6ELEN018W Applied Robotics - Sample In-Class Test Questions

Formatting of this document is not important as the actual test will take place in Blackboard. There are more questions here than the actual test, in order to help the revision and practice of students.

Question 1

Briefly describe what is the pose of a robot.

Correct answer

The position and orientation of the robot.

Question 2

What is true about the following homogeneous transformation matrix in the 3D space?

$$\begin{pmatrix} cos\frac{\pi}{2} & -sin\frac{\pi}{2} & 0 & v_1 \\ sin\frac{\pi}{2} & cos\frac{\pi}{2} & 0 & v_2 \\ 0 & 0 & 1 & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

A: This is a transformation of a rotation of 90^o about the x-axis with translation afterwards

B: This is a transformation of a rotation of 180^o about the y-axis with translation afterwards

C: This is a transformation of a rotation of 90^o about the z-axis with translation afterwards

D: This is a transformation of a translation followed by a rotation about some axis afterwards

(8 marks)

Correct answer: C

Question 3

Which of the following are valid and which ones are invalid? These correspond to a homogeneous transformation of to a rotation followed by a translation in the 3-D space? Justify why each of the invalid choices is not correct.

1.

$$\begin{pmatrix} cos(5\pi) & -sin(5\pi) & 0 & v_1 \\ sin(5\pi) & cos(5\pi) & 0 & v_2 \\ 0 & 0 & 1 & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

2.

$$\begin{pmatrix} 0 & 0 & 1 & v_1 \\ cos(\pi) & -sin(\pi) & 0 & v_2 \\ sin(\pi) & cos(\pi) & 0 & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

3.

$$\begin{pmatrix} cos(\pi) & 0 & -sin(\pi) & v_1 \\ 0 & 1 & 0 & v_2 \\ sin(\pi) & 0 & cos(\pi) & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

4.

$$\begin{pmatrix} cos(\pi) & -sin(\pi) & 0 & v_1 \\ 0 & 1 & 0 & v_2 \\ sin(\pi) & cos(\pi) & 0 & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

5.

$$\begin{pmatrix} cos(\pi) & 0 & sin(\pi) & v_1 \\ 0 & 1 & 0 & v_2 \\ -sin(\pi) & 0 & cos(\pi) & v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6}$$

(10 marks)

Sample answer:

Only options 1 and 5 are valid. Option 2 is invalid because the first row (0,0,1) indicates a rotation about z but this should be in the third row. Option 3 is invalid because of the signs of the two sin functions. They should be reversed. Option 4 is invalid because the second column elements with sin, cos should be located in the third row and also the signs reversed.

Question 4

Give a definition for a rigid body.

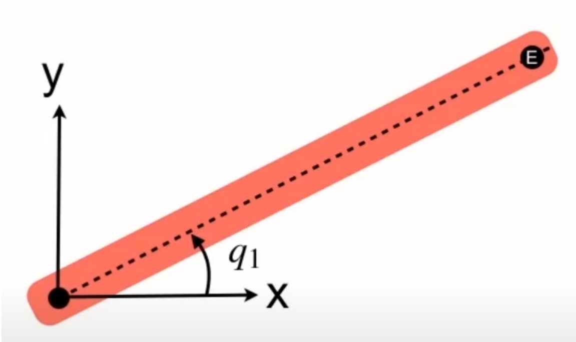
Sample Answer

A solid body which cannot change shape whatever we do to it.

Question 5

Consider the 1-joint robot in the 2D-space shown below.

The length of the single link is a_1.



1. Describe the series of basic transformations needed to calculate the pose of the end-effector of the robot ξ_E using the following notation:

   $R(\theta)$ describes the rotation matrix about angle θ, `T_x(a)` describes a translation along the x axis by a units and `T_y(a)` describes a translation along the y axis by a units. *(5 marks)*

2. Without calculating the overall result (i.e. without making the actual matrix multiplications), write down the full form of each homogeneous matrix corresponding to the basic transformations needed above, using variables q_1, a_1 and trigonometric functions. *(5 marks)*

Correct answer:

1.

$$E = \boldsymbol{R}(q_1) \cdot \boldsymbol{T}_x(a_1)$$

2.

$$E = \begin{pmatrix} cos(q_1) & -sin(q_1) & 0 \\ sin(q_1) & cos(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & a_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Question 6

Consider the following homogeneous transformation matrix:

$$\begin{pmatrix} 0.0000 & -1.0000 & 0 & 1.0000 \\ -1.0000 & 0.0000 & 0 & 3.0000 \\ 0 & 0 & 1.0000 & 2.0000 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

Calculate the inverse transformation and justify your choice (or alternatively show your calculations):

