Quantum Finance:

Using Quantum Methods to Speed-up Monte Carlo Simulations

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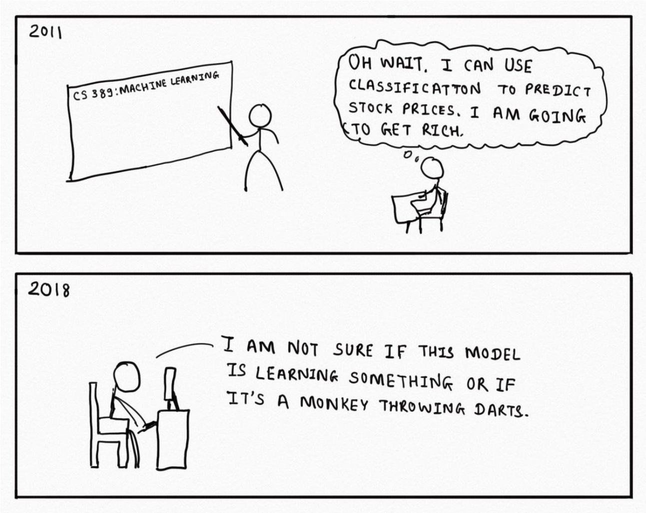
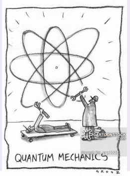
Background

Why did we choose this project?



2019



We can potentially predict prices <u>faster</u> and <u>more accurately</u> using quantum methods!

Finance Introduction

What is a European call option?

$$f(S_T) = \max\{0, S_T - K\}$$

IBM stock has been trading at ~$140 on the NYSE

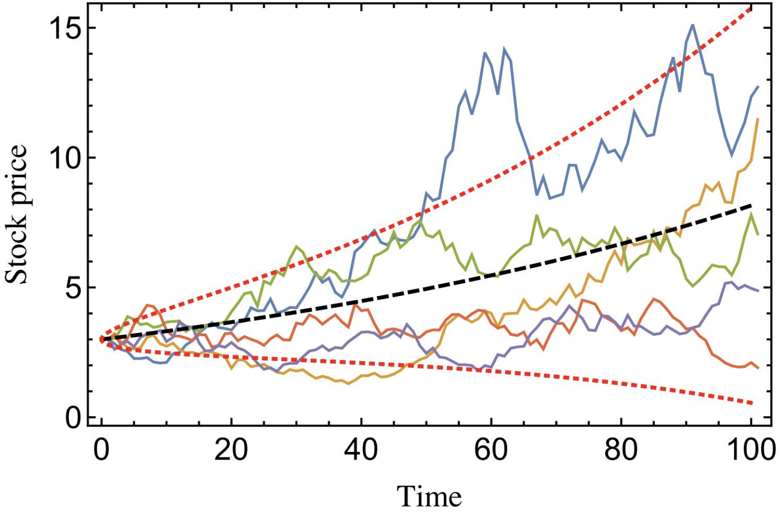
Assume that one month ago, Zach bought a European call option at a strike price of **$130** with an expiration date of today.

He can then sell the stock at its market value of **$140** for a profit of **$10**.

Finance Introduction

Analytical Pricing: The Black Scholes Model

By taking the limit of a binomial tree, underlying assets are modeled by a stochastic process known as Brownian Motion



$$S_t = S_0 e^{\sigma W_t + (\alpha - \sigma^2/2)t}$$

$$f(S_T) = \max\{0, S_T - K\}$$

$$\Pi = e^{-rT}\mathbb{E}_{\mathbb{Q}}[f(S_T)]$$

Improving this Pricing Method

Can a quantum algorithm be utilized in order to improve efficiency of Monte Carlo methods and decrease estimation error?

