Evaluating Deep Learning Uncertainty Measures in Cephalometric Landmark Localization

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Abstract: Cephalometric analysis is a key step in the process of dental treatment diagnosis, planning and surgery. Localization of a set of landmark points is an important but time-consuming and subjective part of this task. Deep learning is able to automate this process but the model predictions are usually given without any uncertainty information which is necessary in medical applications. This work evaluates three uncertainty measures applicable to deep learning models on the task of cephalometric landmark localization. We compare uncertainty estimation based on final network activation with an ensemble-based and a Bayesian-based approach. We conduct two experiments with elastically distorted cephalogram images and images containing undesirable horizontal skull rotation which the models should be able to detect as unfamiliar and unsuitable for automatic evaluation. We show that all three uncertainty measures have this detection capability and are a viable option when landmark localization with uncertainty estimation is required.

1 INTRODUCTION

Cephalometric analysis provides clinicians with the interpretation of the bony, dental and soft tissue structures in patients' dental X-ray images. The analysis results contain relationships between key points in the radiogram. These landmark positions are then used for treatment planning, clinical diagnosis, classification of anatomical abnormalities and for surgery. This procedure is time-consuming if performed manually by experts and high interobserver variability is a significant issue as well. Automatic landmark localization helps to alleviate both of these problems (Wang et al., 2016).

The existing solutions for landmark localization can be classified into knowledge-based, pattern matching-based, statistical learning-based and deep learning-based. Knowledge-based methods automate landmark localization by specifying rules based on expert knowledge (Levy-Mandel et al., 1985). This is problematic since rule complexity increases proportional with image complexity.

Pattern matching-based methods search for a specified pattern within the image. (Cardillo and Sid-Ahmed, 1994) proposed to use template matching and gray-scale morphological operators. (Grau et al., 2001) showed that they can improve detection accuracy by supplementing template matching with edge detection and contour segmentation operators. (Davis and Taylor, 1991) used features extracted from the image to detect a set of candidate positions for landmarks, and then analyzed the spatial relationships among landmarks to select the best candidate points.

Statistical learning-based methods take into account both the local appearance of landmark locations and global constraints specified by some model such as an Active Shape Model (Cootes et al., 1995) or an Active Appearance Model (Cootes et al., 2006). Two public challenges for cephalometric landmark detection were held in 2014 and 2015 at the IEEE ISBI and the solutions were summarized in (Wang et al., 2016). Best-performing methods used random forests for classifying individual landmarks and statistical shape analysis for capturing the spatial relationship among landmarks.

Deep learning-based methods have achieved success in many application domains and their usage in medical image analysis has been consistently increasing since 2015 (Litjens et al., 2017). (Payer et al., 2016) found that convolutional neural networks (CNNs) can be successful in localizing hand landmarks. In the context of cephalometric landmark localization, (Pei et al., 2006) demonstrated the potential of bimodal deep Boltzmann machines and more

recently (Arik et al., 2017) proposed to use deep CNNs in combination with a shape-based model.

Deep learning-based methods show great potential but their shortcoming is that they are usually used as deterministic models providing merely point estimates of predictions and model parameters without any associated measure of uncertainty. Since the models will produce a prediction for any input image, this may lead to situations in which we cannot tell whether the prediction is reasonable or just a random guess (Gal, 2016). That is a problem since information about the reliability of model predictions is a key requirement for their incorporation into the medical diagnostic systems (Widdowson and Taylor, 2016). Deep learning models should thus provide each prediction with an estimate of its uncertainty. This would allow the diagnostic system to distinguish between easy cases which can be handled automatically and problematic ones which may instead be referred to a supervising physician for review.

Models based on probability and uncertainty have been extensively studied in the Bayesian machine learning community. They provide a probabilistic view that offers confidence bounds when performing decision making (Gal, 2016) but usually come with a prohibitive computational cost. To take advantage of the qualities of deep learning models and still have the option of assessing the uncertainty of their predictions, it has been suggested (Gal and Ghahramani, 2016) to recast them as Bayesian models using the popular dropout (Hinton et al., 2012) technique often used for regularization in neural networks. The posterior distribution used by Bayesian models is approximated in deep learning models using Monte Carlo (MC) sampling and model uncertainty is given by the prediction variance of the samples. The MC Dropout method has already been applied in medical imaging applications. (Leibig et al., 2017) used dropout-based uncertainty when diagnosing diabetic retinopathy from fundus images. (Eaton-Rosen et al., 2018) and (Guha Roy et al., 2018) both applied it to semantic segmentation of brain scan images.

