Chapter 1

evaluation and simulation

The first thing to evaluate is the following equation.

$$\tilde{u} = \sum_{k=1}^{\infty} \frac{kP_k}{c} \tilde{u}^{k-1} \tag{1.1}$$

Here $\tilde{u}$ can be solved by finding the fix points of this equation. However, $\tilde{u} = 1$ is always a solution, which can be an absorbing state during the numerical evaluation process. There should be some non-zero solutions when c is larger than a certain value

- k is a integer index range from 1 to 1000 (maybe 500 is enough)
- $P_k = e^{-\lambda} \frac{\lambda^k}{k!}$ is the distribution function in terms of k, λ is a constant parameter which can take value $\{0.5,1,1.5,2\cdots\}$ for different simulations
- c is a constant which can be calculated by $c = \sum_k kP_k$, here $c = \lambda$

The second task is the following iterations:

If we denote by N_d the random variable that a random vertex has N neighbours at d_th shells, then the joint probability of having $\mathbf{k}^{(d)}$ degree structure for a random vertex can be deduced as following:

$$
\begin{aligned}
P(N_1 = n_1) &= p_{n_1} \\
P(N_1 = n_1, N_2 = n_2) &= P(N_2 = n_2 | N_1 = n_1) P(N_1 = n_1) \\
&= \delta_{\sum_{i=1}^{n_1}(\alpha_i - 1), n_2} \left(\sum_{\mathbf{k}^{(1)}} \prod_{i=1}^{n_1} \frac{\alpha_i p_{\alpha_i}}{c} \right) p_{n_1} \\
P(N_1 = n_1, N_2 = n_2, N_3 = n_3) &= P(N_3 = n_3 | N_2 = n_2) P(N_2 = n_2 | N_1 = n_1) P(N_1 = n_1) \\
&= \delta_{\sum_{i=1}^{n_2}(\alpha_i - 1), n_3} \left(\sum_{\mathbf{k}^{(2)}} \prod_{i=1}^{n_2} \frac{\alpha_i p_{\alpha_i}}{c} \right) P(N_1 = n_1, N_2 = n_2) \\
&\vdots \\
P(N_1 = n_1, \cdots, N_d = n_d) &= P(N_d = n_d | N_{d-1} = n_{d-1}) P(N_{d-1} = n_{d-1} | N_{d-2} = n_{d-2}) \cdots P(N_1 = n_1)
\end{aligned}
$$

Here is some explanation for the above equations:

- p_{n_1} is defined by previously introduced Poisson distribution where k is replaced by another initial value n_1
- n_1 can take integer value range from 0 to 10, then for each n_1, n_2 can further take value from 0 to 10 for the second equation, and so on. Keep the record of all results, which is the possible probability distribution.
- $\sum_{\mathbf{k}^{(1)}} = \sum_{\alpha_1} \sum_{\alpha_2} \cdots \sum_{\alpha_{n_1}}$ which is just a simplification of the notation. Therefore, $\sum_{\mathbf{k}^{(2)}} = \sum_{\alpha_1} \sum_{\alpha_2} \cdots \sum_{\alpha_{n_2}}$, $\sum_{\mathbf{k}^{(3)}} = \sum_{\alpha_1} \sum_{\alpha_2} \cdots \sum_{\alpha_{n_3}}$
- we can try the first five equation and see if there is anything interesting.