A:

$$\begin{pmatrix} -0.0000 & -1.0000 & 0 & 3.0000 \\ -1.0000 & -0.0000 & 0 & 1.0000 \\ 0 & 0 & 1.0000 & -2.0000 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

B:

$$\begin{pmatrix} 0.0000 & -1.0000 & 0 & 1.0000 \\ -1.0000 & 0.0000 & 0 & 3.0000 \\ 0 & 0 & 1.0000 & 2.0000 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

C:

$$\begin{pmatrix} -1.6331 & -0.0000 & 0 & 0.0000 \\ 0.0000 & -1.6331 & 0 & 0.0000 \\ 0 & 0 & 1.0000 & 1.0000 \\ 0 & 0 & 0 & 0.5 \end{pmatrix}$$

D:

$$\begin{pmatrix} -0.0000 & -1.0000 & 0 & 1.5 \\ -1.0000 & -0.0000 & 0 & 0.5 \\ 0 & 0 & 1.0000 & -1.0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix}$$

(8 marks)

Correct answer: A

Question 7

Which of the following situations poses an ethical issue relevant to robotics?

A: Unemployment

B: Financial gains due to robot traders

C: Robots performing a surgical operation successfully.

D: Robots creating works of art.

(5 marks)

Correct answer: A

Question 8

Describe briefly what is homogeneous transformation matrix.

(5 marks)

Sample answer

A matrix which combines a rotation followed by a translation. It is a 3x3 for the 2D space and a 4x4 in the 3D space.

Question 9

Write a Python function which accepts 2 matrices (represented as lists) as arguments corresponding to two consecutive rotations in the 2D space. The function returns matrix corresponding to the overall transformation, i.e. the combination of both rotations together.

(10 marks)

Sample answer

Implement this on your own and in Python as a function. You just need to multiply the 2 arguments passed to the function and return it as the result of the function. The arguments could be lists which are converted to `numpy` arrays. You should use the symbol `@` for multiplication of `numpy` arrays.

Question 10

Describe how the Jacobian matrix relates the angular velocities of the joints with the velocity of the end-effector. Give the relevant formula(s).

Sample Answer:

See the lecture slides.

Question 11

A point P with coordinates $[7, 3, 1]^T$ is attached to the end-effector of a robot, a moving frame F, which is subject to the following 3 successive transformations relative to the reference fixed world frame F_{xyz}:

1. Rotation of 90^o about the x-axis.
2. Followed by a rotation of -90^o about the z-axis.
3. Followed by a translation of $[4,\ -3,\ 7]$.

Write some Python code which calculates the coordinates of the point relative to the reference frame after all the transformations have taken place.

(15 marks)

Sample answer:

The transformations are relative to the fixed world frame (not the robot's moving frame) therefore the order of the matrices written need to be reversed.

```python
import numpy as np
from math import *

P = [7, 3, 1, 1]  # 1 added to make calculations valid

# angle 90 degrees
a = radians(pi/2)
tr1 = [[1,    0,       0,    0],
       [0, cos(a), -sin(a), 0],
       [0, sin(a), cos(a),  0],
       [0, 0, 0, 1]]

tr2 = [[cos(-a), -sin(-a), 0, 0],
       [sin(-a), cos(-a), 0, 0],
       [0,        0,       1,  0],
       [0, 0, 0, 1]]
```

```
tr3 = [[1, 0, 0, 4],
       [0, 1, 0, -3],
       [0, 0, 1, 7],
       [0, 0, 0, 1]]

P = np.array(P)
tr1 = np.array(tr1)
tr2 = np.array(tr2)
tr3 = np.array(tr3)

result = tr3@tr2@tr1@P
print(result)
```

Question 12

A point P with coordinates $[7, 3, 1]^T$ is attached to the end-effector of a robot, a moving frame F, which is subject to the following 3 successive transformations relative to the moving frame attached to the robot F_{xyz}:

1. Rotation of 90^o about the x-axis.
2. Followed by a rotation of -90^o about the z-axis.
3. Followed by a translation of $[4,\ -3,\ 7]$.

Write some Python code which calculates the coordinates of the point relative to the reference frame after all the transformations have taken place.

