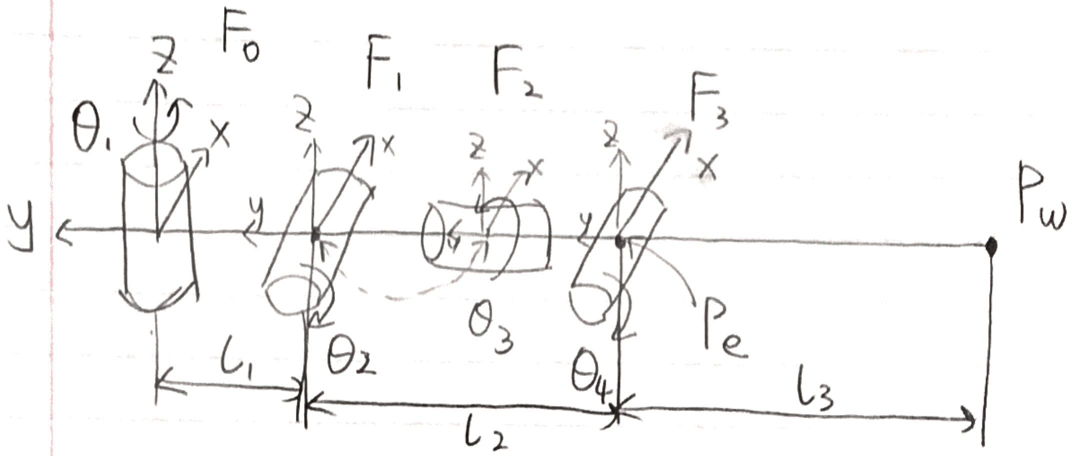
$T_0^1 = rot_z(\theta_1)\, trans_y(-l_1)$

$T_2^1 = rot_x(\theta_2)$

$T_3^2 = rot_y(\theta_3)\, trans_y(-l_2)$

$T_4^3 = rot_x(\theta_4)$

Use P_e, P_w to solve baxter θ_{1-4}



On P_e. P_e depends on theta1, theta2

$$T_1^0\, T_2^1 \begin{bmatrix} 0 \\ -l_2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} P_e \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l_2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & s_1 l_1 \\ s_1 & c_1 & 0 & -c_1 l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -c_2 l_2 \\ -s_2 l_2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} s_1 c_2 l_2 + s_1 l_1 \\ -c_1 c_2 l_2 - c_1 l_1 \\ -s_2 l_2 \end{bmatrix} = \begin{bmatrix} Pex \\ Pey \\ Pez \end{bmatrix}$$

$$\begin{cases} s_1 c_2 l_2 + s_1 l_1 = ex & ① \\ c_1 c_2 l_2 + c_1 l_1 = -ey & ② \\ -s_2 l_2 = ez & ③ \end{cases}$$

$③ \Rightarrow \quad S_2 = -\dfrac{e_z}{l_2}$

$①^2 + ②^2 \Rightarrow \quad (C_2 l_2 + l_1)^2 = e_x^2 + e_y^2$

$\Downarrow$

$C_2^2 l_2^2 + l_1^2 + 2C_2 l_1 l_2 = e_x^2 + e_y^2$

$+ ③^2$ on both side

$C_2^2 l_2^2 + S_2^2 l_2^2 + l_1^2 + 2C_2 l_1 l_2 = e_x^2 + e_y^2 + e_z^2$

$\Rightarrow \quad C_2 = \dfrac{e_x^2 + e_y^2 + e_z^2 - l_1^2 - l_2^2}{2 l_1 l_2}$

$\theta_2 = \operatorname{atan2}(S_2, C_2)$

$① \Rightarrow \quad S_1 = \dfrac{e_x}{l_1 + C_2 l_2}$

$② \Rightarrow \quad C_1 = \dfrac{-e_y}{l_1 + C_2 l_2}$

$\theta_1 = \operatorname{atan2}(S_1, C_1)$

On P_w,

$P_w = T_1^0 T_2^1 T_3^2 T_4^3 \begin{bmatrix} 0 \\ -l_3 \\ 0 \\ 1 \end{bmatrix}$

$T_0^1 T_2^1$ are known

$\Rightarrow \quad T_3^2 T_4^3 \begin{bmatrix} 0 \\ -l_3 \\ 0 \\ 1 \end{bmatrix} = T_2^{1^{-1}} T_1^{0^{-1}} P_w = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$

$$\begin{bmatrix} C_3 & 0 & S_3 & 0 \\ 0 & 1 & 0 & 0 \\ -S_3 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_4 & -S_4 & 0 \\ 0 & S_4 & C_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l_3 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ 0 & 1 & 0 & -l_2 \\ -S_3 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -C_4 l_3 \\ -S_4 l_3 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -S_3 S_4 l_3 \\ -C_4 l_3 - l_2 \\ -C_3 S_4 l_3 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \rightarrow \text{known} \quad \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$$\begin{cases} -S_3 S_4 l_3 = P_1 & ① \\ -C_4 l_3 - l_2 = P_2 & ② \\ -C_3 S_4 l_3 = P_3 & ③ \end{cases}$$

$$② \Rightarrow C_4 = \frac{P_2 + l_2}{-l_3}$$

$$①^2 + ③^2 \Rightarrow P_1^2 + P_3^2 = S_4^2 l_3^2$$

On baxter, θ_4 range is $(-2.864, 150)$

assume here θ_4 range is $(0, 150)$

$$\Rightarrow S_4 = \sqrt{\frac{P_1^2 + P_3^2}{l_3^2}}$$

$$\theta_4 = \operatorname{atan2}(S_4, C_4)$$

① $\Rightarrow$ $s_3 = -P_1 / s_4 \cdot l_3$

③ $\Rightarrow$ $c_3 = -P_3 / s_4 l_3$

$\theta_3 = \operatorname{atan2}(s_3, c_3)$