Quantum Monte Carlo Methods

Loading Uncertainty Models

Given:

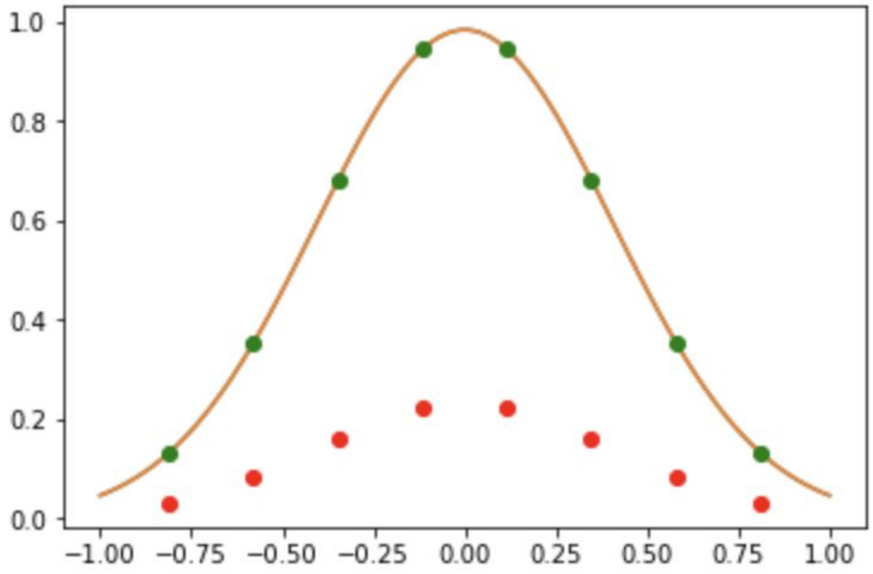
$$W_t - W_s \sim N(0, t - s)$$

We know:

$$p_T(x) = \frac{1}{\sqrt{2\pi T}} e^{-\frac{x^2}{2T}}.$$

Quantum Monte Carlo Methods

Qubits only hold discrete distributions



Quantum Monte Carlo Methods

Create the G gate

$$\mathcal{G}|0^n\rangle = \sum_{j=0}^{2^n-1} \sqrt{p_j}|j\rangle.$$

```
Find G_0:
[[ 0.17747501+0.j]
 [ 0.28963622+0.j]
 [ 0.40147945+0.j]
 [ 0.47268143+0.j]
 [ 0.47268143+0.j]
 [ 0.40147945+0.j]
 [ 0.28963622+0.j]
 [ 0.17747501+0.j]]
```

Quantum Monte Carlo Methods

Create the Chi State

The R gate encodes the payoff function v(x) in an ancillary qubit

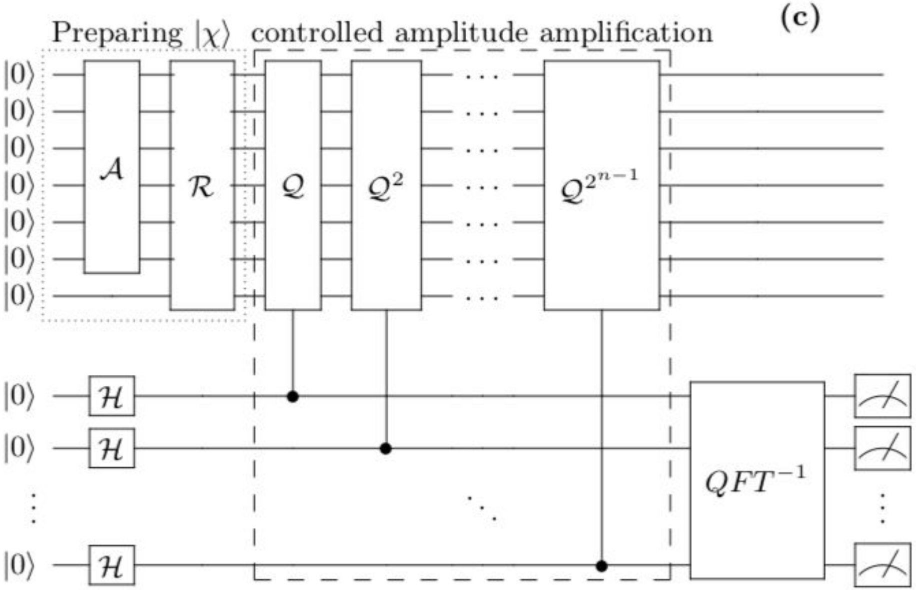
$$\mathcal{R}|x\rangle|0\rangle = |x\rangle\left(\sqrt{1-v(x)}|0\rangle + \sqrt{v(x)}|1\rangle\right).$$

$$\to \sum_{j=0}^{2^n-1} \sqrt{p(x_j)}|j\rangle \left(\sqrt{1-\tilde{v}(x_j)}|0\rangle + \sqrt{\tilde{v}(x_j)}|1\rangle\right) =: |\chi\rangle.$$

