Geometry for NAE-DESC

Magnetic axis:

We are using Cartesian coordinates as our basis (phi is the cylindrical phi on magnetic axis, we shall try to use ϕ_c for the cylindrical angle at x)

In[∘]:=
```
r0[ϕ_] :=
  R0[ϕ] {Cos[ϕ], Sin[ϕ], 0} + Z0[ϕ] {0, 0, 1};
```

where we have used,

In[∘]:=
```
radial = {Cos[ϕ], Sin[ϕ], 0};
toroidal = {-Sin[ϕ], Cos[ϕ], 0};
vertical = {0, 0, 1};
```

The Frenet-Serret basis vectors are then (note here and in rest of document, “Th” is actually referring to phi),

In[∘]:=
```
normal = kR[ϕ] radial +
   kTh[ϕ] toroidal + kZ[ϕ] vertical;
binormal = tR[ϕ] radial +
   tTh[ϕ] toroidal + tZ[ϕ] vertical;
tangent = bR[ϕ] radial +
   bTh[ϕ] toroidal + bZ[ϕ] vertical;
```

First order

- Definition of position:

Here the coordinate ϕ is the cylindrical ϕ on the magnetic axis. Then, the first order displacement with respect to the axis is,

In[ ]:=
```
X1 = X1c[ϕ] Cos[θ] + X1s[ϕ] Sin[θ];
Y1 = Y1c[ϕ] Cos[θ] + Y1s[ϕ] Sin[θ];
x1[ϕ_, θ_] := ϵ X1 normal + ϵ Y1 binormal;
```

so that,

In[ ]:=
```
x[ϕ_, θ_] := r0[ϕ] + x1[ϕ, θ];
R[ϕ_, θ_] := Sqrt[x[ϕ, θ][[1]] × x[ϕ, θ][[1]] +
     x[ϕ, θ][[2]] × x[ϕ, θ][[2]]];
Z[ϕ_, θ_] := x[ϕ, θ][[3]];
TanPhiCyl[ϕ_, θ_] := x[ϕ, θ][[2]] / x[ϕ, θ][[1]];
```

- Expansion of R:

Then let us expand R in powers of ϵ, recalling that ϕ here is parametrising the axis cylindrical angle,

In[ ]:=
```
Clear[R1stExp]
R1stExp =
 FullSimplify[Series[R[ϕ, θ], {ϵ, 0, 2}]]
```

Out[∘]=

$$\sqrt{R0[\phi]^2} + \frac{1}{\sqrt{R0[\phi]^2}} R0[\phi] (kR[\phi] (Cos[\theta] X1c[\phi] + Sin[\theta] X1s[\phi]) + tR[\phi] (Cos[\theta] Y1c[\phi] + Sin[\theta] Y1s[\phi])) \epsilon + \frac{1}{2 \sqrt{R0[\phi]^2}} (kTh[\phi] (Cos[\theta] X1c[\phi] + Sin[\theta] X1s[\phi]) + tTh[\phi] (Cos[\theta] Y1c[\phi] + Sin[\theta] Y1s[\phi]))^2 \epsilon^2 + O[\epsilon]^3$$

Defining a function with the expression to second order,

In[∘]:=

$$R1stExpFun[\phi\_, \theta\_] := R0[\phi] + (kR[\phi] (Cos[\theta] X1c[\phi] + Sin[\theta] X1s[\phi]) + tR[\phi] (Cos[\theta] Y1c[\phi] + Sin[\theta] Y1s[\phi])) \epsilon + \frac{1}{2 R0[\phi]} (kTh[\phi] (Cos[\theta] X1c[\phi] + Sin[\theta] X1s[\phi]) + tTh[\phi] (Cos[\theta] Y1c[\phi] + Sin[\theta] Y1s[\phi]))^2 \epsilon^2;$$

- Expansion of the cylindrical angle:

It is now the turn to compute the cylindrical angle at the point of the surface that corresponds to the particular parametrisation used for the axis,

In[ ]:=
```
TanPhi1stExp =
  Series[TanPhiCyl[ϕ, θ], {ϵ, 0, 2}];
```

Which eliminating the secular ϕ piece leaves, to second order

In[ ]:=
```
Phi1stExp = FullSimplify[
  ArcTan[TanPhi1stExp] - ArcTan[Tan[ϕ]]]
```

Out[ ]=

$$\frac{1}{R0[\phi]}(Cos[\theta]\,kTh[\phi]\times X1c[\phi]+kTh[\phi]\,Sin[\theta]\,X1s[\phi]+Cos[\theta]\,tTh[\phi]\times Y1c[\phi]+Sin[\theta]\,tTh[\phi]\times Y1s[\phi])\,\epsilon-\frac{1}{R0[\phi]^2}(kR[\phi]\,(Cos[\theta]\,X1c[\phi]+Sin[\theta]\,X1s[\phi])+tR[\phi]\,(Cos[\theta]\,Y1c[\phi]+Sin[\theta]\,Y1s[\phi]))\,(kTh[\phi]\,(Cos[\theta]\,X1c[\phi]+Sin[\theta]\,X1s[\phi])+tTh[\phi]\,(Cos[\theta]\,Y1c[\phi]+Sin[\theta]\,Y1s[\phi]))\,\epsilon^2+O[\epsilon]^3$$

From which we construct the cylindrical angle difference $\phi_c - \phi_{c,\mathrm{axis}}$,