(15 marks)

Sample answer:

The transformations are relative to the robot's moving frame therefore the order of the matrices written need to be in the same sequence that they occur.

```
import numpy as np
from math import *

P = [7, 3, 1, 1]  # 1 added to make calculations valid

# angle 90 degrees
a = radians(pi/2)
tr1 = [[1,    0,       0,    0],
       [0, cos(a), -sin(a), 0],
       [0, sin(a), cos(a),  0],
       [0, 0, 0, 1]]

tr2 = [[cos(-a), -sin(-a), 0, 0],
       [sin(-a), cos(-a), 0, 0],
       [0,       0,       1,  0],
       [0, 0, 0, 1]]
```

```
tr3 = [[1, 0, 0, 4],
       [0, 1, 0, -3],
       [0, 0, 1, 7],
       [0, 0, 0, 1]]

P = np.array(P)
tr1 = np.array(tr1)
tr2 = np.array(tr2)
tr3 = np.array(tr3)

result = tr1@tr2@tr3@P
print(result)
```

Question 13

Describe the Newtonian mechanics method and the Euler-Lagrange method for the calculation of the dynamic equations of motion for a robot, subject to (generalised) forces, i.e. linear forces and angular forces. What are the advantages/disadvantages of each method?

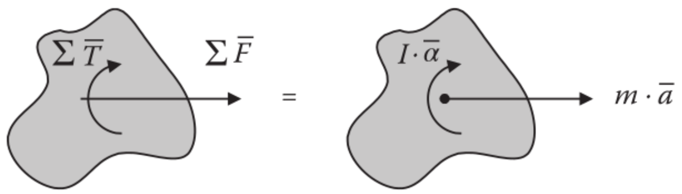
(10 marks)

Sample answer

Newtonian based on the Newton's 3 laws.

- **Newtonian Mechanics**
  - Easier for simpler systems.
  - More familiar for some people
  - Calculate the sum of all linear forces and sum of all torques:

$$\sum \bar{F} = m\bar{a}, \quad \sum \bar{T} = I\bar{\alpha}$$



1. Lagrangian is the difference between kinetic and potential energies of the system.

$$\mathcal{L} = \mathcal{K} - \mathcal{P} \tag{7}$$

   To calculate the dynamic equations of motion for a system such as a serial-link robot according to the Euler Lagrange method: write the kinetic and potential energies of the system in terms of a set of *generalised coordinates* $(q_1, q_2, \ldots q_n)$ where n is the degrees of freedom of the system. q_k can be linear distances or angles:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_k} - \frac{\partial \mathcal{L}}{\partial q_k} = \tau_k, \quad k = 1, \ldots, n$$

   where τ_k is the (generalised) force (linear force or torque) associated with q_k

2. • Easier for more complicated systems.
   • Based on system's energies
   • Systematic approach

Question 14

Describe how would you calculate the velocity of the end-effector of a 2-D robot manipulator given its joints angles.

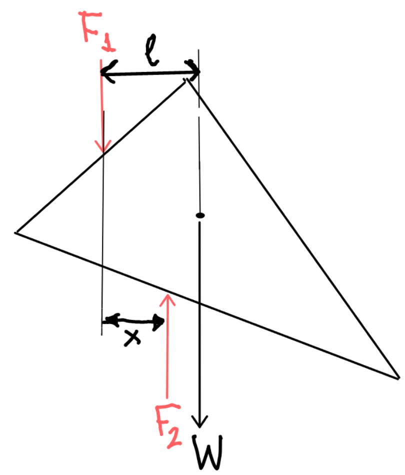
(8 marks)

Sample answer:

We can solve the forward kinematics problem and find the position $(x, y$ of the end-effector of the robot as a function of its joint angles. Then we can take the derivative of $(x, y$ to calculate the velocity (speed) of the end-effector.

Question 15

Consider a robot which tries to hold an object of a rectangular shape with its 2 fingers. The weight force W is applied at the centre of mass. The 2 fingers apply 2 forces F_1 and F_2 at the top and the bottom of the object respectively and these forces are at a distance x as shown in the figure below.



1. Describe the necessary condition(s) so that the rectangular rigid body block reaches equilibrium (it does not move). Give the formulas required in order to achieve this, based on the problem parameters shown in the figure. *(6 marks)*
2. Derive the equations for forces F_1, F_2 which are required to achieve the rigid body equilibrium. Show all the steps of your derivation. *(9 marks)*

Sample answer:

1. Apply the equilibrium principles for a rigid body, balance both forces and torques:

   To balance the forces:

$$F_2 = F_1 + W$$

   To balance the torques:

$$F_1 * l - F_2 * (l - x) = 0$$

2. Solve this system of equations for the forces of the 2 robot fingers.

Solution:

$$F_1 + W - F_2 = 0 \Rightarrow F_1 = F_2 - W$$

$$F_1 * \ell - F_2 * (\ell - x) = 0 \quad (2)$$

$$(2) \xrightarrow{(1)} (F_2 - W) * \ell - F_2 * (\ell - x) = 0 \Rightarrow$$

$$F_2 * \ell - W * \ell - F_2 * \ell + F_2 * x = 0$$

$$\Longrightarrow$$

$$\cancel{F_2 * \ell} - W * \ell - \cancel{F_2 * \ell} + F_2 * x = 0$$

$$\Longrightarrow$$

$$-W * \ell + F_2 * x = 0 \Rightarrow$$

$$\boxed{\left.\begin{aligned} F_2 &= \frac{W * \ell}{x} \quad (3) \\ \xrightarrow{(1)} F_1 &= \frac{W * \ell}{x} - W \quad (4) \end{aligned}\right\}}$$

1) What is the industry which robots have currently the most widely usage?

