

# 6ELEN018W - Applied Robotics

## Lecture 9: Robot Control - Intelligent Control Algorithms - Part II

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- ▶ Transitions (dynamics): how the system moves from the current state  $s_t$  to the next state  $s_{t+1}$ .
- ▶ The transitions satisfy the first order Markov property:

$$P(s_{t+1}|s_t, s_{t-1}, \dots, s_0) = P_1(s_{t+1}|s_t) \quad (1)$$

## Markov Chains (cont'd)

Markov chains define a stochastic system which generates a sequence of states:

$$s_0 \longrightarrow s_1 \longrightarrow s_2 \longrightarrow \dots$$

where  $s_0$  is drawn from  $P_0(s_0)$  and each  $s_{t+1}$  from one step transition probabilities  $P_1(s_{t+1}|s_t)$ .

- ▶ A Markov chain can be represented as a state transition diagram.

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- ▶ Since  $p_{ij}$  are probabilities:

$$0 \leq p_{ij} \leq 1 \quad (3)$$

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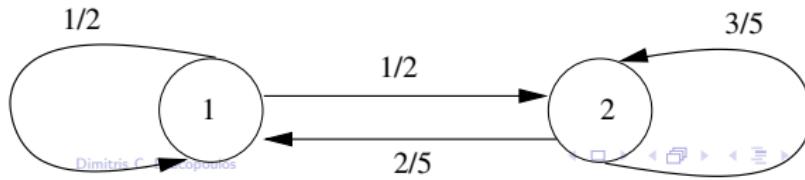
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$$P = [p_{ij}] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix} \quad (4)$$



# Markov Chain Problems

- ▶ *Prediction:* Probabilities that the system will be in state  $s_k$  after  $n$  transitions, given that at  $n = 0$  is it in a known state.
- ▶ *Estimation:* Calculation of transition probabilities given some observed sequences of state transitions.

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$$\pi_j(n+1) = \sum_{i=1}^N \pi_i(n)p_{ij} \quad n = 0, 1, 2, \dots \quad (6)$$

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In general:

$$\pi(n) = \pi(0)P^n \quad n = 0, 1, 2, \dots \quad (9)$$

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After 1 week the robot maker is equally likely to be successful or unsuccessful.

After 2 weeks:

$$\pi(2) = \pi(1)P = \left[ \frac{1}{2} \ \frac{1}{2} \right] \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{5} & \frac{3}{5} \end{bmatrix} = \left[ \frac{9}{20} \ \frac{11}{20} \right]$$

so that the robot maker is slightly more likely to be unsuccessful.

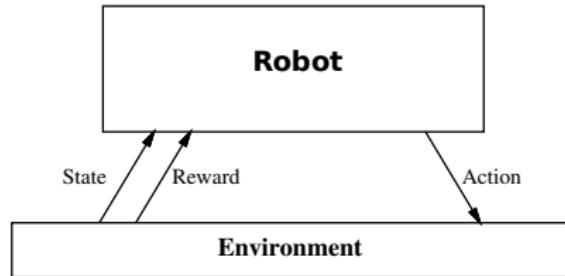
## Example: Successive State Probabilities Starting with a Successful Robot

$n$	0	1	2	3	4	5	...
$\pi_1(n)$	1	0.5	0.45	0.445	0.4445	0.44445	...
$\pi_2(n)$	0	0.5	0.55	0.555	0.5555	0.55555	...

As  $n$  becomes very large:

- ▶  $\pi_1(n)$  approaches  $\frac{4}{9}$
- ▶  $\pi_2(n)$  approaches  $\frac{5}{9}$

# The Reinforcement Learning Problem for a Robot



$$s_0 \xrightarrow[a_0]{r_0} s_1 \xrightarrow[a_1]{r_1} s_2 \xrightarrow[a_2]{r_2} \dots$$

Goal: Learn to choose actions that maximise:

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \dots,$$

where  $0 \leq \gamma < 1$

# Robot's Learning Task

Execute actions in environment, observe results, and

- ▶ learn action policy  $\pi : S \rightarrow A$  that maximises

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots]$$

from any starting state in  $S$

- ▶ here  $0 \leq \gamma < 1$  is the discount factor for future rewards

## Value Function

*How can a robot calculate the optimum action at each state?*

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$$\begin{aligned} V^\pi(s) &\equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots \\ &\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i} \end{aligned}$$