In[ ]:=
```
Phi1stExpFun[ϕ_, θ_] :=
  1/R0[ϕ] (Cos[θ] kTh[ϕ] × X1c[ϕ] + kTh[ϕ] Sin[θ] X1s[ϕ] + Cos[θ] tTh[ϕ] × Y1c[ϕ] +
      Sin[θ] tTh[ϕ] × Y1s[ϕ]) ϵ - 1/R0[ϕ]^2
    (kR[ϕ] (Cos[θ] X1c[ϕ] + Sin[θ] X1s[ϕ]) +
      tR[ϕ] (Cos[θ] Y1c[ϕ] + Sin[θ] Y1s[ϕ]))
    (kTh[ϕ] (Cos[θ] X1c[ϕ] + Sin[θ] X1s[ϕ]) +
      tTh[ϕ]
       (Cos[θ] Y1c[ϕ] + Sin[θ] Y1s[ϕ])) ϵ^2;
```

- Evaluating R as a function of ϕ_c:
- Expansion of Z:
- Define $\nu = \phi_b - \phi_c$ (in terms of ϕ on axis):

We can also define $\nu = \phi_b - \phi_c$, between the boozer angle and the cylindrical angle. Of course, the cylindrical angle here is that at x. Thus, $\nu = \phi_b - \phi_c(x) = \phi_b - \phi_c(0) - (\phi_c(x) - \phi_c(0))$. Now the latter is just what we computed above for

In[ ]:=
```
(*nuOrder1=-ϵ x1Th[ϕ,θ]/R0[ϕ];*)
```

Second order:

- Defining position:

As we did at first order, we define the position once again, but this time, with the same basis

In[ ]:=
```
ClearAll["Global'*"]
radial = {Cos[ϕ], Sin[ϕ], 0};
toroidal = {-Sin[ϕ], Cos[ϕ], 0};
vertical = {0, 0, 1};
```

defining more abstractly,

In[ ]:=
```
r0[ϕ_] := R0[ϕ] radial + Z0[ϕ] vertical
x1[ϕ_, θ_] :=
 ϵ ( x1R[ϕ, θ] radial + x1Th[ϕ, θ] toroidal +
     x1Z[ϕ, θ] vertical)
x2[ϕ_, θ_] :=
 ϵ^2 (x2R[ϕ, θ] radial + x2Th[ϕ, θ] toroidal +
     x2Z[ϕ, θ] vertical)
```

Instead of,

In[ ]:=
```
(*x2[ϕ_,θ_]:=
 ϵ^2 (X20[ϕ]+X2c[ϕ]Cos[2θ]+X2s[ϕ]Sin[2θ])
   {kR[ϕ]Cos[ϕ]-Sin[ϕ]kTh[ϕ],
    kR[ϕ]Sin[ϕ]+Cos[ϕ]kTh[ϕ],kZ[ϕ]}+
  ϵ^2 (Y20[ϕ]+Y2c[ϕ]Cos[2θ]+Y2s[ϕ]Sin[2θ])
   {tR[ϕ]Cos[ϕ]-Sin[ϕ]tTh[ϕ],
    tR[ϕ]Sin[ϕ]+Cos[ϕ]tTh[ϕ],tZ[ϕ]}+
  ϵ^2 (Z20[ϕ]+Z2c[ϕ]Cos[2θ]+Z2s[ϕ]Sin[2θ])
   {bR[ϕ]Cos[ϕ]-Sin[ϕ]bTh[ϕ],
    bR[ϕ]Sin[ϕ]+Cos[ϕ]bTh[ϕ],bZ[ϕ]}*)
```

Then, as before

In[ ]:=
```
x[ϕ_, θ_] := r0[ϕ] + x1[ϕ, θ] + x2[ϕ, θ]
R[ϕ_, θ_] := Sqrt[x[ϕ, θ][[1]] × x[ϕ, θ][[1]] +
   x[ϕ, θ][[2]] × x[ϕ, θ][[2]]]
Z[ϕ_, θ_] := x[ϕ, θ][[3]];
TanPhiCyl[ϕ_, θ_] := x[ϕ, θ][[2]] / x[ϕ, θ][[1]]
```

- Expansion R (in terms of the cylindrical angle on axis):

Let us expand in ϵ^2,

In[ ]:=
```
R2ndExp = Normal[
  FullSimplify[Series[R[ϕ, θ], {ϵ, 0, 3}]]]
```

Out[ ]=

$$\sqrt{\text{R0}[\phi]^2} + \frac{\epsilon\,\text{R0}[\phi]\times\text{x1R}[\phi,\theta]}{\sqrt{\text{R0}[\phi]^2}} + \frac{\epsilon^2\left(\text{x1Th}[\phi,\theta]^2 + 2\,\text{R0}[\phi]\times\text{x2R}[\phi,\theta]\right)}{2\sqrt{\text{R0}[\phi]^2}} + \frac{1}{2\,\text{R0}[\phi]^3}\epsilon^3\sqrt{\text{R0}[\phi]^2}\,\text{x1Th}[\phi,\theta]\left(-\text{x1R}[\phi,\theta]\times\text{x1Th}[\phi,\theta] + 2\,\text{R0}[\phi]\times\text{x2Th}[\phi,\theta]\right)$$

In[ ]:=

```
Simplify[R2ndExp]
```

Out[ ]=

$$\frac{1}{2\,\text{R0}[\phi]\sqrt{\text{R0}[\phi]^2}}\left(2\,\text{R0}[\phi]^3 - \epsilon^3\,\text{x1R}[\phi,\theta]\,\text{x1Th}[\phi,\theta]^2 + 2\,\epsilon\,\text{R0}[\phi]^2\left(\text{x1R}[\phi,\theta] + \epsilon\,\text{x2R}[\phi,\theta]\right) + \epsilon^2\,\text{R0}[\phi]\times\text{x1Th}[\phi,\theta]\left(\text{x1Th}[\phi,\theta] + 2\,\epsilon\,\text{x2Th}[\phi,\theta]\right)\right)$$

which we may cast into a function,

In[ ]:=
```
R2ndExpFun[ϕ_, θ_] := R0[ϕ] + ϵ x1R[ϕ, θ] +
    ϵ^2 (x1Th[ϕ, θ]^2 + 2 R0[ϕ] × x2R[ϕ, θ]) / (2 R0[ϕ]) +
    1/(2 R0[ϕ]^2) ϵ^3 x1Th[ϕ, θ] (-x1R[ϕ, θ] ×
        x1Th[ϕ, θ] + 2 R0[ϕ] × x2Th[ϕ, θ]);
```