Choose all that apply:

A. Health
B. Manufacturing
C. Army
D. Package delivery

2) What is the most common form of a modern robot?

Choose all that apply:

A. Dog
B. Humanoid
C. Manipulator
D. Drone

3) A robotic manipulator consists of?

Choose all that apply:

A. Joints and Links
B. Joints and Rigid Bodies
C. Sensors and Joints and Links and Actuators
D. Little robots
E. A human who manipulates the robot remotely

4) A rigid body is:

A. A robot which has a fixed operation that cannot be altered
B. A robot performing its task perfectly
C. A solid body which cannot change shape whatever we do to it
D. A body of a robot

5) What is the configuration space of a robot?

A. The set of all possible ways to assemble a robot
B. The configuration file that we save the software settings of a programmable robot
C. The combination of all possible positions of all of the points of a robot
D. This question is tough out of my reach

6) What is a revolute joint in a robot?

A. A joint that can rotate
B. A joint that can move in a linear direction
C. A joint that can both move linearly and rotate
D. A joint that moves like a human shoulder

7) What is a prismatic joint in a robot?

A. A joint that can rotate
B. A joint that can move in a linear direction
C. A joint that can both move linearly and rotate
D. A joint that moves like a human shoulder

8) What is *pose* in Robotics?

A. The position and orientation of a robot
B. The position and orientation of one coordinate frame with respect to another reference coordinate frame
C. The angles of the joints of a robot
D. The "looks" of a robot when demonstrating it in a robot fashion show
E. The performance of a robot in a specific task as measured by the corresponding error

9) The rotation matrix:

Choose all that apply:

A. transforms the coordinates of a vector from a new frame to an old frame
B. transforms the coordinates of a vector from an old frame to a new frame
C. transforms the coordinates of a vector from a frame rotated by an angle with respect to another frame with the same origin.
D. transforms the coordinates of a vector from a frame rotated by an angle with respect to another frame with the a different origin.

10) What is true regarding the properties of a rotation matrix?

A. Its inverse matrix is 1
B. Its inverse matrix is equal to itself
C. Its inverse matrix is equal to its transpose
D. Its inverse matrix is equal to its determinant
E. Its inverse matrix is equal to its eigenvectors

11) What is true about the following matrix assuming θ is positive?

$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

A. This is the rotation matrix of a new coordinate frame with an angle θ rotated counter-clockwise with respect to an original frame
B. This is the rotation matrix of a new coordinate frame with an angle θ rotated clockwise with respect to an original frame
C. This is the translation matrix of a new coordinate frame with respect to an original frame
D. I don't have a clue

$f(x)$

12) The homogeneous transform:

$$\begin{pmatrix} cos\theta & -sin\theta & V_x \\ sin\theta & cos\theta & V_y \\ 0 & 0 & 1 \end{pmatrix}$$

represents:

A. Translation only
B. Rotation only
C. Translation followed by rotation
D. Rotation followed by translation
E. Rotation followed by translation and translation followed by rotation. The order of transformations does not matter.

13) What is true about the following homogeneous transformation matrix in the 3D space?

$$\begin{pmatrix} 1 & 0 & 0 & v_x \\ 0 & cos\theta & -sin\theta & v_y \\ 0 & sin\theta & cos\theta & v_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A. This is a transformation of a rotation about the y-axis with translation afterwards
B. This is a transformation of a rotation about the z-axis with translation afterwards
C. This is a transformation of a rotation about the x-axis with translation afterwards
D. This is a transformation of a translation about the x-axis with rotation afterwards

14) A differential equation is an equation which:

Choose all that apply:

A. Involves speed and acceleration and position
B. Involves time-series
C. Involves integrals of a function
D. Involves derivatives of a function

15) Consider the following homogeneous transformation matrix:

$$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Calculate the inverse transformation:

A. $\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

C. $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & -2 & 1 \end{pmatrix}$

D. $\begin{pmatrix} -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

16) Which of the following situations poses an ethical issue relevant to robotics?

A. A human harming another human
B. A human harming an animal
C. A human harming a robot
D. Robots creating works of art

Key:
1. B
2. C
3. B, A, C
4. C
5. C
6. A
7. B
8. B
9. A, C
10. C
11. A
12. D
13. C
14. D
15. A
16. C