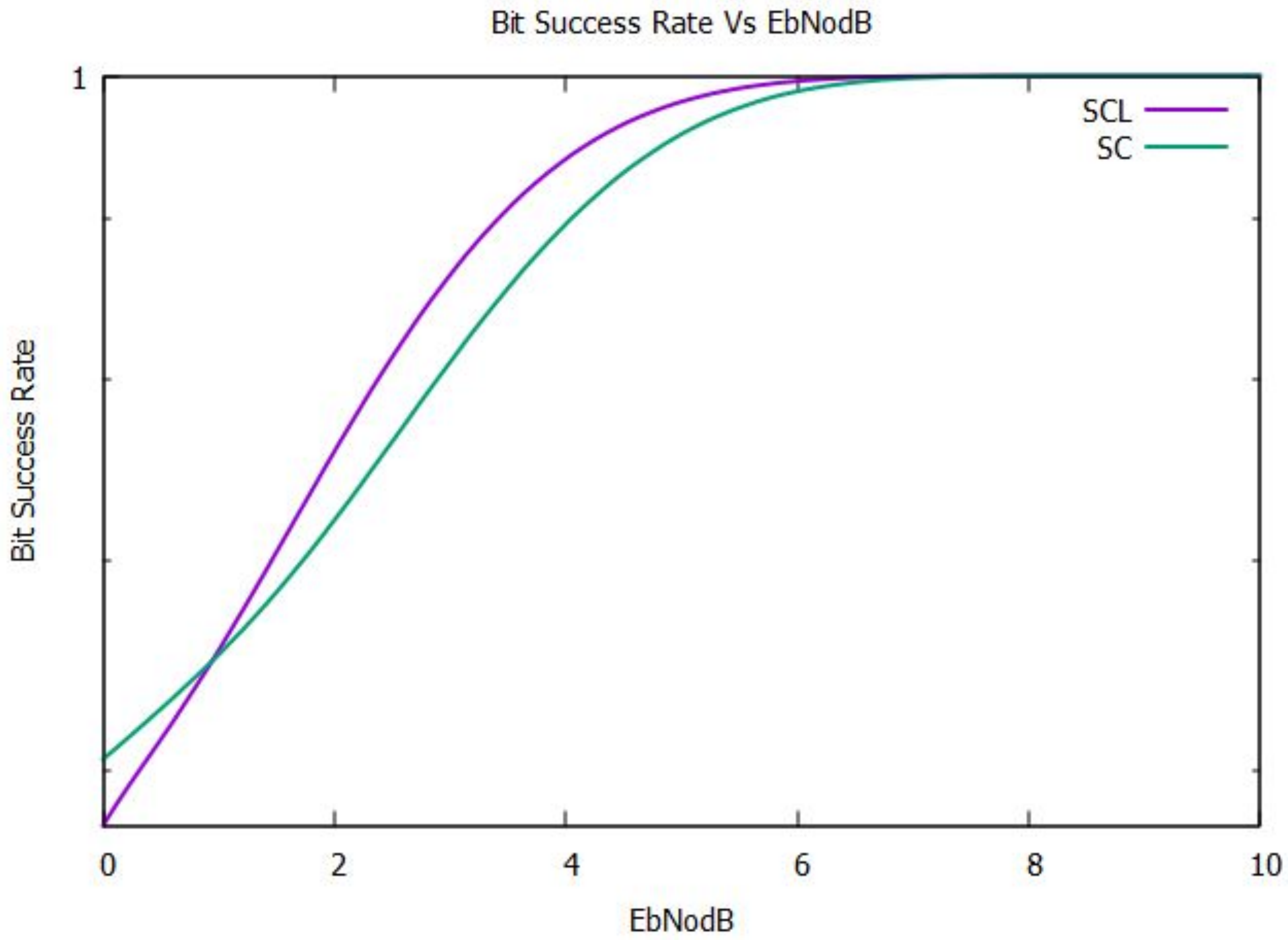


❖ Bit success rate: SCL vs SC