- Z expansion (in terms of the cylindrical angle on axis):

In[ ]:=
```
Z2ndExp = Normal[
   FullSimplify[Series[Z[ϕ, θ], {ϵ, 0, 3}]]]
```

Out[ ]=

$$\epsilon\, x1Z[\phi, \theta] + \epsilon^2\, x2Z[\phi, \theta] + Z0[\phi]$$

and constructing a function out of it,

In[ ]:=
```
Z2ndExpFun[ϕ_, θ_] :=
  ϵ x1Z[ϕ, θ] + ϵ^2 x2Z[ϕ, θ] + Z0[ϕ]
```

- ϕ expansion (in terms of the cylindrical angle on axis):

Doing the same for the cylindrical angle at the surface,

In[ ]:=
```
Phi2ndExp = Normal[FullSimplify[
    ArcTan[Series[TanPhiCyl[ϕ, θ],
       {ϵ, 0, 3}]] - ArcTan[Tan[ϕ]]]]
```

Out[∘]=

$$\frac{\epsilon\, \text{x1Th}[\phi,\theta]}{\text{R0}[\phi]} + \frac{1}{\text{R0}[\phi]^2} \epsilon^2 (-\text{x1R}[\phi,\theta] \times \text{x1Th}[\phi,\theta] + \text{R0}[\phi] \times \text{x2Th}[\phi,\theta]) - \frac{1}{3\,\text{R0}[\phi]^3} \epsilon^3 \left(-3\,\text{x1R}[\phi,\theta]^2\, \text{x1Th}[\phi,\theta] + \text{x1Th}[\phi,\theta]^3 + 3\,\text{R0}[\phi] \times \text{x1Th}[\phi,\theta] \times \text{x2R}[\phi,\theta] + 3\,\text{R0}[\phi] \times \text{x1R}[\phi,\theta] \times \text{x2Th}[\phi,\theta]\right)$$

which cast into a function,

In[∘]:=

$$\text{Phi2ndExpFun}[\phi\_,\theta\_] := \frac{\epsilon\, \text{x1Th}[\phi,\theta]}{\text{R0}[\phi]} + \frac{1}{\text{R0}[\phi]^2} \epsilon^2 (-\text{x1R}[\phi,\theta] \times \text{x1Th}[\phi,\theta] + \text{R0}[\phi] \times \text{x2Th}[\phi,\theta]) - \frac{1}{3\,\text{R0}[\phi]^3} \epsilon^3 \left(-3\,\text{x1R}[\phi,\theta]^2\, \text{x1Th}[\phi,\theta] + \text{x1Th}[\phi,\theta]^3 + 3\,\text{R0}[\phi] \times \text{x1Th}[\phi,\theta] \times \text{x2R}[\phi,\theta] + 3\,\text{R0}[\phi] \times \text{x1R}[\phi,\theta] \times \text{x2Th}[\phi,\theta]\right);$$

- ϕ expansion (in terms of ϕ_c):

We now do, for evaluating the difference in ϕ but at ϕ_c (using ϕ in the expression as the true cylindrical angle)

In[•]:=
```
deltaPhi2 =
  Series[Phi2ndExpFun[ϕc - Phi2ndExpFun[ϕc -
       Phi2ndExpFun[ϕc, θ], θ], θ], {ϵ, 0, 2}]
```

Out[•]=

$$\frac{\text{x1Th}[\phi_c,\theta]\,\epsilon}{\text{R0}[\phi_c]}+\frac{1}{\text{R0}[\phi_c]^3}\left(-\text{R0}[\phi_c]\times\text{x1R}[\phi_c,\theta]\times\text{x1Th}[\phi_c,\theta]+\text{R0}[\phi_c]^2\,\text{x2Th}[\phi_c,\theta]+\text{x1Th}[\phi_c,\theta]^2\,\text{R0}'[\phi_c]-\text{R0}[\phi_c]\times\text{x1Th}[\phi_c,\theta]\,\text{x1Th}^{(1,0)}[\phi_c,\theta]\right)\epsilon^2+O[\epsilon]^3$$

- v expansion (in terms of the cylindrical angle on axis):
- v expansion (in terms of ϕ_c, the true on-surface cylindrical angle)
- R expansion (in terms of ϕ_c, the true on-surface cylindrical angle):

To find the form of the function $R(\phi_c)$, where of course ϕ_c is the cylindrical angle of x, we note that at this point we have things as a function of $\phi = \phi_c$ –Phi1stExp. So,

In[•]:=
```
DeltaR = Series[R2ndExpFun[
     ϕc - Phi2ndExpFun[ϕc - Phi2ndExpFun[
          ϕc - Phi2ndExpFun[ϕc, θ], θ], θ],
     θ] - R0[ϕc], {ϵ, 0, 2}]
```

