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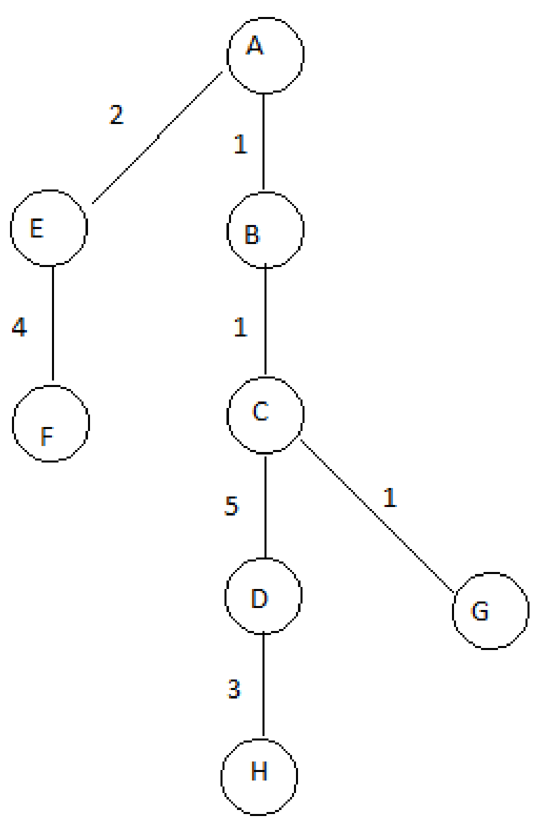
CMPS 102
Homework 4

Problem 1:

(a) The chart below show's Dijkstra's algorithm on the graph. The letter next to each weight keeps track of the parent node of the shortest path tree for that iteration. Assume vertex A is the source.

	A	B	C	D	E	F	G	H
A	0	1 (A)	∞	∞	2 (A)	10 (A)	∞	∞
B	0	1 (A)	2 (B)	∞	2 (A)	9 (B)	4 (B)	∞
C	0	1 (A)	2 (B)	7 (C)	2 (A)	9 (B)	3 (C)	∞
E	0	1 (A)	2 (B)	7 (C)	2 (A)	6 (E)	3 (C)	∞
G	0	1 (A)	2 (B)	7 (C)	2 (A)	6(E)	3 (C)	13 (G)
F	0	1 (A)	2 (B)	7 (C)	2 (A)	6 (E)	3 (C)	13 (G)
D	0	1 (A)	2 (B)	7 (C)	2 (A)	6 (E)	3 (C)	10 (D)
H	0	1 (A)	2 (B)	7 (C)	2 (A)	6 (E)	3 (C)	10 (D)

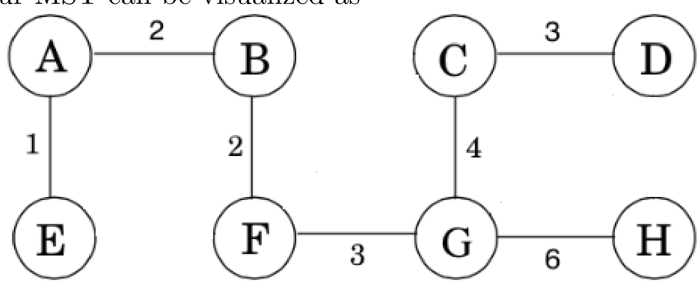
(b) Show the shortest path tree corresponding to running Dijkstra on this graph.
From the table, we can derive the shortest-path tree by looking at the top-most row and bottom-most row. In this case, the edge set for our shortest-path tree is $\{AB, BC, CD, AE, EF, CG, DH\}$. Thus the tree can be visualized as:



Problem 2: Trace Kruskal's algorithm on the graph.
First we sort the edges by weight and we construct a tree by taking the edges in order so long as it does not introduce a cycle.

Edge	Weight	Take?
A - E	1	Yes
A - B	2	Yes
B - F	2	Yes
C - D	3	Yes
F - G	3	Yes
E - F	4	No, introduces cycle ABEF.
C - G	4	Yes
B - E	5	No, introduces cycle ABE
C - F	5	No, introduces cycle CFG
D - G	5	No, introduces cycle CGD
G - H	6	Yes
B - C	7	No, the tree is spanning
D - H	7	No, the tree is spanning

Our MST can be visualized as



Problem 3:

1. Consider the edge (B, E) in the graph of Problem 2. Either show that there is a minimum cost spanning tree using this edge by applying the cut property, or prove that this edge is not used in any minimum cost spanning tree using the cycle property.

2. Ditto for edge (C, G).

part 1: Consider the cycle $A - B$, $B - E$, $E - A$. Out of all these edges $B - E$ has maximal weight. By the cycle property, edge $B - E$ is not part of any MST of the graph in problem 2.

part 2: Consider the cut $\{C, D\}$. The corresponding cut-set is the set of edges $\{C - B$, $C - F$, $C - G$, $D - G$, $H - D\}$. Out of these set of edges, edge $C - G$ has the minimal weight of 4, so by the cut property, the MST of problem 2 must contain edge $C - G$.

Problem 4: Prove or disprove the following statements about an arbitrary undirected graph $G = (V, E)$:

1. If e is part of some MST of G, then it must be a lightest edge in some cutset of G.
2. If graph G has more than $|V| - 1$ edges, and there is a unique heaviest edge, then this edge cannot be part of a minimum spanning tree.
3. The shortest path between two nodes is necessarily part of some MST.