Another option for estimating the uncertainty of deep learning models comes from a recent non-Bayesian line of research by (Lakshminarayanan et al., 2017). While ensembles of machine learning models have long been known to increase performance in terms of predictive accuracy, the authors also suggest using the prediction variance of the ensemble members as measure of the ensemble's uncertainty.

While (Gal, 2016) criticized the use of raw model outputs as a measure of uncertainty estimation, that conclusion was not based on experiments conducted on a heatmap regression task (Payer et al., 2016). Since we use that method to localize cephalometric landmarks in this work, we also determine whether a useful uncertainty measure can be derived from the predictions of a CNN trained for the task of heatmap regression.

The contribution of our work is in evaluating the MC Dropout and ensemble methods of estimating deep learning model uncertainty on the cephalometric landmark localization task. To the best of our knowledge, deep learning model uncertainty estimation has not been studied on this task before. We further evaluate whether CNN activations can be used for estimation of landmark uncertainty without multiple forward passes required by other methods. We show that all three uncertainty measures are able to detect out-of-distribution data unsuitable for automatic evaluation. Our experiments also hint at the possibility of applying models trained on X-ray images to 2D CT projections.

2 MATERIALS AND METHODS

2.1 Dataset

The dataset used for the landmark localization experiments was released as a part of the 2014 and 2015 IEEE ISBI challenges (Wang et al., 2016). It consists of 400 cephalograms from 400 subjects. All cephalograms were acquired in the same format and from an identical scanning machine. The resolution of the images is 1935 x 2400 pixels with a pixel spacing of 0.1 mm. Two orthodontists provided ground truth manual annotations of 19 cephalometric landmark positions and we used only the one from the senior physician accuracy evaluation. For consistency with the protocol designed for the competition, we used only 150 images for training and the rest (which is split by the competition authors into split test1 and test2) for evaluation.

2.2 Landmark Localization

We implemented landmark localization using heatmap regression (Payer et al., 2016). In this approach, the landmark positions are not regressed directly as a pair of real coordinates but the model learns to regress a separate heatmap for each landmark instead. For each training example, the CNN receives a single-channel gray-scale image rescaled to $d \times d$ dimensions. The corresponding ground truth is a $19 \times d \times d$ volume of heatmaps. Each heatmap corresponds to a single landmark and contains a

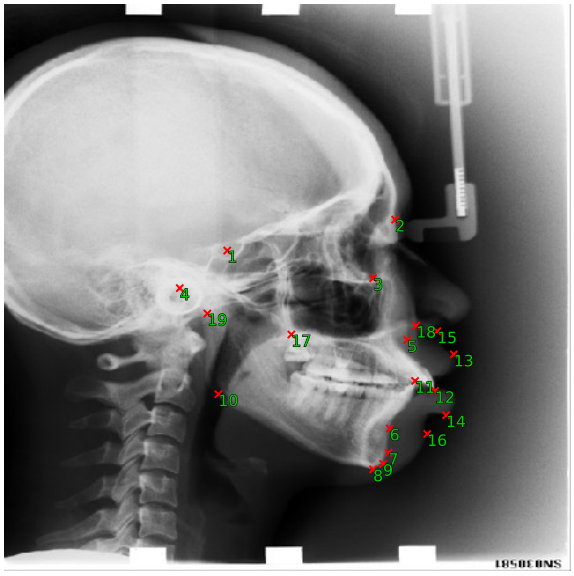

Figure 1: A rescaled image from the 2015 IEEE ISBI challenge dataset with 19 ground truth landmarks visualized.

Gaussian with a fixed variance and amplitude centered on the landmark position as annotated by the physician. The output of the CNN is a $19 \times d \times d$ volume of predicted heatmaps minimizing the mean squared error loss. As a post-processing step, each heatmap is convolved with a Gaussian filter of the same variance as was used when creating the ground truth heatmap. The position of the maximum value in this activation map is chosen as the final predicted landmark position.