Out[ ]=

$$\left(\text{x1R}[\phi_c, \Theta] - \frac{\text{x1Th}[\phi_c, \Theta]\ \text{R0}'[\phi_c]}{\text{R0}[\phi_c]}\right) \in + \left(\frac{\text{x1Th}[\phi_c, \Theta]^2 + 2\ \text{R0}[\phi_c] \times \text{x2R}[\phi_c, \Theta]}{2\ \text{R0}[\phi_c]} + \frac{\text{x1Th}[\phi_c, \Theta]^2\ \text{R0}''[\phi_c]}{2\ \text{R0}[\phi_c]^2} - \frac{\text{x1Th}[\phi_c, \Theta]\ \text{x1R}^{(1,0)}[\phi_c, \Theta]}{\text{R0}[\phi_c]} + \frac{1}{\text{R0}[\phi_c]^3}\ \text{R0}'[\phi_c]\ \left(\text{R0}[\phi_c] \times \text{x1R}[\phi_c, \Theta] \times \text{x1Th}[\phi_c, \Theta] - \text{R0}[\phi_c]^2\ \text{x2Th}[\phi_c, \Theta] - \text{x1Th}[\phi_c, \Theta]^2\ \text{R0}'[\phi_c] + \text{R0}[\phi_c] \times \text{x1Th}[\phi_c, \Theta]\ \text{x1Th}^{(1,0)}[\phi_c, \Theta]\right)\right) \in^2 + O[\in]^3$$

So,

In[ ]:=

```
DeltaR2 = Coefficient[DeltaR, ϵ^2];
```

We may rewrite it in the following more succinct form

In[∘]:=
```
DeltaR2man =
  -1/2 D[D[R0[ϕc], ϕc], ϕc] x1Th[ϕc, θ]^2/R0[ϕc]^2 -
   (x2Th[ϕc, θ]/R0[ϕc] - x1R[ϕc, θ] × x1Th[ϕc, θ]/R0[ϕc]^2)
    D[R0[ϕc], ϕc] - x1Th[ϕc, θ]/R0[ϕc]
    D[x1R[ϕc, θ] - x1Th[ϕc, θ]/R0[ϕc] D[R0[ϕc], ϕc],
     ϕc] + (x2R[ϕc, θ] + x1Th[ϕc, θ]^2/(2 R0[ϕc]));
```

Which may be shown to be identical,

In[∘]:=
```
Simplify[DeltaR2 - DeltaR2man]
```

Out[∘]=
```
0
```

- Z expansion (in terms of ϕ_c, the true on-surface cylindrical angle):

In[∘]:=
```
DeltaZ = Series[Z2ndExpFun[
    ϕc - Phi2ndExpFun[ϕc - Phi2ndExpFun[
         ϕc - Phi2ndExpFun[ϕc, θ], θ], θ],
    θ] - Z0[ϕc], {ϵ, 0, 2}]
```

Out[ ]=

$$\left(\text{x1Z}[\phi_c,\Theta] - \frac{\text{x1Th}[\phi_c,\Theta]\,\text{Z0}'[\phi_c]}{\text{R0}[\phi_c]}\right)\epsilon + \left(\text{x2Z}[\phi_c,\Theta] + \frac{\text{x1Th}[\phi_c,\Theta]^2\,\text{Z0}''[\phi_c]}{2\,\text{R0}[\phi_c]^2} + \frac{1}{\text{R0}[\phi_c]^3}\,\text{Z0}'[\phi_c]\,\big(\text{R0}[\phi_c]\times\text{x1R}[\phi_c,\Theta]\times\text{x1Th}[\phi_c,\Theta] - \text{R0}[\phi_c]^2\,\text{x2Th}[\phi_c,\Theta] - \text{x1Th}[\phi_c,\Theta]^2\,\text{R0}'[\phi_c] + \text{R0}[\phi_c]\times\text{x1Th}[\phi_c,\Theta]\,\text{x1Th}^{(1,0)}[\phi_c,\Theta]\big) - \frac{\text{x1Th}[\phi_c,\Theta]\,\text{x1Z}^{(1,0)}[\phi_c,\Theta]}{\text{R0}[\phi_c]}\right)\epsilon^2 + O[\epsilon]^3$$

So,

In[ ]:=
```
DeltaZ2 = Coefficient[DeltaZ, ϵ^2];
```

Which may be more succinctly rewritten as,

In[ ]:=
```
DeltaZ2man =
  x2Z[ϕc, θ] - x1Th[ϕc, θ]/R0[ϕc] D[x1Z[ϕc, θ] -
      x1Th[ϕc, θ]/R0[ϕc] D[Z0[ϕc], ϕc], ϕc] -
    (x2Th[ϕc, θ]/R0[ϕc] - x1R[ϕc, θ] × x1Th[ϕc, θ]/R0[ϕc]^2)
     D[Z0[ϕc], ϕc] -
    x1Th[ϕc, θ]^2/(2 R0[ϕc]^2) D[D[Z0[ϕc], ϕc], ϕc];
```

$$\mathrm{DeltaZ2man} = \mathrm{x2Z}[\phi_c,\theta] - \frac{\mathrm{x1Th}[\phi_c,\theta]}{\mathrm{R0}[\phi_c]} D\left[\mathrm{x1Z}[\phi_c,\theta] - \frac{\mathrm{x1Th}[\phi_c,\theta]}{\mathrm{R0}[\phi_c]} D[\mathrm{Z0}[\phi_c],\phi_c],\phi_c\right] - \left(\frac{\mathrm{x2Th}[\phi_c,\theta]}{\mathrm{R0}[\phi_c]} - \frac{\mathrm{x1R}[\phi_c,\theta]\times\mathrm{x1Th}[\phi_c,\theta]}{\mathrm{R0}[\phi_c]^2}\right) D[\mathrm{Z0}[\phi_c],\phi_c] - \frac{\mathrm{x1Th}[\phi_c,\theta]^2}{2\,\mathrm{R0}[\phi_c]^2} D[D[\mathrm{Z0}[\phi_c],\phi_c],\phi_c];$$

