20170082 Dongwon Kim EE412 HW#2

Please note that I use one late day token for this assignment.

1

1-(a)

Suppose the set is $S = \{a, aa, aaaa, aaaaaaaa, aaaaaaaaaaaaaaaa\}$ Then the editdistance for each element to the rest of the element is:

$$
\begin{aligned}
&editdistance(a, aa) = 1\\
&editdistance(a, aaaa) = 3\\
&editdistance(a, aaaaaaaa) = 7\\
&editdistance(a, aaaaaaaaaaaaaaaa) = 15\\
&editdistance(aa, aaaa) = 2\\
&editdistance(aa, aaaaaaaa) = 6\\
&editdistance(aa, aaaaaaaaaaaaaaaa) = 14\\
&editdistance(aaaa, aaaaaaaa) = 4\\
&editdistance(aaaa, aaaaaaaaaaaaaaaa) = 12\\
&editdistance(aaaaaaaa, aaaaaaaaaaaaaaaa) = 8
\end{aligned}
$$

So the sum of the editdistance for each element is,

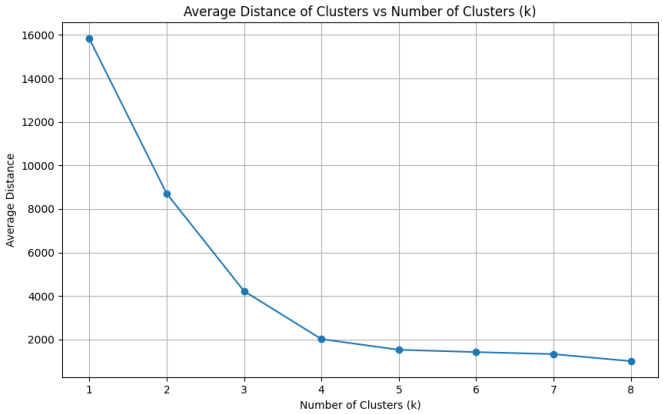
$$
\begin{aligned}
a &\rightarrow 1 + 3 + 7 + 15 = 26\\
aa &\rightarrow 1 + 2 + 6 + 14 = 23\\
aaaa &\rightarrow 3 + 2 + 4 + 12 = 21\\
aaaaaaaa &\rightarrow 7 + 6 + 4 + 8 = 25\\
aaaaaaaaaaaaaaaa &\rightarrow 15 + 14 + 12 + 8 = 49
\end{aligned}
$$

Where the clustroid becomes aaaa with the minimum sum of editdistance, 21. The maximum of the editdistance for each element is,

$$
\begin{aligned}
a &\rightarrow 15\\
aa &\rightarrow 14\\
aaaa &\rightarrow 12\\
aaaaaaaa &\rightarrow 8\\
aaaaaaaaaaaaaaaa &\rightarrow 15
\end{aligned}
$$

Where the clustroid becomes aaaaaaaa with the minimum of 15.

1-(b)



The k value with an explanation why it is good for this data. The answer is 5, as the difference between the average distance begins to decrease insignificantly (i.e. smaller than 10%) after k = 5. The difference is over 10% in case of k = 8, but this requires more clusters, increasing model complexity. If higher complexity (i.e. more clusters) is desirable, then k = 8 is also a good choice.

2

11.1.7

Following is the code for the problem.

```
import numpy as np


def power_iteration(A, B, nsim):
    # Choose a random starting vector
    b_k = B

    for _ in range(nsim):
        # Calculate the matrix-by-vector product Ab
        b_k1 = np.dot(A, b_k)

        # Re normalize the vector
        b_k = b_k1 / np.linalg.norm(b_k1)

    return b_k


def main():
    A = np.array([[1, 1, 1], [1, 2, 3], [1, 3, 6]])
    B = np.array([1, 1, 1])

    eigvec = power_iteration(A, B, nsim=100)
```

```
        eigval = np.dot(np.dot(A, eigvec), eigvec)

        print("eigvec: ", np.around(eigvec, decimals=3))
        print("eigval: ", np.around(eigval, decimals=3))

        # second eigenvalue
        A1 = A - eigval * np.outer(eigvec, eigvec)
        print("A: ", np.around(A1, decimals=3))

        eigvec = power_iteration(A1, B, nsim=100)
        eigval = np.dot(np.dot(A1, eigvec), eigvec)

        print("eigvec: ", np.around(eigvec, decimals=3))
        print("eigval: ", np.around(eigval, decimals=3))

        # third eigenvalue
        A2 = A1 - eigval * np.outer(eigvec, eigvec)
        print("A: ", np.around(A2, decimals=3))

        eigvec = power_iteration(A2, B, nsim=100)
        eigval = np.dot(np.dot(A2, eigvec), eigvec)

        print("eigvec: ", np.around(eigvec, decimals=3))
        print("eigval: ", np.around(eigval, decimals=3))


    if __name__ == "__main__":
        main()
```

The result for the problem is as follows.