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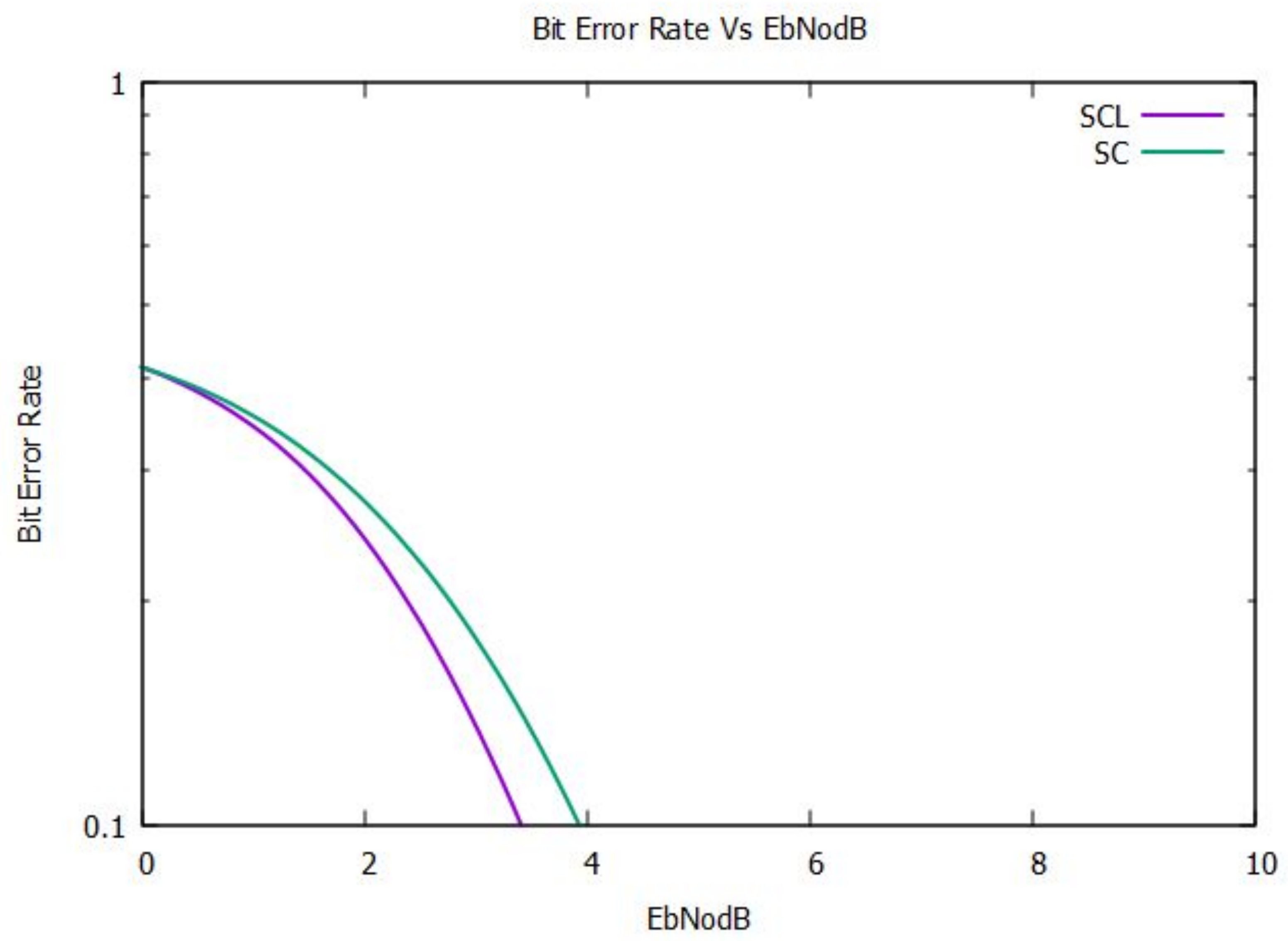
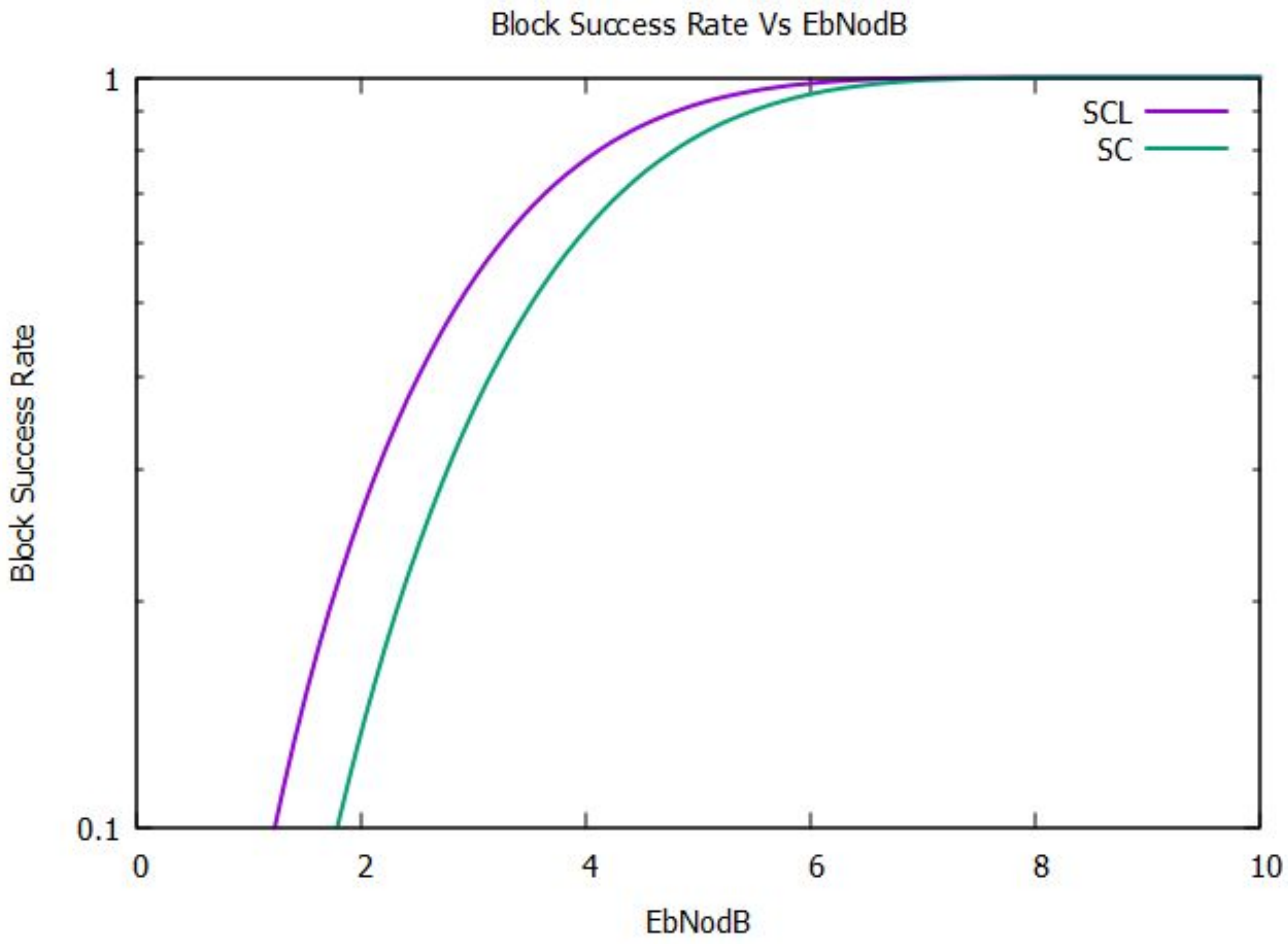