Which can be shown explicitly to be equivalent,

In[ ]:=
```
Simplify[DeltaZ2 - DeltaZ2man]
```

Out[ ]=
```
0
```

- Other (not checked)

To separate the different harmonics,

In[ ]:=
```
x1th = X1thc[ϕ] Cos[θ] + X1ths[ϕ] Sin[θ];
x1R = X1Rc[ϕ] Cos[θ] + X1Rs[ϕ] Sin[θ];
x2th = X2thc[ϕ] Cos[2 θ] +
    X2ths[ϕ] Sin[2 θ] + X2th0[ϕ];
x2R = X2Rc[ϕ] Cos[2 θ] +
    X2Rs[ϕ] Sin[2 θ] + X2R0[ϕ];
x1z = X1zc[ϕ] Cos[θ] + X1zs[ϕ] Sin[θ];
x2z = X2zc[ϕ] Cos[2 θ] +
    X2zs[ϕ] Sin[2 θ] + X2z0[ϕ];
```

In[ ]:=

$$\mathrm{delR2} = -\frac{1}{2}\,\mathrm{D}[\mathrm{D}[\mathrm{R0}[\phi],\phi],\phi]\,\frac{\mathrm{x1th}^2}{\mathrm{R0}[\phi]^2} - \left(\frac{\mathrm{x2th}}{\mathrm{R0}[\phi]} - \frac{\mathrm{x1R}\,\mathrm{x1th}}{\mathrm{R0}[\phi]^2}\right)\mathrm{D}[\mathrm{R0}[\phi],\phi] - \frac{\mathrm{x1th}}{\mathrm{R0}[\phi]}\,\mathrm{D}\left[\mathrm{x1R} - \frac{\mathrm{x1th}}{\mathrm{R0}[\phi]}\,\mathrm{D}[\mathrm{R0}[\phi],\phi],\phi\right] + \left(\mathrm{x2R} + \frac{\mathrm{x1th}^2}{2\,\mathrm{R0}[\phi]}\right);$$

In[ ]:=

$$\text{delZ2} = -\frac{1}{2}\,\text{D}[\text{D}[\text{Z0}[\phi],\phi],\phi]\,\frac{\text{x1th}^2}{\text{R0}[\phi]^2} - \left(\frac{\text{x2th}}{\text{R0}[\phi]} - \frac{\text{x1R x1th}}{\text{R0}[\phi]^2}\right)\text{D}[\text{Z0}[\phi],\phi] - \frac{\text{x1th}}{\text{R0}[\phi]}\,\text{D}\left[\text{x1z} - \frac{\text{x1th}}{\text{R0}[\phi]}\,\text{D}[\text{Z0}[\phi],\phi],\phi\right] + \text{x2z};$$

In[ ]:=

```
Collect[TrigReduce[delR2],
 {Cos[2 θ], Sin[2 θ]}]
```

Out[ ]=

$$\frac{\text{X1thc}[\phi]^2}{4\,\text{R0}[\phi]} + \frac{\text{X1ths}[\phi]^2}{4\,\text{R0}[\phi]} + \text{X2R0}[\phi] + \frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X2th0}[\phi]\,\text{R0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3} - \frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3} - \frac{\text{X1thc}[\phi]\,\text{X1Rc}'[\phi]}{2\,\text{R0}[\phi]} - \frac{\text{X1ths}[\phi]\,\text{X1Rs}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1thc}[\phi]\,\text{R0}'[\phi]\,\text{X1thc}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]\,\text{R0}'[\phi]\,\text{X1ths}'[\phi]}{2\,\text{R0}[\phi]^2} +$$

$$\frac{\text{X1thc}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}+\frac{\text{X1ths}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}+$$

$$\text{Sin}[2\,\theta]\left(\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]}{2\,\text{R0}[\phi]}+\right.$$

$$\text{X2Rs}[\phi]+\frac{\text{X1Rs}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}+$$

$$\frac{\text{X1Rc}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-$$

$$\frac{\text{X2ths}[\phi]\,\text{R0}'[\phi]}{\text{R0}[\phi]}-$$

$$\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]^2}{\text{R0}[\phi]^3}-$$

$$\frac{\text{X1ths}[\phi]\,\text{X1Rc}'[\phi]}{2\,\text{R0}[\phi]}-\frac{\text{X1thc}[\phi]\,\text{X1Rs}'[\phi]}{2\,\text{R0}[\phi]}+$$

$$\frac{\text{X1ths}[\phi]\,\text{R0}'[\phi]\,\text{X1thc}'[\phi]}{2\,\text{R0}[\phi]^2}+$$

$$\frac{\text{X1thc}[\phi]\,\text{R0}'[\phi]\,\text{X1ths}'[\phi]}{2\,\text{R0}[\phi]^2}+$$

$$\left.\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]\,\text{R0}''[\phi]}{2\,\text{R0}[\phi]^2}\right)+$$

$$\text{Cos}[2\,\theta]\left(\frac{\text{X1thc}[\phi]^2}{4\,\text{R0}[\phi]}-\frac{\text{X1ths}[\phi]^2}{4\,\text{R0}[\phi]}+\right.$$

$$\text{X2Rc}[\phi]+\frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-$$

$$\frac{\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\ \text{R0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X2thc}[\phi]\ \text{R0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\ \text{R0}'[\phi]^2}{2\ \text{R0}[\phi]^3} + \frac{\text{X1ths}[\phi]^2\ \text{R0}'[\phi]^2}{2\ \text{R0}[\phi]^3} - \frac{\text{X1thc}[\phi]\ \text{X1Rc}'[\phi]}{2\ \text{R0}[\phi]} + \frac{\text{X1ths}[\phi]\ \text{X1Rs}'[\phi]}{2\ \text{R0}[\phi]} + \frac{\text{X1thc}[\phi]\ \text{R0}'[\phi]\ \text{X1thc}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]\ \text{R0}'[\phi]\ \text{X1ths}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\ \text{R0}''[\phi]}{4\ \text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]^2\ \text{R0}''[\phi]}{4\ \text{R0}[\phi]^2} \Bigg)$$