(a) $\begin{bmatrix} 0.194 \\ 0.472 \\ 0.86 \end{bmatrix}$

(b) 7.873

(c) $A = \begin{bmatrix} 0.704 & 0.279 & -0.312 \\ 0.279 & 0.244 & -0.197 \\ -0.312 & -0.197 & 0.179 \end{bmatrix}$

(d) $\begin{bmatrix} 0.816 \\ 0.408 \\ -0.408 \end{bmatrix}, 1.0$

(e) $A = \begin{bmatrix} 0.038 & -0.054 & 0.021 \\ -0.054 & 0.078 & -0.03 \\ 0.021 & -0.03 & 0.012 \end{bmatrix}$

eigvec: $\begin{bmatrix} 0.544 \\ -0.781 \\ 0.306 \end{bmatrix}$

eigval: 0.127

11.1.7

Following is the code for the problem.

```
import numpy as np

def main():
    M = [[1, 2, 3], [3, 4, 5], [5, 4, 3], [0, 2, 4], [1, 3, 5]]
    M = np.array(M)
    transpose_M = np.transpose(M)

    MtM = np.matmul(transpose_M, M)
    print("transpose(M)*M:")
    print(MtM)

    MMt = np.matmul(M, transpose_M)
    print("M*transpose(M):")
    print(MMt)

    # Find eigenpairs
    eigval_MtM, eigvec_MtM = np.linalg.eig(MtM)
    # sort eigenpairs
    idx = eigval_MtM.argsort()[::-1]
    eigval_MtM = eigval_MtM[idx]
    eigvec_MtM = eigvec_MtM[:, idx]

    eigval_MMt, eigvec_MMt = np.linalg.eig(MMt)
    # sort eigenpairs
    idx = eigval_MMt.argsort()[::-1]
    eigval_MMt = eigval_MMt[idx]
    eigvec_MMt = eigvec_MMt[:, idx]

    print("Eigenvalues of Transpose(M)*M:")
    print(np.around(eigval_MtM, decimals=3))
    print("Eigenvectors of Transpose(M)*M:")
    print(np.around(eigvec_MtM, decimals=3))

    print("Eigenvalues of M*Transpose(M):")
    print(np.around(eigval_MMt, decimals=3))
    print("Eigenvectors of M*Transpose(M):")
    print(np.around(eigvec_MMt, decimals=3))

    # find SVD using above only two eigenpairs
    V = eigvec_MtM[:, :2]
    S = np.sqrt(np.diag(eigval_MtM[:2]))

    # calculate U using V and S
    U = np.matmul(np.matmul(M, V), np.linalg.inv(S))

    print("U:")
    print(np.around(U, decimals=3))
    print("S:")
    print(np.around(S, decimals=3))
    print("V:")
    print(np.around(V, decimals=3))
    print("U*S*V^T:")
    print(np.around(np.matmul(np.matmul(U, S), V.T), decimals=3))
    print("M:")
```

```
    print(np.around(M, decimals=3))

    # rank 1 approximation
    U1 = U[:, :1]
    S1 = S[:1, :1]
    V1 = V[:, :1]
    M1 = np.matmul(np.matmul(U1, S1), V1.T)
    print("rank 1 approximation of M:")
    print(np.around(M1, decimals=3))

    print("energy of the original singular values:")
    print(np.around(np.sum(S**2), decimals=3))
    print("energy of the one-dimensional approximation:")
    print(np.around(np.sum(S1**2), decimals=3))


if __name__ == "__main__":
    main()
```

The result for the problem is as follows.

(a) The matrices M^TM and MM^T are given by:

$$M^TM = \begin{bmatrix} 36 & 37 & 38 \\ 37 & 49 & 61 \\ 38 & 61 & 84 \end{bmatrix}$$

and

$$MM^T = \begin{bmatrix} 14 & 26 & 22 & 16 & 22 \\ 26 & 50 & 46 & 28 & 40 \\ 22 & 46 & 50 & 20 & 32 \\ 16 & 28 & 20 & 20 & 26 \\ 22 & 40 & 32 & 26 & 35 \end{bmatrix}$$

(b) The eigenpairs for the matrices M^TM and MM^T are:

For M^TM:

Eigenvalues: $[153.567, 15.433, 0.0]$
Eigenvectors:

$$\begin{bmatrix} -0.409 & -0.816 & 0.408 \\ -0.563 & -0.126 & -0.816 \\ -0.718 & 0.564 & 0.408 \end{bmatrix}$$

For MM^T:

Eigenvalues: $[153.567, 15.433, 0.0, -0.0, -0.0]$

Eigenvectors:

$$\begin{bmatrix} 0.298 & -0.159 & 0.125 & 0.075 & 0.941 \\ 0.571 & 0.033 & -0.453 & -0.073 & -0.175 \\ 0.521 & 0.736 & 0.326 & -0.106 & -0.04 \\ 0.323 & -0.51 & 0.72 & -0.726 & -0.188 \\ 0.459 & -0.414 & -0.393 & 0.672 & -0.215 \end{bmatrix}$$