2.3 CNN Architecture

The CNN architecture we used closely follows the U-Net (Ronneberger et al., 2015) with some minor modifications. U-Net contains a down-sampling path followed by a symmetric up-sampling path and is designed to be able to learn both global context (relative landmark positions) and local characteristics of each landmark.

The down-sampling path contains 3×3 double convolutions with filter sizes of 64, 128, 256, 512 and 1024, each followed by a 2×2 max pooling layer. Width and height of the feature map are then progressively increased back to the original 128×128 size in the up-sampling path via transpose convolution which halves the filter dimension. Feature map from the corresponding down-sampling level is concatenated to the result and this is followed by a double convolution whose filter size decreases from the bottom level towards the top (1024, 512, 256, 128 and 64). The final double convolution uses 19 filters to produce the prediction heatmaps.

For the model based on Monte Carlo dropout (see Section 2.4.3), dropout layers are added just before each max pooling layer and right after the transposed convolution in the up-sampling path.

2.4 Uncertainty Measures

We train three models, all based on the same CNN architecture. Baseline model uses the activation heatmap produced by the CNN when estimating uncertainty while the Ensemble and MC-Dropout models both use prediction variance of the ensemble members and MC samples respectively.

2.4.1 Maximum Heatmap Activation (MHA)

Baseline is a single CNN without dropout layers. Recall from Section 2.2 that the heatmap predicted by the CNN is convolved with a Gaussian filter as a post-processing step. The position of the maximum value in the activation map produced this way is chosen as the predicted landmark position. The Baseline model additionally uses the maximum activation value (not just the position) as a measure of uncertainty associated with the prediction. We hypothesized that there is an inverse correlation between the maximum activation and the uncertainty of the model. The CNN is trained to output a heatmap which has a strong maximum at the correct position. Consequently, when the predicted maximum is low, it might be a good indicator that the network is not sure about the prediction. Note that maximum heatmap activation (MHA) is technically a measure of model certainty since it should increase proportional to model's confidence in its predictions.

For the purpose of experiment analysis in Section 3, this quantity was normalized to a unit range. The upper bound of one for normalization was chosen based on the maximum value of this uncertainty measure observed for all of the landmarks in the test set. Note that the other two models described in this section ignore the value of the MHA (and only use its position) and do not use it for uncertainty estimation.

2.4.2 Ensemble Prediction Variance

Ensemble is an ensemble model consisting of 15 CNNs trained independently using the same CNN architecture as the Baseline model. To predict landmark positions for an input image, each CNN in the ensemble is first evaluated as described in Section 2.2 and produces its individual predictions of the landmark positions. Predictions of all networks are then averaged to produce the final position (see Equation 1). While it is well-known that forming an ensemble of

machine learning models improves prediction accuracy, (Lakshminarayanan et al., 2017) suggested treating the variance of the ensemble members' predictions (see Equation 2) as a measure of uncertainty. Greater variance indicates discord in the ensemble predictions. The member models were trained using random initialization so they all ended up with different parameter values at the end of training. Since they were trained using the same data, it is reasonable to assume that there will not be a large difference between their predictions on data coming from a similar distribution like the one they observed during training. On the other hand, when being evaluated on out-of-distribution data (such as a misaligned X-ray, or an X-ray from a different scanner) the difference between predictions will be larger since each model will take a different guess on the unfamiliar data based on its final parameters.

2.4.3 Monte Carlo Dropout Prediction Variance

The Monte Carlo (MC) Dropout technique is based on the Bayesian assumption that neural network weights $\mathbf{W}$ have probability distributions instead of being point estimates as is common in deep learning. The goal of Bayesian modelling is to approximate the posterior distribution $p(\mathbf{W}|\mathbf{X},\mathbf{Y})$ given the training data $\{\mathbf{X},\mathbf{Y}\}$. While true Bayesian neural networks are computationally expensive, (Gal and Ghahramani, 2016) suggested approximating them with dropout (Hinton et al., 2012). When applying a dropout layer in a CNN, a randomly selected subset of neurons in the previous layer is dropped at each iteration. Since the number of CNN parameters is usually in the millions, this essentially leads to a different network being sampled at each iteration. The resulting stochasticity of the network can be used to approximate a Bayesian neural network. In practice, evaluating the prediction of a an MC Dropout based network amounts to computing the mean of T stochastic forward passes through the network, which sample from T network architectures (different neurons are dropped for each one). The predictive uncertainty over a prediction is obtained by computing the sample variance of the T forward passes.