- R20

In[ ]:=

$$\frac{\text{X1thc}[\phi]^2}{4\ \text{R0}[\phi]} + \frac{\text{X1ths}[\phi]^2}{4\ \text{R0}[\phi]} + \text{X2R0}[\phi] + \frac{\text{X1Rc}[\phi] \times \text{X1thc}[\phi]\ \text{R0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\ \text{R0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X2th0}[\phi]\ \text{R0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\ \text{R0}'[\phi]^2}{2\ \text{R0}[\phi]^3} - \frac{\text{X1ths}[\phi]^2\ \text{R0}'[\phi]^2}{2\ \text{R0}[\phi]^3} - \frac{\text{X1thc}[\phi]\ \text{X1Rc}'[\phi]}{2\ \text{R0}[\phi]} - \frac{\text{X1ths}[\phi]\ \text{X1Rs}'[\phi]}{2\ \text{R0}[\phi]} + \frac{\text{X1thc}[\phi]\ \text{R0}'[\phi]\ \text{X1thc}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]\ \text{R0}'[\phi]\ \text{X1ths}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\ \text{R0}''[\phi]}{4\ \text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]^2\ \text{R0}''[\phi]}{4\ \text{R0}[\phi]^2}$$

Out[•]=

$$\frac{\text{X1thc}[\phi]^2}{4\,\text{R0}[\phi]}+\frac{\text{X1ths}[\phi]^2}{4\,\text{R0}[\phi]}+\text{X2R0}[\phi]+\frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}+\frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X2th0}[\phi]\,\text{R0}'[\phi]}{\text{R0}[\phi]}-\frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}-\frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}-\frac{\text{X1thc}[\phi]\,\text{X1Rc}'[\phi]}{2\,\text{R0}[\phi]}-\frac{\text{X1ths}[\phi]\,\text{X1Rs}'[\phi]}{2\,\text{R0}[\phi]}+\frac{\text{X1thc}[\phi]\,\text{R0}'[\phi]\,\text{X1thc}'[\phi]}{2\,\text{R0}[\phi]^2}+\frac{\text{X1ths}[\phi]\,\text{R0}'[\phi]\,\text{X1ths}'[\phi]}{2\,\text{R0}[\phi]^2}+\frac{\text{X1thc}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}+\frac{\text{X1ths}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

In[•]:=

- R2c

In[•]:=

$$\frac{\text{X1thc}[\phi]^2}{4\,\text{R0}[\phi]}-\frac{\text{X1ths}[\phi]^2}{4\,\text{R0}[\phi]}+\text{X2Rc}[\phi]+\frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X2thc}[\phi]\,\text{R0}'[\phi]}{\text{R0}[\phi]}-\frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}+\frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}-\frac{\text{X1thc}[\phi]\,\text{X1Rc}'[\phi]}{2\,\text{R0}[\phi]}+\frac{\text{X1ths}[\phi]\,\text{X1Rs}'[\phi]}{2\,\text{R0}[\phi]}+\frac{\text{X1thc}[\phi]\,\text{R0}'[\phi]\,\text{X1thc}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X1ths}[\phi]\,\text{R0}'[\phi]\,\text{X1ths}'[\phi]}{2\,\text{R0}[\phi]^2}+\frac{\text{X1thc}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}-\frac{\text{X1ths}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

Out[•]=

$$\frac{\text{X1thc}[\phi]^2}{4\,\text{R0}[\phi]}-\frac{\text{X1ths}[\phi]^2}{4\,\text{R0}[\phi]}+\text{X2Rc}[\phi]+\frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{R0}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X2thc}[\phi]\,\text{R0}'[\phi]}{\text{R0}[\phi]}-\frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}+\frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]^2}{2\,\text{R0}[\phi]^3}-\frac{\text{X1thc}[\phi]\,\text{X1Rc}'[\phi]}{2\,\text{R0}[\phi]}+\frac{\text{X1ths}[\phi]\,\text{X1Rs}'[\phi]}{2\,\text{R0}[\phi]}+\frac{\text{X1thc}[\phi]\,\text{R0}'[\phi]\,\text{X1thc}'[\phi]}{2\,\text{R0}[\phi]^2}-\frac{\text{X1ths}[\phi]\,\text{R0}'[\phi]\,\text{X1ths}'[\phi]}{2\,\text{R0}[\phi]^2}+\frac{\text{X1thc}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}-\frac{\text{X1ths}[\phi]^2\,\text{R0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

