5ELEN018W - Tutorial 9 Exercises

1 Finishing all the Exercises from Tutorial 8

Based on the last tutorial sessions, none of the students finished all the tutorial exercises.

Make sure that you finish them during the tutorial of this Week as well as all of the exercises since the beginning of the module.

2 The Inverted Pendulum Control Problem

The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies. It is implemented with the pivot point mounted on a cart that can move horizontally with the usage of a force F produced by a motor.

The objective is to control the pendulum so that it is in its upright position at all times ($\theta = 0$).

The problem is shown in Figure 1.

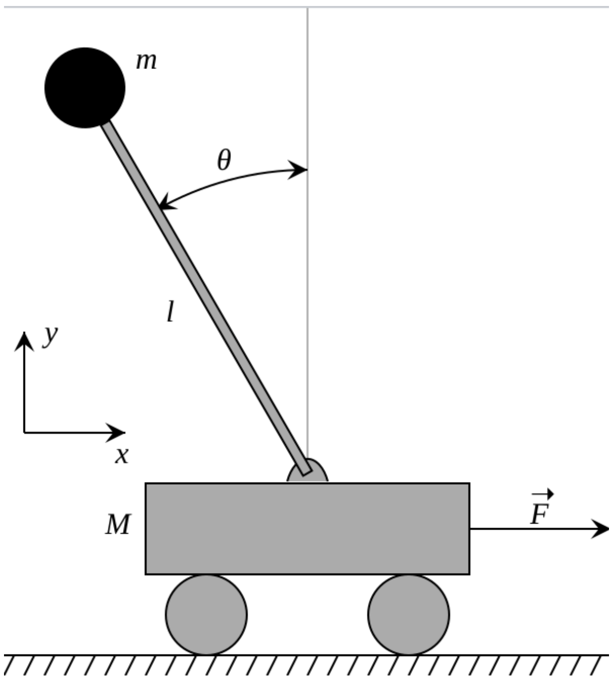

Figure 1: The inverted pendulum control problem.

The equations of motion for the system are:

$$(M + m)\,\ddot{x} + m\ell\cos\theta\,\ddot{\theta} - m\ell\dot{\theta}^2\sin\theta = F - b_c\dot{x}, \tag{1}$$

$$m\ell\cos\theta\,\ddot{x} + m\ell^2\,\ddot{\theta} - mg\ell\sin\theta = -b_p\dot{\theta}. \tag{2}$$

where

- Cart mass: $M = 1kg$
- Pole mass: $m = 0.1kg$
- Pole length: $l = 0.5m$
- Gravity: $g = 9.81m/s^2$
- Actuator (force) limits (typical): $F_{min} = -10N, F_{max} = +10N$
- Viscous friction on cart: $b_c = 0.10N \cdot s/m$
- Viscous friction at the pendulum pivot: $b_p = 0.01N \cdot m \cdot s/rad$

1. Model the dynamic system using Simulink.
2. Attempt to control the system use a PID controller. Can you find a set of parameters which solves the balancing problem?
3. Plot the Bode diagrams for the system based on your PID controller. What can you say about the stability of the system?