Measuring the final qubit gives the expectation

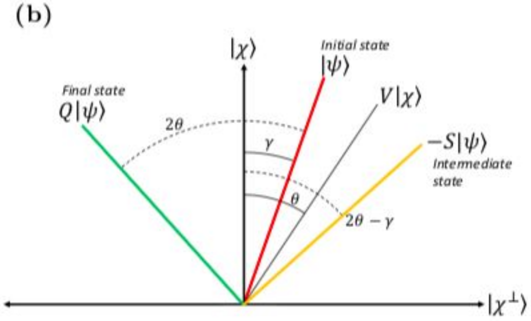
$$\mu = \langle\chi|(\mathcal{I}_{2^n} \otimes |1\rangle\langle 1|)|\chi\rangle = \sum_{j=0}^{2^n-1} p_T(x_j)\tilde{v}(x_j).$$

Quantum Circuit Overview



Q Gate and amplitude amplification

$$\mathcal{V}|\chi\rangle = \cos(\theta/2)|\chi\rangle + e^{i\phi}\sin(\theta/2)|\chi^{\perp}\rangle$$



Expectation value from multi-qubit state |χ⟩ can be calculated if we find θ

$$1 - 2\mu = \cos(\theta/2).$$

Our Simulation

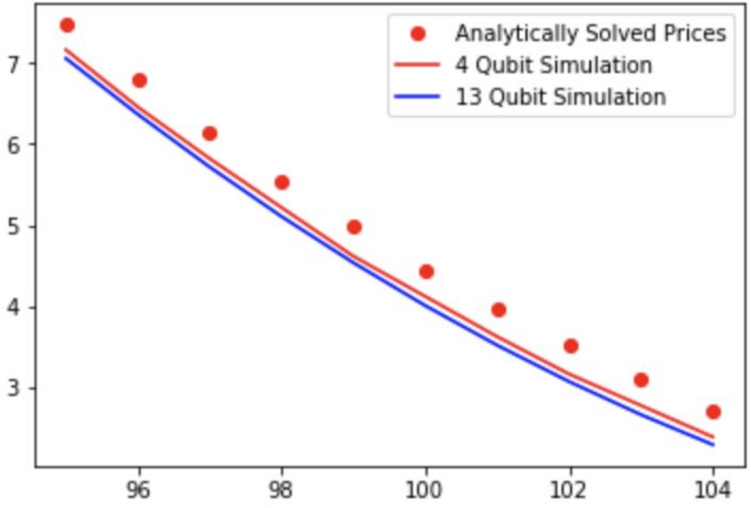
In [834]:

```python
# One Function to Price Option

def Quantum_Price(S, K, T, r, sigma, qubits):
    def v_euro(x):
        return max(0, S * np.exp((sigma * x) + (r - 0.5 * sigma * sigma)*T)
    dp = disc_points(T, sigma, 2, qubits)
    norm_probs = norm_disc_probs(dp, T)
    G_0 = G_on_zero(norm_probs, qubits)
    chi = get_Chi(G_0, dp, norm_probs, qubits, v_euro)
    eq = mu(chi, qubits)
    return eq * np.e ** (-r * T)

print(Quantum_Price(100, 95, 60/365, 0.05, 0.3, 3))
print(Quantum_Price(100, 95, 60/365, 0.05, 0.3, 6))
print(bs_price(100, 95, 60/365, 0.05, 0.3))
```

```
7.87276420958
7.65438447923
8.15000633124
```

Our Simulation

Quantum Generated Prices (brown) vs. Analytically Calculated Prices (blue)