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Now, the task is to learn the optimal policy  $\pi^*$ :

$$\pi^*(s) = \arg \max_a [r(s, a) + \gamma V^*(\delta(s, a))]$$

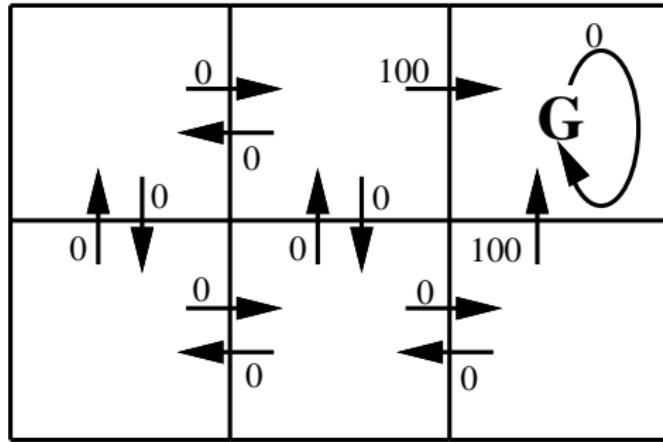


Figure 1:  $r(s, a)$  (immediate reward) values.

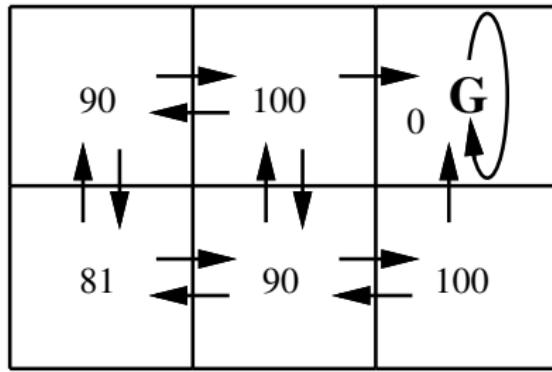


Figure 2:  $V^*(s)$  values for  $\gamma = 0.9$ .

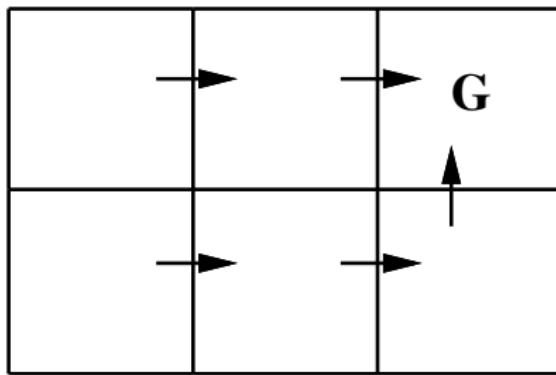


Figure 3: One optimal policy.

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How to do iteratively? Update the  $V$  for only greedy moves according to the formula:

$$V(S_t) \leftarrow V(S_t) + \alpha[V(S_{t+1}) - V(S_t)] \quad (10)$$

where  $\alpha$  is a small positive number (in the range between 0 and 1), which affects the rate of learning.

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# $\epsilon$ -Greedy Methods for Exploration vs Exploitation

- ▶ To make sure that we explore while we exploit as well,  $\epsilon$ -greedy actions can be applied:
- ▶ Most of the time a greedy action is selected (i.e. the one leading to the maximum  $V$  value estimated so far).
- ▶ With probability  $\epsilon$  we apply an action which is selected randomly from all the actions (**including the greedy action**) with equal probability.

# $Q$ Function - An Alternative to choose Robot Actions

Define new function very similar to  $V^*$

$$Q(s, a) \equiv r(s, a) + \gamma V^*(\delta(s, a))$$

If agent learns  $Q$ , it can choose optimal action even without knowing  $\delta$ !