2.4.4 Prediction Mean and Prediction Variance

The Ensemble and MC-Dropout models both use prediction variance as a measure of their uncertainty. For the task of landmark localization, we compute the prediction variance of a vector $\vec{y}$ containing T prediction samples as the mean Euclidean distance between the prediction samples y_i and the prediction mean $\hat{y}$:

$$\hat{y} = \frac{1}{T}\sum_{i=1}^{T} y_i \tag{1}$$

$$\mathrm{Var}(\vec{y}) = \frac{1}{T}\sum_{i=1}^{T} \|y_i - \hat{y}\| \tag{2}$$

Note that the prediction mean $\hat{y}$ is also used as the landmark location predicted by the Ensemble and MC-Dropout models.

2.5 Implementation Details

All training images were resized to 128×128 size to speed up training and allow for faster experimentation. The predictions and prediction variance of the Ensemble and MC-Dropout models was computed using 15 ensemble CNNs and MC samples respectively. The probability of a unit being dropped in the MC-Dropout model was set uniformly to $p = 0.4$. All models along with the training process were implemented using the PyTorch (Paszke et al., 2017) library.

3 EXPERIMENTS AND RESULTS

We first shortly evaluate the landmark localization accuracy of the trained models. We then describe two experiments which aimed to assess whether the evaluated uncertainty measures are able to reliably detect out-of-distribution data on the cephalometric landmark localization task.

3.1 Landmark Localization Accuracy

We first verified that the performance of our models was comparable to that of the best previous solutions on the dataset we used (see Table 1). Due to computational reasons, we trained on images resized to the 128×128 size from the original 1935×2400. While it was sufficient for the purpose of our study, image sub-sampling reduced the accuracy of the models and direct clinical application would require it to be less aggressive.

3.2 Elastically Distorted Out-of-Distribution Data Detection

Elastic distortion was applied to the entire test set to evaluate the ability of the uncertainty measures to detect out-of-distribution data examples. Forty versions of the test set were created in total, and each copy had an elastic distortion of progressively stronger magnitude applied to it. First row in Figure 2 shows a