❖ Block success rate: SCL vs SC



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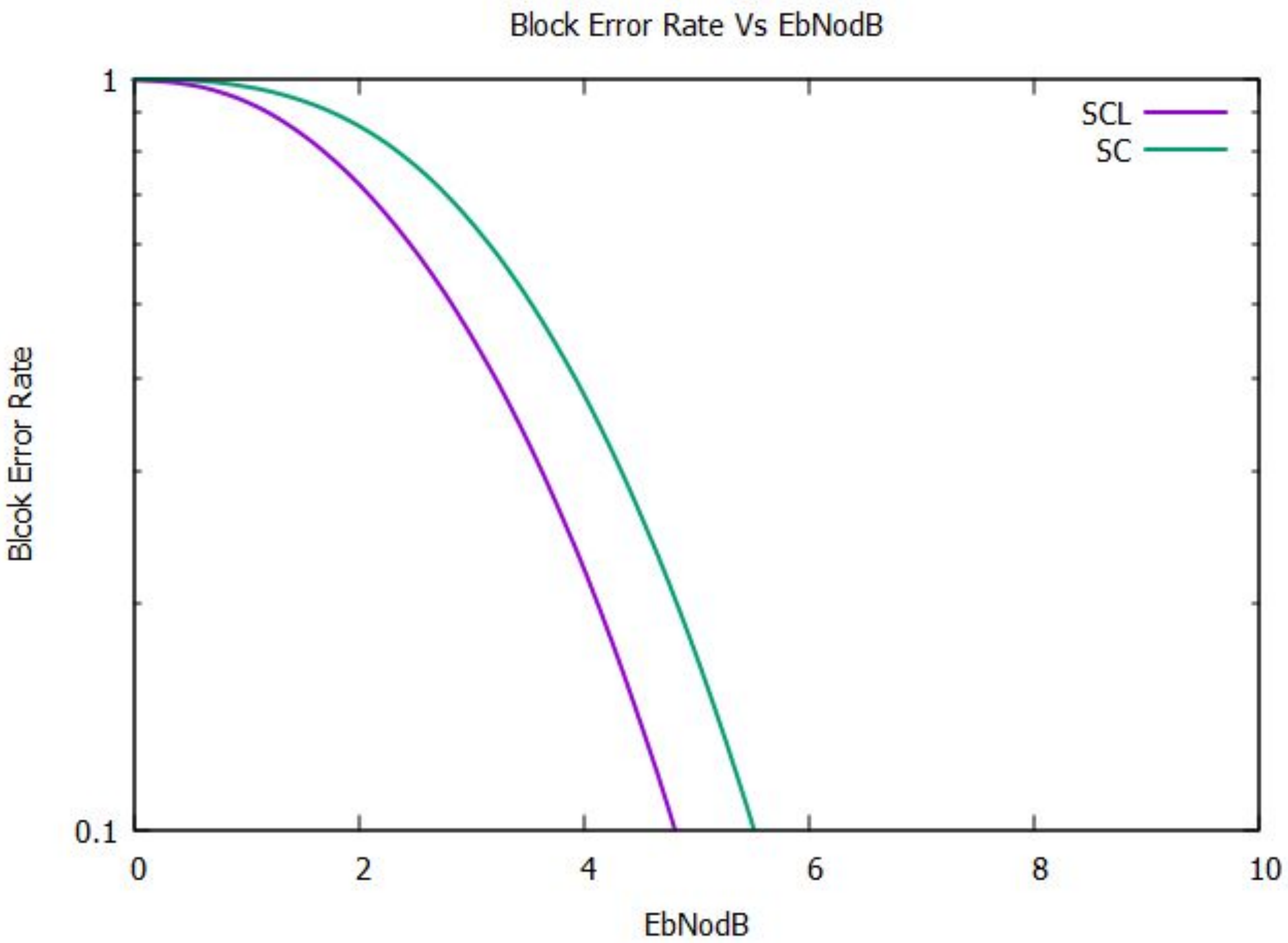

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