Q2

Given a function $f(x)$ in one-dimension

First order derivative is $\frac{\partial f}{\partial x} = f(x+1) - f(x)$

Second order " " $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$

The image is a 2D function of two variables, $f(x,y)$

We have partial derivatives.

First order $\nabla f = \frac{\partial f(x,y)}{\partial x \partial y} = \frac{\partial f(x,y)}{\partial x} + \frac{\partial f(x,y)}{\partial y}$

Laplacian (Second order) $\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$

In the discrete form

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y) \right]$$

Image - Laplacian $= f(x,y) - \nabla^2 f(x,y)$

$= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$

$= 5f(x,y) - [f(x+1) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$

$= 6f(x,y) - 6f(x,y) - [\quad "\quad]$

$= 6f(x,y) - [f(x+1) + f(x-1,y) + f(x,y+1) + f(x,y-1) + f(x,y)]$

$= 5 \cdot \frac{6}{5} f(x,y) - \frac{5}{5} [\quad "\quad]$

$= 5 \left[\frac{6}{5} f(x,y) - \frac{1}{5} \bar{f}(x,y) \right]$