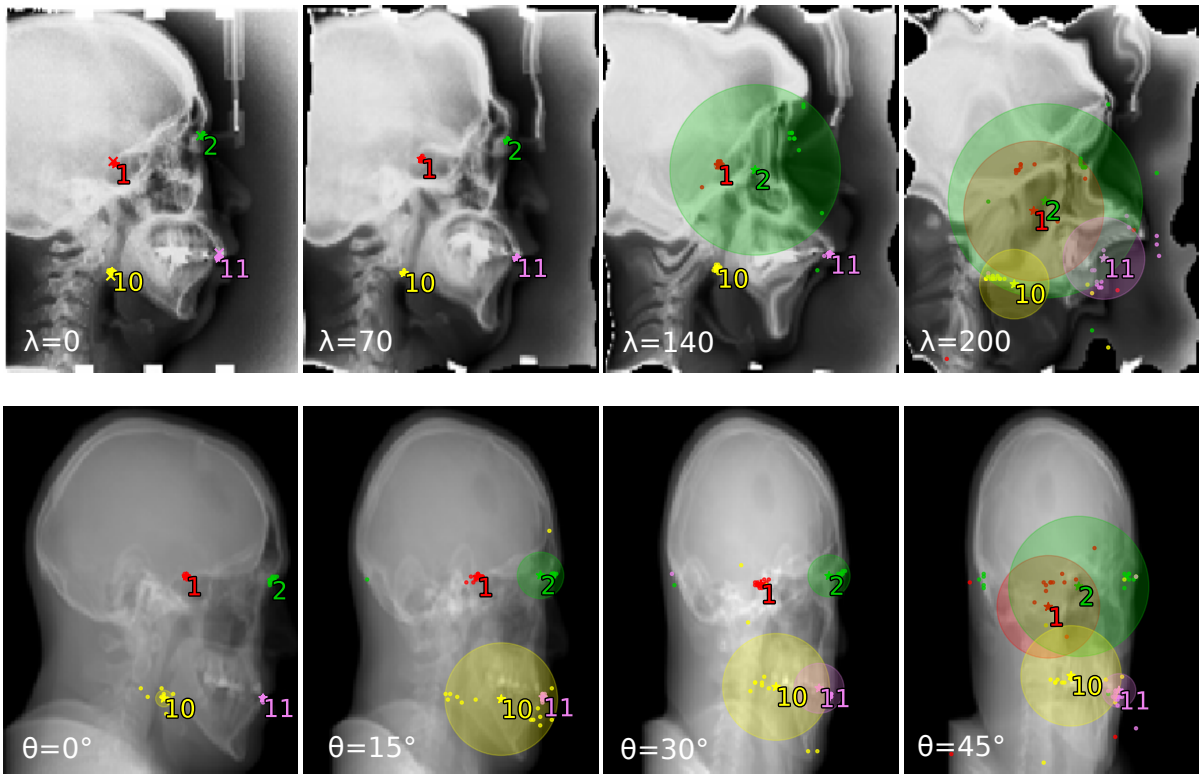

Figure 2: Visualization of the Ensemble model's predictions and uncertainty values. Top row shows an image from the test set transformed with elastic distortion of increasing magnitude λ. Bottom row shows a skull CT scan rotated in the horizontal plane by angle θ and projected onto the sagittal plane. The individual ensemble members' predictions (dots) are combined into a final position prediction (star), and the ground truth is marked by a cross (only applicable to the top left undistorted image with known ground truth). Only four landmarks are shown for clarity. As the magnitude of elastic distortion and rotation increases, so does the model uncertainty (prediction variance).

Table 1: Accuracy of the proposed models on the test1 split compared with the best solution from the 2015 IEEE ISBI challenge (Wang et al., 2016). Mean Radial Error gives the mean error in landmark detection. Success Detection Rate gives the percentage of predictions within that radius of the ground truth.

	MRE	SDR 2.5 mm
Lindner et al.	1.67 mm	80.2 %
Baseline	2.05 mm	74.4 %
Ensemble	1.79 mm	78.5 %
MC-Dropout	1.92 mm	74.7 %

test image transformed with an elastic distortion of varying strength, along with the landmark predictions of the Ensemble model for that image. Uncertainty for each predicted landmark position (variance of the prediction samples) is visualized by a circle superimposed upon the predicted location.

Each model's predictions and uncertainty estimates for every version of the distorted test set was then computed. Left column in Figure 3 shows the correlation between the mean uncertainty measure value for all landmark position predictions for a given version of the test set, and the elastic distortion magnitude applied to that version of the test set. The analysis shows that a strong correlation exists between the mean value of each uncertainty measure and the strength of the elastic distortion being applied on the data.

3.3 Rotated Out-of-Distribution Data Detection

During the process of X-ray scanning for the purpose of cephalometric analysis, patient's head in the scanner should be perfectly aligned with the sagittal plane. However, patients sometimes rotate their head in the horizontal plane which distorts the resulting image and may even lead to some of the landmarks overlapping. A model should detect such data by being uncertain about its predictions.

Since a dataset of cephalograms containing horizontal head rotation is not publicly available, we used a volumetric CT scan of a single skull to create one.