- R2s

In[•]:=

$$\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]}{2\,\text{R0}[\phi]}+\text{X2Rs}[\phi]+$$

$$(\text{X1Rs}[\phi] \times \text{X1thc}[\phi]\, \text{R0}'[\phi]) \Big/ \left(2\, \text{R0}[\phi]^2\right) +$$
$$(\text{X1Rc}[\phi] \times \text{X1ths}[\phi]\, \text{R0}'[\phi]) \Big/ \left(2\, \text{R0}[\phi]^2\right) -$$
$$\frac{\text{X2ths}[\phi]\, \text{R0}'[\phi]}{\text{R0}[\phi]} - \frac{1}{\text{R0}[\phi]^3}$$
$$\text{X1thc}[\phi] \times \text{X1ths}[\phi]\, \text{R0}'[\phi]^2 -$$
$$\frac{\text{X1ths}[\phi]\, \text{X1Rc}'[\phi]}{2\, \text{R0}[\phi]} -$$
$$\frac{\text{X1thc}[\phi]\, \text{X1Rs}'[\phi]}{2\, \text{R0}[\phi]} +$$
$$(\text{X1ths}[\phi]\, \text{R0}'[\phi]\, \text{X1thc}'[\phi]) \Big/ \left(2\, \text{R0}[\phi]^2\right) +$$
$$(\text{X1thc}[\phi]\, \text{R0}'[\phi]\, \text{X1ths}'[\phi]) \Big/ \left(2\, \text{R0}[\phi]^2\right) +$$

$$(\text{X1thc}[\phi]\times\text{X1ths}[\phi]\ \text{R0}''[\phi])\Big/\left(2\ \text{R0}[\phi]^2\right)$$

Out[ ]=

$$\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]}{2\ \text{R0}[\phi]}+\text{X2Rs}[\phi]+\frac{\text{X1Rs}[\phi]\times\text{X1thc}[\phi]\ \text{R0}'[\phi]}{2\ \text{R0}[\phi]^2}+\frac{\text{X1Rc}[\phi]\times\text{X1ths}[\phi]\ \text{R0}'[\phi]}{2\ \text{R0}[\phi]^2}-\frac{\text{X2ths}[\phi]\ \text{R0}'[\phi]}{\text{R0}[\phi]}-\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]\ \text{R0}'[\phi]^2}{\text{R0}[\phi]^3}-\frac{\text{X1ths}[\phi]\ \text{X1Rc}'[\phi]}{2\ \text{R0}[\phi]}-\frac{\text{X1thc}[\phi]\ \text{X1Rs}'[\phi]}{2\ \text{R0}[\phi]}+\frac{\text{X1ths}[\phi]\ \text{R0}'[\phi]\ \text{X1thc}'[\phi]}{2\ \text{R0}[\phi]^2}+\frac{\text{X1thc}[\phi]\ \text{R0}'[\phi]\ \text{X1ths}'[\phi]}{2\ \text{R0}[\phi]^2}+\frac{\text{X1thc}[\phi]\times\text{X1ths}[\phi]\ \text{R0}''[\phi]}{2\ \text{R0}[\phi]^2}$$

In[ ]:=

```
Collect[TrigReduce[delZ2],
 {Cos[2 θ], Sin[2 θ]}]
```

Out[ ]=

$$
\begin{aligned}
&\text{X2z0}[\phi] - \frac{\text{X1thc}[\phi]\ \text{X1zc}'[\phi]}{2\ \text{R0}[\phi]} - \frac{\text{X1ths}[\phi]\ \text{X1zs}'[\phi]}{2\ \text{R0}[\phi]} + \\
&\frac{\text{X1Rc}[\phi] \times \text{X1thc}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X2th0}[\phi]\ \text{Z0}'[\phi]}{\text{R0}[\phi]} - \\
&\frac{\text{X1thc}[\phi]^2\ \text{R0}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^3} - \frac{\text{X1ths}[\phi]^2\ \text{R0}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^3} + \frac{\text{X1thc}[\phi]\ \text{X1thc}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \\
&\frac{\text{X1ths}[\phi]\ \text{X1ths}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\ \text{Z0}''[\phi]}{4\ \text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]^2\ \text{Z0}''[\phi]}{4\ \text{R0}[\phi]^2} + \\
&\text{Sin}[2\,\theta] \left(\text{X2zs}[\phi] - \frac{\text{X1ths}[\phi]\ \text{X1zc}'[\phi]}{2\ \text{R0}[\phi]} - \frac{\text{X1thc}[\phi]\ \text{X1zs}'[\phi]}{2\ \text{R0}[\phi]} + \right. \\
&\quad \frac{\text{X1Rs}[\phi] \times \text{X1thc}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1Rc}[\phi] \times \text{X1ths}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X2ths}[\phi]\ \text{Z0}'[\phi]}{\text{R0}[\phi]} - \\
&\quad \frac{\text{X1thc}[\phi] \times \text{X1ths}[\phi]\ \text{R0}'[\phi]\ \text{Z0}'[\phi]}{\text{R0}[\phi]^3} + \frac{\text{X1ths}[\phi]\ \text{X1thc}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \\
&\quad \left. \frac{\text{X1thc}[\phi]\ \text{X1ths}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi] \times \text{X1ths}[\phi]\ \text{Z0}''[\phi]}{2\ \text{R0}[\phi]^2} \right) + \\
&\text{Cos}[2\,\theta] \left(\text{X2zc}[\phi] - \frac{\text{X1thc}[\phi]\ \text{X1zc}'[\phi]}{2\ \text{R0}[\phi]} + \frac{\text{X1ths}[\phi]\ \text{X1zs}'[\phi]}{2\ \text{R0}[\phi]} + \right. \\
&\quad \frac{\text{X1Rc}[\phi] \times \text{X1thc}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} - \frac{\text{X2thc}[\phi]\ \text{Z0}'[\phi]}{\text{R0}[\phi]} - \\
&\quad \frac{\text{X1thc}[\phi]^2\ \text{R0}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^3} + \frac{\text{X1ths}[\phi]^2\ \text{R0}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^3} + \frac{\text{X1thc}[\phi]\ \text{X1thc}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} - \\
&\quad \left. \frac{\text{X1ths}[\phi]\ \text{X1ths}'[\phi]\ \text{Z0}'[\phi]}{2\ \text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\ \text{Z0}''[\phi]}{4\ \text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]^2\ \text{Z0}''[\phi]}{4\ \text{R0}[\phi]^2} \right)
\end{aligned}
$$