Running on a Real Device

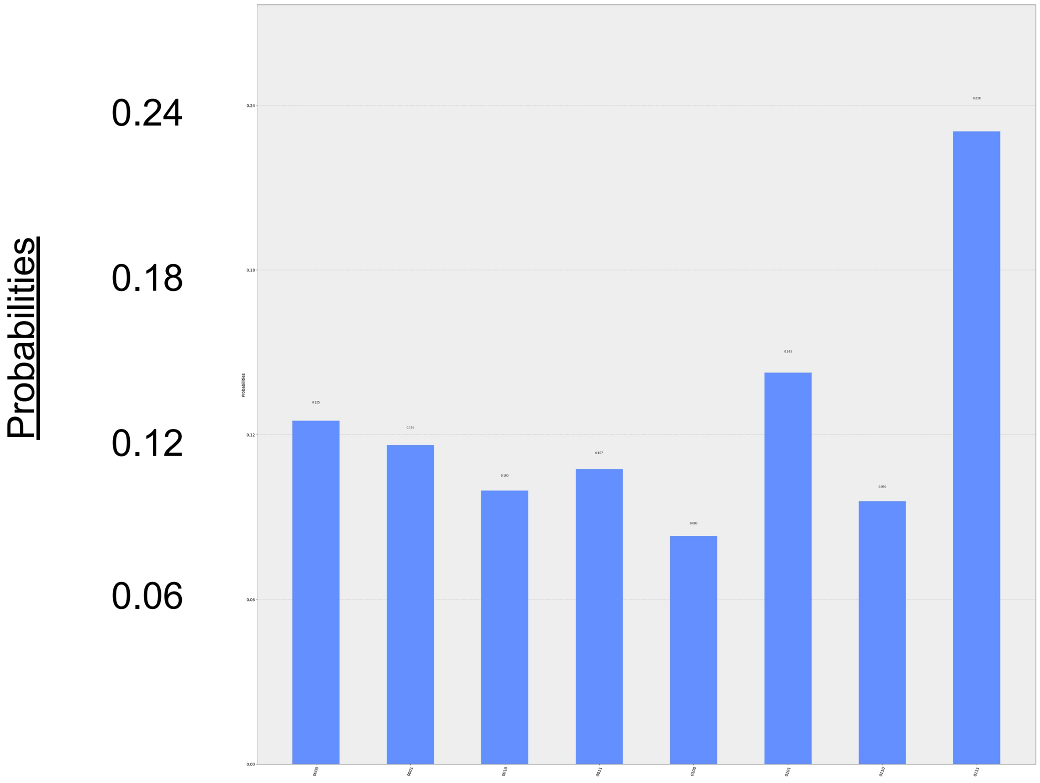
- **Qiskit Aqua Finance Package**

```
# construct circuit factory for payoff function
european_call = EuropeanCallExpectedValue(
    uncertainty_model,
    strike_price=strike_price,
    c_approx=c_approx
)
```

```
# number of ancillary qubits = number in uncertainty model+2
anc_qubits = num_uncertainty_qubits+2
N = num_uncertainty_qubits + anc_qubits

q = QuantumRegister(N)
c = ClassicalRegister(N)
qc = QuantumCircuit(q,c)
european_call.build(qc,q, q_ancillas=[q[i] for i in range(anc_qubits, N)])
qc.measure(q,c)
for i in range (0,7):
   qc.measure(q[i],c[i])
```

