5ELEN018W - Robotic Principles
Lecture 5: Inverse Kinematics

Dr Dimitris C. Dracopoulos

The problem of Forward kinematics

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where $\theta_i, \alpha_i, r_i, d_i$ are the DH parameters.

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The problem of forward kinematics is expressed as the calculation of the transformation between a coordinate frame fixed in the *end-effector* and another coordinate frame fixed in the base. For example, for 6-joint manipulator:

$$ {}^{0}\boldsymbol{T}_6 = {}^{0}\boldsymbol{T}_1 \cdot {}^{1}\boldsymbol{T}_2 \cdot {}^{2}\boldsymbol{T}_3 \cdot {}^{3}\boldsymbol{T}_4 \cdot {}^{4}\boldsymbol{T}_5 \cdot {}^{5}\boldsymbol{T}_6 \qquad (2) $$

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3. It is possible that no solutions exist

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- Surgery

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- Surgery
- Search and rescue

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- They are not general, but robot dependent.
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- As the number of joints increases, this becomes increasingly difficult.
- For some serial-link robot manipulators, no analytical (closed form) solution exists!

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- Algebraic methods

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- Geometric methods

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Solution:

$$\theta_i = 2\tan^{-1}\left(\frac{C_2 \pm \sqrt{C_2^2 - C_3^2 + C_1^2}}{C_1 - C_3}\right)$$

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only one solution:

$$\theta_i = \operatorname{atan2}(-C_1 C_4 - C_2 C_3, C_2 C_4 - C_1 C_3)$$

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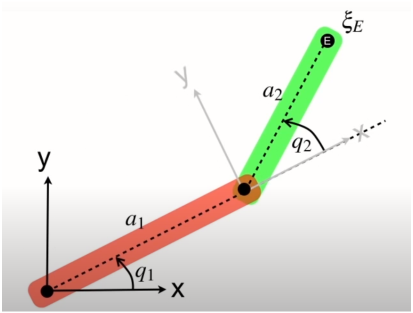
- Decomposition of the full problem into inverse position kinematics and inverse orientation kinematics.
- The solution is derived by rewriting equation (2) as:

$$^0\boldsymbol{T}_6 \cdot {}^6\boldsymbol{T}_5 \cdot {}^5\boldsymbol{T}_4 \cdot {}^4\boldsymbol{T}_3 = {}^0\boldsymbol{T}_1 \cdot {}^1\boldsymbol{T}_2 \cdot {}^2\boldsymbol{T}_3 \tag{3}$$

Calculating Analytical Solutions in the Python Robotics Toolbox

Consider a 2-joint Planar (2D) Robot



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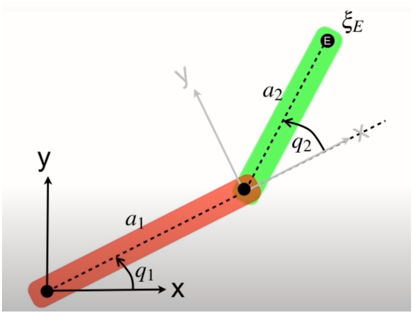

Given the position of the end-effector (x_E, y_E) calculate the required joint angles to achieve this position.

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# create symbols for lengths of the 2 links
>>> a1 = Symbol('a1')
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# calculate forward kinematics matrix for end-effector
>>> TE = e.fkine([q1, q2])

# translation part of matrix gives the position (x_fk, y_fk) of the
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# x_fk = x and y_fk = y
# then x_fk**2 + y_fk**2 - x**2 - y**2 = 0
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>>> eq1 = (x_fk**2 + y_fk**2 - x**2 - y**2).trigsimp()
```

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>>> print(q2_sol)
[-acos(-(a1**2 + a2**2 - x**2 - y**2)/(2*a1*a2)) + 2*pi,
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# expand the two equations x_fk=x,  y_fk=y
>>> eq2 = tuple(map(sympy.expand_trig, [x_fk - x,  y_fk - y]))
>>> print(eq2)
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(a1*cos(q1) + a2*(-sin(q1)*sin(q2) + cos(q1)*cos(q2)) - x,
a1*sin(q1) + a2*(sin(q1)*cos(q2) + sin(q2)*cos(q1)) - y)

# solve for sin(q1), cos(q1)
>>> q1_sol = sympy.solve(eq2, [sympy.sin(q1), sympy.cos(q1)])
>>> print(q1_sol)  # dictionary containing sin(q1) and cos(q1)
```

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# tan(q1) = sin(q1)/cos(q1)
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>>> print(q1_sol[sin(q1)]/q1_sol[cos(q1)])

# solve for q1
>>> sympy.atan2(q1_sol[sin(q1)], q1_sol[cos(q1)]).simplify()
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How? Minimise the error between the forward kinematics solution and the desired end-effector pose $\boldsymbol{\xi}_E$:

$$\boldsymbol{q}^* = \arg\min_{\boldsymbol{q}} (FK(\boldsymbol{q}) - \boldsymbol{\xi}_E)$$

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Various classical numerical methods can be applied, including among others:

- Newton-Raphson: first order approximation of original equations
- Levenberg–Marquardt optimisation: using the second order derivative for the approximation of the original system.

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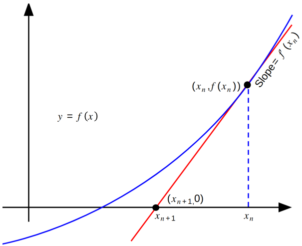

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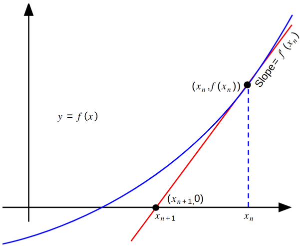

Then the approximated solution for finding the root of f (where $f(x) = 0$) can be calculated iteratively by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{5}$$

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# define the error (E) function
>>> def E(q):
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Other topics/issues in Robotics

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