- Z20

In[ ]:=

$$\text{X2z0}[\phi] - \frac{\text{X1thc}[\phi]\,\text{X1zc}'[\phi]}{2\,\text{R0}[\phi]} - \frac{\text{X1ths}[\phi]\,\text{X1zs}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X2th0}[\phi]\,\text{Z0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} - \frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + \frac{\text{X1thc}[\phi]\,\text{X1thc}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]\,\text{X1ths}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

Out[ ]=

$$\text{X2z0}[\phi] - \frac{\text{X1thc}[\phi]\,\text{X1zc}'[\phi]}{2\,\text{R0}[\phi]} - \frac{\text{X1ths}[\phi]\,\text{X1zs}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1Rc}[\phi]\times\text{X1thc}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1Rs}[\phi]\times\text{X1ths}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X2th0}[\phi]\,\text{Z0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} - \frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + \frac{\text{X1thc}[\phi]\,\text{X1thc}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]\,\text{X1ths}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2} + \frac{\text{X1ths}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

- Z2s

In[ ]:=

$$X2zs[\phi] - \frac{X1ths[\phi]\, X1zc'[\phi]}{2\, R0[\phi]} - \frac{X1thc[\phi]\, X1zs'[\phi]}{2\, R0[\phi]} + (X1Rs[\phi] \times X1thc[\phi]\, Z0'[\phi]) \Big/ \left(2\, R0[\phi]^2\right) + (X1Rc[\phi] \times X1ths[\phi]\, Z0'[\phi]) \Big/ \left(2\, R0[\phi]^2\right) - \frac{X2ths[\phi]\, Z0'[\phi]}{R0[\phi]} - (X1thc[\phi] \times X1ths[\phi]\, R0'[\phi]\, Z0'[\phi]) \Big/ R0[\phi]^3 + (X1ths[\phi]\, X1thc'[\phi]\, Z0'[\phi]) \Big/ \left(2\, R0[\phi]^2\right) + (X1thc[\phi]\, X1ths'[\phi]\, Z0'[\phi]) \Big/ \left(2\, R0[\phi]^2\right) + (X1thc[\phi] \times X1ths[\phi]\, Z0''[\phi]) \Big/ \left(2\, R0[\phi]^2\right)$$

Out[ ]=

$$X2zs[\phi] - \frac{X1ths[\phi]\, X1zc'[\phi]}{2\, R0[\phi]} - \frac{X1thc[\phi]\, X1zs'[\phi]}{2\, R0[\phi]} + \frac{X1Rs[\phi] \times X1thc[\phi]\, Z0'[\phi]}{2\, R0[\phi]^2} + \frac{X1Rc[\phi] \times X1ths[\phi]\, Z0'[\phi]}{2\, R0[\phi]^2} - \frac{X2ths[\phi]\, Z0'[\phi]}{R0[\phi]} - \frac{X1thc[\phi] \times X1ths[\phi]\, R0'[\phi]\, Z0'[\phi]}{R0[\phi]^3} + \frac{X1ths[\phi]\, X1thc'[\phi]\, Z0'[\phi]}{2\, R0[\phi]^2} + \frac{X1thc[\phi]\, X1ths'[\phi]\, Z0'[\phi]}{2\, R0[\phi]^2} + \frac{X1thc[\phi] \times X1ths[\phi]\, Z0''[\phi]}{2\, R0[\phi]^2}$$

- Z2c

In[∘]:=

$$\text{X2zc}[\phi] - \frac{\text{X1thc}[\phi]\,\text{X1zc}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1ths}[\phi]\,\text{X1zs}'[\phi]}{2\,\text{R0}[\phi]} + (\text{X1Rc}[\phi] \times \text{X1thc}[\phi]\,\text{Z0}'[\phi]) \Big/ \left(2\,\text{R0}[\phi]^2\right) - (\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\,\text{Z0}'[\phi]) \Big/ \left(2\,\text{R0}[\phi]^2\right) - \frac{\text{X2thc}[\phi]\,\text{Z0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + \frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + (\text{X1thc}[\phi]\,\text{X1thc}'[\phi]\,\text{Z0}'[\phi]) \Big/ \left(2\,\text{R0}[\phi]^2\right) - (\text{X1ths}[\phi]\,\text{X1ths}'[\phi]\,\text{Z0}'[\phi]) \Big/ \left(2\,\text{R0}[\phi]^2\right) + \frac{\text{X1thc}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2}$$

Out[∘]=

$$\text{X2zc}[\phi] - \frac{\text{X1thc}[\phi]\,\text{X1zc}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1ths}[\phi]\,\text{X1zs}'[\phi]}{2\,\text{R0}[\phi]} + \frac{\text{X1Rc}[\phi] \times \text{X1thc}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X1Rs}[\phi] \times \text{X1ths}[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X2thc}[\phi]\,\text{Z0}'[\phi]}{\text{R0}[\phi]} - \frac{\text{X1thc}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + \frac{\text{X1ths}[\phi]^2\,\text{R0}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^3} + \frac{\text{X1thc}[\phi]\,\text{X1thc}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]\,\text{X1ths}'[\phi]\,\text{Z0}'[\phi]}{2\,\text{R0}[\phi]^2} + \frac{\text{X1thc}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2} - \frac{\text{X1ths}[\phi]^2\,\text{Z0}''[\phi]}{4\,\text{R0}[\phi]^2}$$