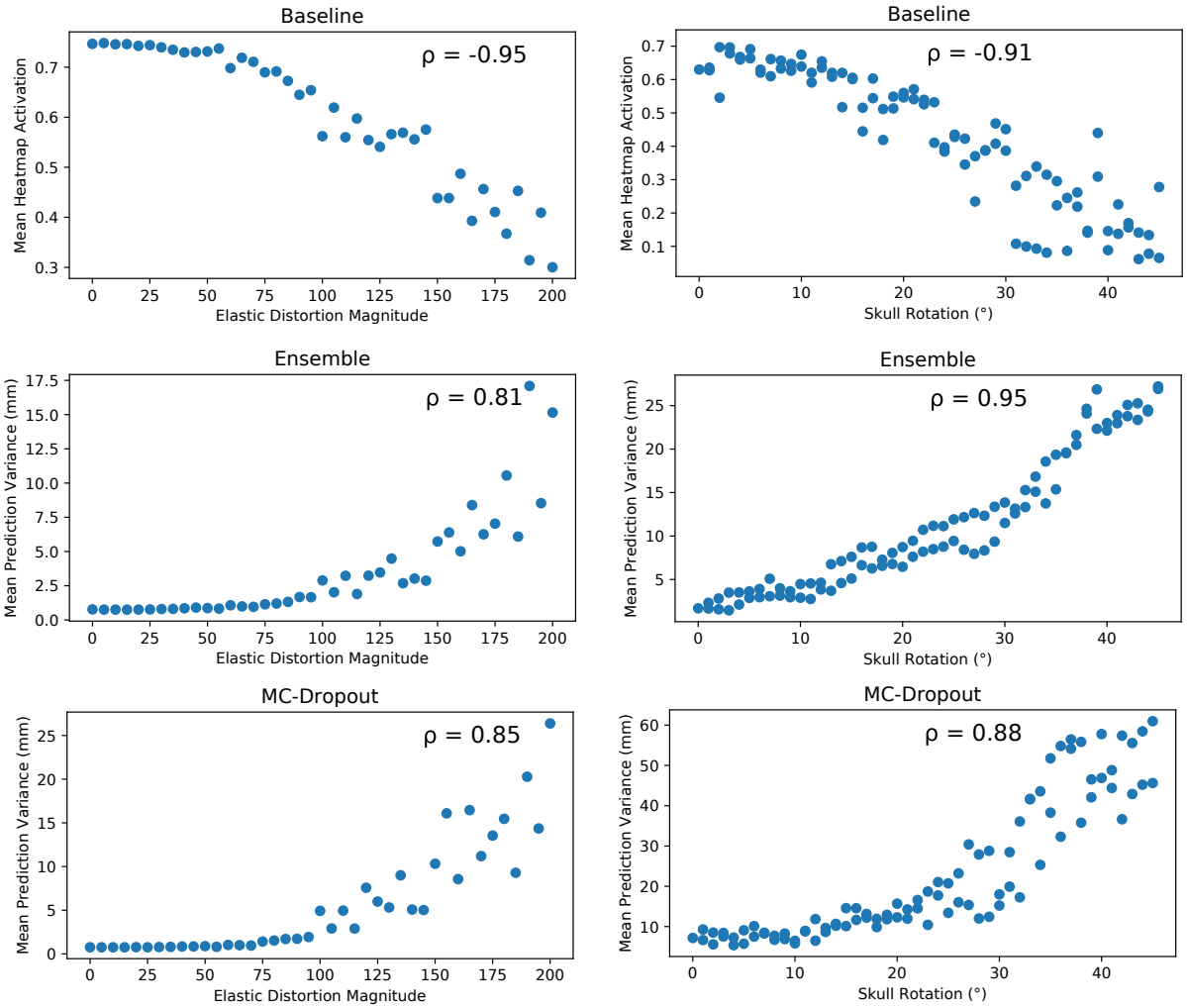

Figure 3: Correlation of the three uncertainty measure values with elastic distortion magnitude (left) and skull rotation magnitude (right). In the first experiment, each of the models along with its uncertainty measure was evaluated on forty versions of the test set modified by elastic distortion of varying magnitude. In the second experiment, the models were evaluated on 91 images of a skull CT scan projected onto the sagittal plane. The skull was transformed before projection with different magnitudes of rotation. As distortion and rotation magnitude increase, so does model uncertainty for all three measures. Note that maximum heatmap activation (top row) is actually a measure of model certainty so the correlation is negative as expected.

The skull volume (originally aligned with the sagittal plane) was first rotated by θ degrees in the horizontal plane to simulate a patient's undesirable movement in the scanner. To simulate X-ray acquisition process, the resulting volume was then projected onto the sagittal plane by summing the intensity values of overlapping voxels. Pixel values in the resulting 2D image were then normalized by dividing them with the maximum pixel intensity present in the image. The resulting dataset contains 91 images with θ ranging from $-45°$ to $45°$ including a rotation of $0°$.