(c) The Singular Value Decomposition (SVD) for the matrix M is:
U:

$$\begin{bmatrix} -0.298 & 0.159 \\ -0.571 & -0.033 \\ -0.521 & -0.736 \\ -0.323 & 0.51 \\ -0.459 & 0.414 \end{bmatrix}$$

Σ:

$$\begin{bmatrix} 12.392 & 0 \\ 0 & 3.928 \end{bmatrix}$$

V:

$$\begin{bmatrix} -0.409 & -0.816 \\ -0.563 & -0.126 \\ -0.718 & 0.564 \end{bmatrix}$$

(d) The one-dimensional approximation to the matrix M is:

$$\begin{bmatrix} 1.51 & 2.079 & 2.647 \\ 2.894 & 3.984 & 5.074 \\ 2.641 & 3.636 & 4.631 \\ 1.636 & 2.252 & 2.869 \\ 2.328 & 3.205 & 4.082 \end{bmatrix}$$

(e) The energy of the original singular values is 169.0. The energy of the one-dimensional approximation is 153.567. The fraction of energy retained is given by the ratio of the energy of the approximation to the original energy.

3-(a)

9.3.1

(a)

	a	b	c	d	e	f	g	h
A	1	1	0	1	1	0	1	1
B	0	1	1	1	1	1	1	0
C	1	0	1	1	0	1	1	1

$$D_J(A, B) = 1 - \frac{|A \cap B|}{|A \cup B|}$$

Computing for each pair:

$D_J(A,B) = 1 - \frac{4}{8} = \frac{1}{2}$
$D_J(A,C) = 1 - \frac{4}{8} = \frac{1}{2}$
$D_J(B,C) = 1 - \frac{4}{8} = \frac{1}{2}$

(b) Cosine distance is given by:

$$D_C(A,B) = acos(\frac{A \cdot B}{\|A\|\|B\|})$$

Computing for each pair:

$D_C(A,B) = acos(\frac{4}{6}) = 0.841$
$D_C(A,C) = acos(\frac{4}{6}) = 0.841$
$D_C(B,C) = acos(\frac{4}{6}) = 0.841$

(c)

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

$D_J(A,B) = 1 - \frac{2}{5} = \frac{3}{5}$
$D_J(A,C) = 1 - \frac{2}{6} = \frac{2}{3}$
$D_J(B,C) = 1 - \frac{1}{6} = \frac{5}{6}$

(d) $D_C(A,B) = acos(\frac{2}{2\sqrt{3}}) = 0.955$
$D_C(A,C) = acos(\frac{2}{4}) = 1.047$
$D_C(B,C) = acos(\frac{1}{2\sqrt{3}}) = 1.278$

(e) $AVG(A) = (4+5+5+1+3+2)/6 = 20/6 = 10/3$
$AVG(B) = (3+4+3+1+2+1)/6 = 14/6 = 7/3$
$AVG(C) = (2+1+3+4+5+3)/6 = 18/6 = 3$

The utility matrix becomes:

	a	b	c	d	e	f	g	h
A	$2/3$	$5/3$	0	$5/3$	$-7/3$	0	$-1/3$	$-4/3$
B	0	$2/3$	$5/3$	$2/3$	$-4/3$	$-1/3$	$-4/3$	0
C	-1	0	-2	0	0	1	2	0

(f) $D_C(A,B) = acos(\frac{52/9}{\sqrt{120/9}\sqrt{66/9}}) = 0.9468$

$D_C(A,C) = acos(\frac{-4/3}{\sqrt{120/9}\sqrt{10}}) = 1.687$

$D_C(B,C) = acos(\frac{-19/3}{\sqrt{66/9}\sqrt{10}}) = 2.403$

9.3.2

(a) The utility matrix becomes,

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

The smallest Jaccard distance is between f and h, whose distance is 0. So the first cluster is {f, h}.
The second smallest is between b and d, whose distance is 1/3, so the second cluster is {b, d}.
The third smallest is between {b, d} and g, whose distance is 1/3, so the third cluster is {b, d, g}.
The fourth smallest is between {b, d, g} and c, whose distance is 1/2, so the fourth cluster is {b, c, d, g}.
Therefore, the final clusters are {a}, {e}, {f, h}, {b, c, d, g}.

(b) For user A, the average of {f, h} is 2, the average of {b, c, d, g} is 13/3.
For user B, the average of {f, h} is 2, the average of {b, c, d, g} is 11/3.
For user C, the average of {f, h} is 7/2, the average of {b, c, d, g} is 3.
The utility matrix becomes,

	$\{a\}$	$\{e\}$	$\{f,h\}$	$\{b,c,d,g\}$
A	4	1	2	13/3
B	*null*	1	2	11/3
C	2	*null*	7/2	3

(c) Considering null is 0, we have,

$D_C(A,B) = acos(\frac{188/9}{\sqrt{358/9}\sqrt{166/9}}) = 0.690$

$D_C(B,C) = acos(\frac{18}{\sqrt{166/9}\sqrt{101/4}}) = 0.584$

$D_C(C,A) = acos(\frac{28}{\sqrt{101/4}\sqrt{358/9}}) = 0.487$