

Algorithm Analysis

Assignment 1 Order the following functions by growth rate: N , \sqrt{N} , $N^{1.5}$, N^2 , $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, 2^N , $2^{N/2}$, 37 , $N^2 \log N$, N^3 . Indicate which functions grow at the same rate.

Assignment 2. Give an analysis of running time of each following code (Big-Oh)

1. Sum = 0;

for (i = 1; i <= N; i++) $\rightarrow N$ $O(N)$
 Sum += i*i*i; $\rightarrow 3$

2. Sum = 0;

for (i = 1; i <= N; i++) $\rightarrow N$ $O(N^3)$
 for(j=0; j <= N*N; j++) $\rightarrow N^2$
 Sum ++;

3. Sum = 0;

for (i = 1; i <= N; i++)
 for(j=0; j <= i; j++) $O(N^2)$
 Sum ++;

4. Sum = 0;

for (i = 1; i <= N; i++) $O(N^2)$
 for(j=0; j <= i; j++)
 Sum ++;

5. Sum = 0;

for (i = 1; i <= N; i++) $O(N)$
 for(j=0; j <= i*i; j++) $O(N^2)$ $O(N^5)$
 \downarrow \downarrow
 k k
 for(k=0; k <= j; k++) $O(N^2)$
 Sum ++;

6. Sum = 0;

for (i = 1; i <= N; i++) $O(N)$

for(j=0; j <= i*i; j++) $O(N)$

if (j%i == 0) $O(N^4)$

for(k=0; k <= j; k++) $O(N^2)$

Sum ++;

7.

i=1;

s=1;

while(s <= N) $S = 1 + 2 + \dots + k = N + 1 = k(k+1) / 2$

{ $k(k+1) / 2 = N + 1$
 $k^2 + k = 2(N+1)$
 while loop run k times Then $k = \sqrt{N}$

i++;

s+=i; $O(\sqrt{N})$

}

Assignment 3, $O(N^2)$

```
for (int i = 0; i <= N; i++) {
    double term = 1.0;
    for (int j = 0; j < i; j++) {
        term *= X;
    }
    result += alpha[i] * term;
}

printf("F(X) = %lf\n", result);
```

Assignment 3, $O(N)$

```
double evaluate_polynomial(double x, int N) {
    double result = 0.0;
    for (int i = 0; i <= N; ++i) {
        result += coefficients[i] * pow(x, i);
    }
    return result;
}
```

Assignment 3 Write a program to evaluate the function $F(X) = \sum_{i=0}^N a_i X^i$. After that, calculate your running time?

b. If your running time is $O(N^2)$, please find an algorithm with linear complexity.

Assignment 4.

- Write a program to determine if a positive integer, N, is prime.
- In terms of N, what is the worst-case running time of your program?

```
#include <stdio.h>
#include <stdbool.h>
bool isPrime(long long int N){
    if (N <= 1){
        return false;
    }
    for (int i = 2; i < sqrt(N); i++){
        if (N % i == 0){
            return false;
        }
    }
    return true;
}

int main(){
    long long int N;
    scanf("%lld", &N);
    isPrime(N) ? printf("True") : printf("False");
    return 0;
}
```

for (i = N; i >= 1){
 i = i/2
 }
 N -> N/2 -> N/4 -> ...
 i = N / (2^k) = 1
 2^k = N
 k = log(N) / log(2)

k times