We first verified that the model predictions for the CT volume projection without any rotation were reasonably accurate. The models provided acceptable predictions but their mean uncertainty increased compared with the predictions from the X-ray images in the test set (compare the model uncertainty in the first image in the top row with the first image in the bottom row of Figure 2). This is not unexpected since even a CT projection created from an unrotated skull volume is an out-of-distribution data point for a network trained on X-ray images. However, the sensible predictions of the models indicate that it is plausible to apply X-ray trained models on CT projections without a substantial loss of performance. A more confident conclusion would require further research using more CT scans. Also note that the viability of an inverse knowledge transfer (i.e., applying models

trained on CT projections to X-ray data) was previously observed by (Bier et al., 2018).

The models' prediction and uncertainty measure values were then evaluated for each image in the created dataset. Right column in Figure 3 shows the correlation between the mean uncertainty value for a given image (computed as the mean of uncertainty estimates for all of the landmarks predicted for the image) and the magnitude of rotation corresponding to that image. For each evaluated uncertainty measure, there is a very strong correlation with the rotation magnitude.

It is noteworthy that the ensemble uncertainty increases more stably than MHA uncertainty as the rotation applied to the image intensifies. For most steps in rotation increase (an increase of 1°, e.g., from 10° to 11°), there is a corresponding increase in uncertainty. Additionally, this increase has a consistent magnitude between all rotation steps. On the other hand, the MHA uncertainty values increase on the whole, but the difference in the uncertainty values between successive rotation steps oscillates. Moreover, for some consecutive rotation steps, the MHA uncertainty actually decreases significantly for such a small change in the input image. The MC Dropout method suffers from a similar instability.

We hypothesize that the superior stability of the ensemble prediction variance is due to the fact that the Ensemble model consists of 15 unique CNNs while the other two measures only have a single CNN available. A single network might have a weak spot in its parameters for some inputs, which then also affects the associated uncertainty estimate. Multiple networks will possibly different weak spots and the average of their predictions will be more reasonable which will positively affect uncertainty estimate as well.

A visualization of the Ensemble model's predictions and corresponding uncertainty values for a skull projection rotated by different angle θ are in the bottom row of Figure 2.

4 CONCLUSION

In this paper, we evaluated three measures for estimating deep learning model uncertainty on the cephalometric landmark localization task. We compared uncertainty estimation based on the maximum heatmap activation (MHA) of a heatmap regression CNN with an ensemble-based and a Bayesian-based approach.

Our experiments with out-of-distribution data showed a strong correlation between the uncertainty estimates accompanying model predictions and the distance of the data from the training distribution for all measures. This suggests their usability in detecting images unsuitable for automatic evaluation. When individually comparing the measures' performance, MHA showed the strongest correlation with image distance from training distribution when both experiments are taken into account. On the other hand, both MHA and the MC Dropout uncertainty values increased inconsistently in the rotation experiment while the ensemble uncertainty was very stable in this regard.

The usability of MHA is an interesting finding because this uncertainty measure is directly available when using a CNN trained for heatmap regression. Conversely, the other two measures require the model to contain dropout layers or necessitate the training of multiple networks. Additionally, while MHA requires a single forward pass of the image, the other examined methods both need multiple passes and are more computationally expensive.

Although MHA could be used as a strong baseline uncertainty estimation method on its own, due to its observed instability it might be useful to combine it (e.g., by a weighted average) with one of the ensemble (preferrably) or MC Dropout methods when their requirements and the increase in computation time are not a problem.

To further verify that the uncertainty measures we explored in this work are able to detect the failure cases when a model is being applied on data distant from its training distribution, it would be desirable to train the CNN on cephalograms from one set of scanners and then evaluate it on images from a different set of scanners. Another experiment could target a more confident result regarding the potential of applying X-ray trained deep learning models on CT projection images by using a larger dataset of CT volumes. Since this issue is not necessarily restricted to cephalometry or landmark localization, it would also be preferable to expand the experiments to include other machine learning tasks and other type of structures beside skulls.

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