

1

$$\textcircled{a} \quad |\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin \theta$$

$$= 3 \times 5 \times \sin 90^\circ$$

$$= 15$$

$\theta = 90^\circ$  as  $\vec{u}$  and  $\vec{v}$  are orthogonal

$\textcircled{b} \quad \vec{u} \times \vec{v}$  is orthogonal to  $\vec{u}$   
 $\vec{u} \times \vec{v}$  is orthogonal to  $\vec{v}$

$x\text{-coordinate} > 0$

$\textcircled{c} \quad y\text{-coordinate} = 0$

$\textcircled{d} \quad z\text{-coordinate} > 0$

$$0 = (0-09)2 + (0-02)3 - (81-08)1 \quad \Leftarrow$$

2

Given

$$|\vec{u}| = 2\sqrt{2}, |\vec{v}| = 2\sqrt{2}$$

$$|\vec{u} - \vec{v}| = 2\sqrt{2}$$

$$|\vec{u} + \vec{v}| = ? , (\vec{u}, \vec{v}) = ?$$

(a)

$$|\vec{u} - \vec{v}| = \sqrt{u^2 + v^2 - 2uv \cos \theta}$$

$$\Rightarrow 2\sqrt{2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2 - 2(2\sqrt{2})(2\sqrt{2}) \cos \theta}$$

$$\Rightarrow 2\sqrt{2} = \sqrt{8 + 8 - 16 \cos \theta}$$

$$= \sqrt{16 - 16 \cos \theta}$$

$$\Rightarrow 2\sqrt{2} = 4\sqrt{1 - \cos \theta}$$

$$\Rightarrow 18 = 16^2 (1 - \cos \theta)$$

$$\Rightarrow \frac{1}{2} = 1 - \cos \theta$$

$$\Rightarrow \cos \theta = 1 - 1/2 = 1/2$$

$$\Rightarrow \theta = 60^\circ$$

$$|\vec{u} + \vec{v}| = \sqrt{u^2 + v^2 + 2uv \cos \theta}$$

$$= \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2 + 2(2\sqrt{2})(2\sqrt{2}) \cos 60^\circ}$$

$$= \sqrt{8 + 8 + 2(8) \times \frac{1}{2}}$$

$$= \sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$$

(b)

$$\theta = \pi/3$$



3

(a)

$$a = 10$$

Leading entry of first row is 1  
Row echelon form has multiple properties.

- (P<sub>1</sub>) All the leading entries in each of the rows of the matrix are 1

$$\begin{bmatrix} 1 & 3 & 2 \\ a & 6 & 2 \\ 0 & 9 & 5 \end{bmatrix}$$

- (P<sub>2</sub>) If a column contains a leading entry then all entries below the leading entry are zero.

$$\therefore \underline{a = 0}$$

- (b) For what values of  $a$  is the matrix singular?

- A square matrix is singular if and only if its determinant is zero.

$$\Rightarrow 1(30-18) - 3(5a-0) + 2(9a-0) = 0$$

$$\Rightarrow 12 - 15a + 18a = 0$$

$$\Rightarrow 12 + 3a = 0$$

$$\Rightarrow \underline{a = -4}$$