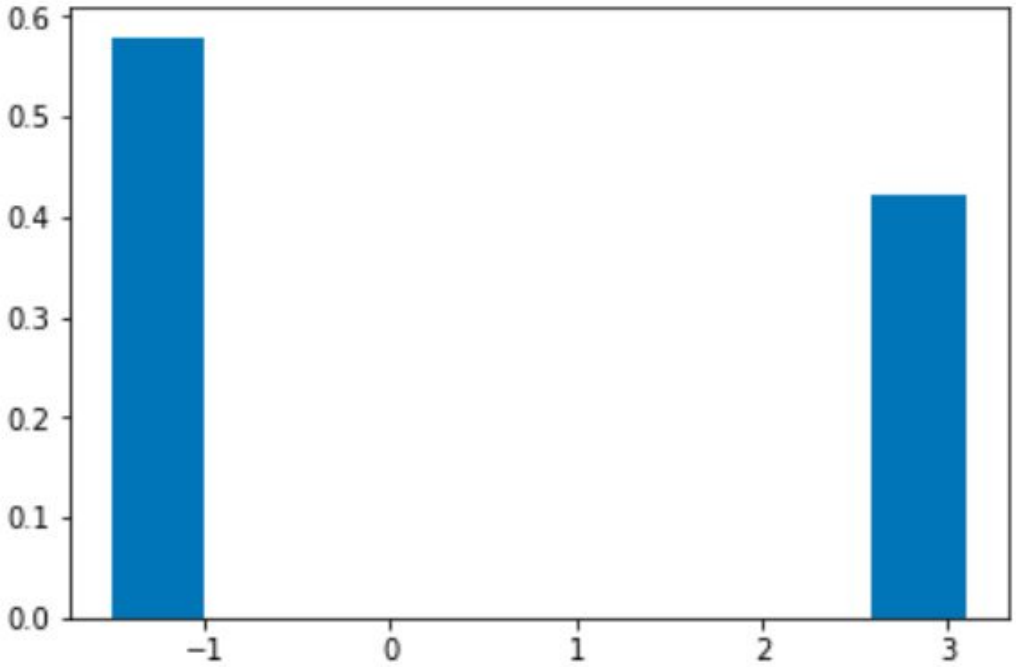
Real Device Results

5 Qubit Device (3 ancillary qubits measured)



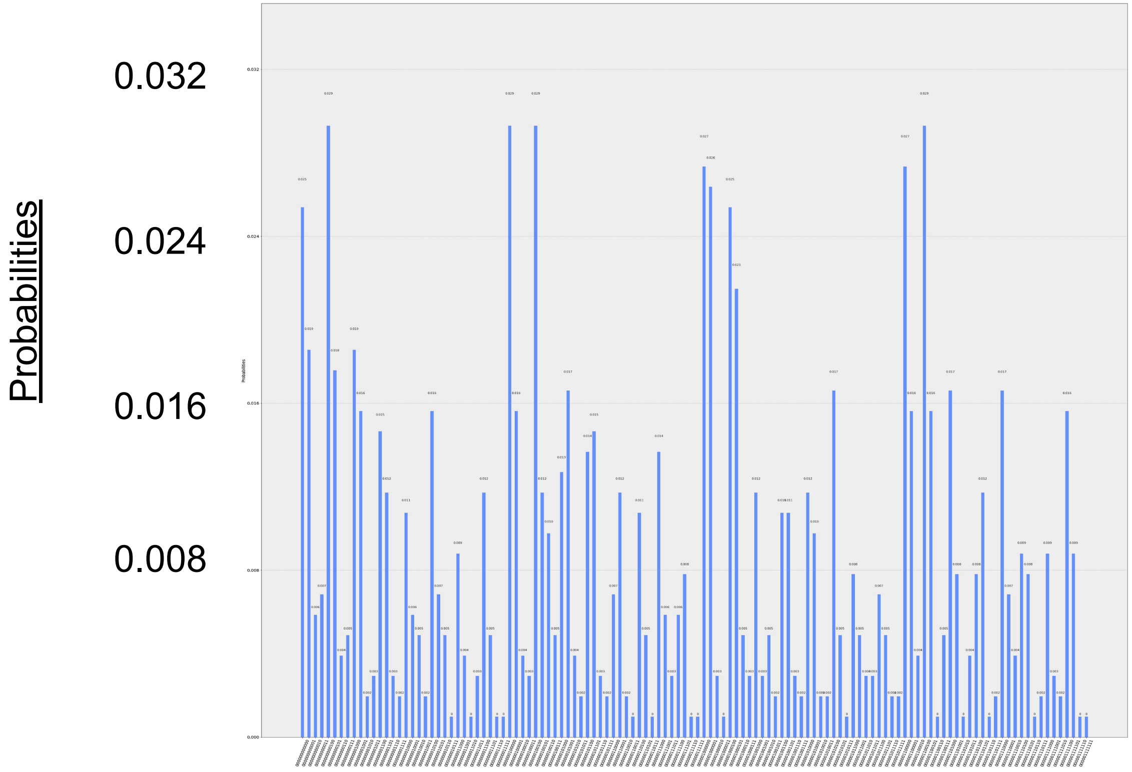
Real Device Results - Amplitude Amplification

With a strike price of \$2 and payoff price estimated at ~\$3, we can see a calculated payoff of \$1



Real Device Results

14 Qubit Device (7 ancillary qubits measured)



Future Work

- Future experimentation and analysis of results on the real device, along with successful amplitude estimation, could lead to meaningful conclusions about the speed-up

- Current theoretical speed-up is nearly quadratic, but robustness-to-noise is unknown

- Modifying “G” gate and changing payoff functions can allow for speed-up of pricing of more complex derivatives