# $Q$ Function - An Alternative to choose Robot Actions

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If agent learns  $Q$ , it can choose optimal action even without knowing  $\delta$ ! (the function which describes the transition between the current state and the next one if the robot takes a specific action)

$$\pi^*(s) = \arg \max_a [r(s, a) + \gamma V^*(\delta(s, a))]$$

$$\pi^*(s) = \arg \max_a Q(s, a)$$

$Q$  is the evaluation function the agent will learn

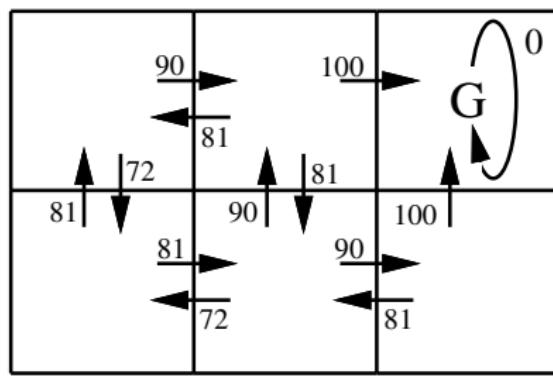


Figure 4:  $Q(s, a)$  values for the grid problem previously seen.

## Training Rule to Learn $Q$

Note  $Q$  and  $V^*$  closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write  $Q$  recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t)) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

Let  $\hat{Q}$  denote learner's current approximation to  $Q$ . Consider training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

where  $s'$  is the state resulting from applying action  $a$  in state  $s$ .

# *Q* Learning Pseudocode for Deterministic Worlds

For each  $s, a$  initialise table entry  $\hat{Q}(s, a) \leftarrow 0$

Observe current state  $s$

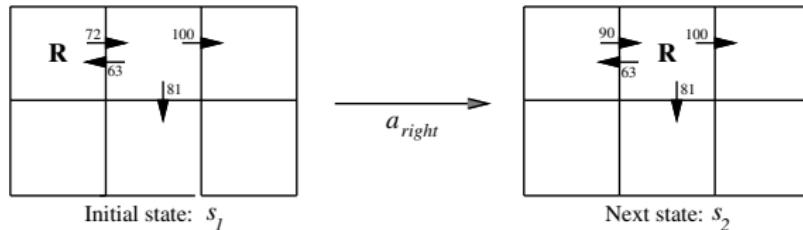
Do forever:

- ▶ Select an action  $a$  and execute it
- ▶ Receive immediate reward  $r$
- ▶ Observe the new state  $s'$
- ▶ Update the table entry for  $\hat{Q}(s, a)$  as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

- ▶  $s \leftarrow s'$

## Updating $\hat{Q}$



$$\begin{aligned}\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\ &\leftarrow 0 + 0.9 \max\{63, 81, 100\} \\ &\leftarrow 90\end{aligned}$$

notice if rewards non-negative, then

$$(\forall s, a, n) \quad \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n) \quad 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

## Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine  $V, Q$  by taking expected values

$$\begin{aligned} V^\pi(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots] \\ &\equiv E\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i}\right] \\ &\equiv E[r_t + \gamma V^\pi(s+1)] \end{aligned} \tag{11}$$

$$Q(s, a) \equiv E[r(s, a) + \gamma V^*(\delta(s, a))]$$

## Nondeterministic Case

$Q$  learning generalises to nondeterministic worlds

Alter training rule to

$$\hat{Q}_n(s, a) \leftarrow (1 - \alpha_n)\hat{Q}_{n-1}(s, a) + \alpha_n[r + \max_{a'} \hat{Q}_{n-1}(s', a')]$$

where

$$\alpha_n = \frac{1}{1 + visits_n(s, a)}$$

Can still prove convergence of  $\hat{Q}$  to  $Q$ .

# Temporal Difference Learning

$Q$  learning (TD(0) algorithm): reduce discrepancy between successive  $Q$  estimates

One step time difference:

$$Q^{(1)}(s_t, a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1}, a)$$

Why not two steps?

$$Q^{(2)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2}, a)$$

Or  $n$ ?

$$Q^{(n)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \cdots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n}, a)$$

Blend all of these:

$$Q^\lambda(s_t, a_t) \equiv (1-\lambda) \left[ Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots \right]$$

# Families of Reinforcement Learning Algorithms

1. *Dynamic Programming*: based on Bellman equation (11), well developed mathematically, but require a complete and accurate model of the environment.
2. *Monte Carlo Methods*: do not require a model but not appropriate for step-by-step incremental learning.
3. *Temporal Difference Methods* (e.g. Q-learning (which is the TD(0) algorithm), temporal difference learning): require no model, they are fully incremental, but are more complex to analyse.

## Other Improvements on what was discussed

- ▶ Extend to continuous action, state: Replace  $\hat{Q}$  table with neural net or other generaliser
- ▶ Learn and use  $\hat{\delta} : S \times A \longrightarrow S$