

## 1.1 Kinematics

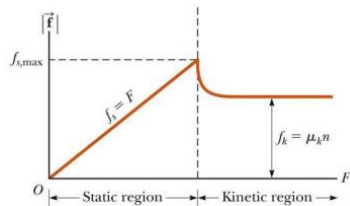
$$v = \frac{\Delta x}{\Delta t} \quad a = \frac{\Delta v}{\Delta t}$$

### Newtonian mechanics

$$1^{\text{st}} + 2^{\text{nd}}: \vec{F} = m \times \vec{a}$$

$$3^{\text{rd}}: \vec{F1} = -\vec{F2}$$

### Friction force



### Circular motion

$$F = m \frac{v^2}{r} = m \omega^2 r$$

### Gravitation

$$F = G \frac{m_1 m_2}{r^2} \quad (G = 6.673 \times 10^{-11})$$

### Energy

$$KE = \frac{1}{2} m v^2$$

$$PE = mgh$$

## 1.2 Forces & Motion in Liquids

### Pressure

$$P = \frac{F}{A} \text{ (pa)}$$

$$P = \rho gh \text{ (In liquid of depth h)}$$

**Pascal Principle:** A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

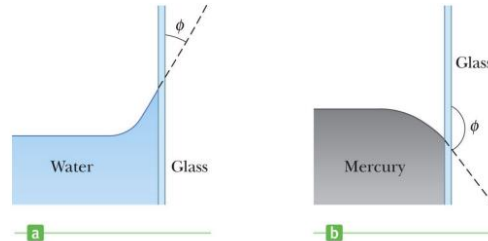
**One atmosphere (1 atm)** = 76.0 cm of mercury

$$= 1.013 \times 10^5 \text{ Pa}$$

### Buoyant force

$$F = \rho g V = \text{weight displaced}$$

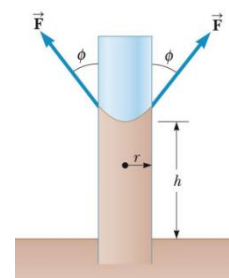
### Surface Tension



$\Phi$  is called the contact angle

- In a,  $\phi < 90^\circ$  and adhesive forces are greater than cohesive forces
- In b,  $\phi > 90^\circ$  and cohesive forces are greater than adhesive forces
- A lifting or pressing force directed along the contact angle

### Capillary action



$$F = \gamma(2\pi r)$$

$\gamma$  is a constant decided by liquid & solid

$$h = \frac{2\gamma}{\rho g r} \cos \phi$$

The weight of the lifted liquid

$$w = Mg = \rho V g \approx \rho g(\pi r^2 h)$$

### Flow of liquid

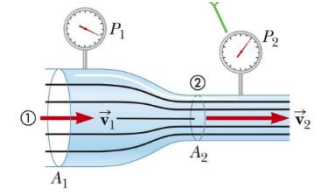
Poiseuille's Law: the volume flow rate of a fluid in a tube is proportional to the pressure difference

$$\text{Rate of flow:} \quad \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8 \eta L}$$

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

### Bernoulli's Equation

$$P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant}$$



$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

### Liquid viscosity

A fluid between two solid surfaces (the bottom surface is fixed and the top moveable) A force moves upper surface by a constant speed  $v$  so that the force equals to friction

$$F = \eta \frac{A v}{d}$$

$A$  is contact area between the upper surface and liquid  
 $\eta$  is the coefficient of viscosity ( N s/m<sup>2</sup> )

### Sedimentation rate & centrifuge

Frictional coefficient for a small, spherical object (good approximation for cells & biomolecules)

$$K_r = 6\pi\eta r$$

The speed at which materials fall through a fluid to reach the bottom of a container is called the **sedimentation rate**, which is often determined by the terminal speed of the materials in the fluid

$$v_t = \frac{mg}{k_r} \left( 1 - \frac{\rho_f}{\rho} \right) \quad \begin{matrix} m & \text{- mass of the object} \\ \rho(\rho_f) & \text{- density of the object (fluid)} \end{matrix}$$

For an object in centrifuge,  $g$  is replaced by  $\omega^2 r$   
 ( $\omega$  - angular speed of the rotation, here combines into centripetal acceleration)

### Reynolds number

Consider an object (e.g., a cubic fluid parcel) of size  $L$  moving at speed  $U$  in liquid

$$Re = \frac{\rho U^2 / L}{\eta U / L^2} = \rho L U / \eta = \frac{\text{kinetic energy}}{\text{dissipated energy}}$$

$(C_2 - C_1)/L$  is concentration change per unit of distance along the flow direction

$D$  is the diffusion coefficient

## Diffusion

$A$  is cross-sectional area of the tube

Diffusion rate is proportional to concentration difference

$$\text{Diffusion rate} = \frac{\text{Mass}}{\text{time}} = DA \left( \frac{C_2 - C_1}{L} \right)$$

## 1.3 Electric-magnetic force & motion

**Coulomb's Law**  $K_e = 8.9875 \times 10^9 \text{ N} \frac{\text{m}^2}{\text{C}^2}$

$$F = k_e \frac{|q_1| |q_2|}{r^2} \quad \text{Like charges repel, unlike charges attract}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

## Electric Field (N / C)

Field at the position of the test charge is defined as the electric force on it divided by its charge

$$\vec{E} = \frac{\vec{F}}{q_0} \quad E = |\vec{E}| = \frac{k_e q}{r^2} \quad \text{Direction of the field is that of the force on the positive test charge}$$

## Electric Flux (N m<sup>2</sup> / C)

Electric flux is a measure of how much the electric field vectors penetrate through a given surface.

$\Phi_E = E A \cos \theta$ , when the perpendicular to the area  $A$  is at an angle  $\theta$  to the field

## Gauss' Law

The electric flux passing through any closed surface from inside to outside is equal to the net charge  $Q$  inside the surface divided by  $\epsilon_0$  (permittivity of free space and equals  $8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ )

$$\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_0} \quad \text{Coulomb constant} \quad k_e = \frac{1}{4\pi\epsilon_0}$$

## Electric Potential Energy

The change of electric potential energy of a charge  $q$  making a displacement  $\Delta x$  in the direction of a constant field  $E$  is defined as

$$\Delta PE = -q E \Delta x \quad PE = k \frac{Q}{r} \text{ (point charge)}$$

$$C = \kappa C_0 = \kappa \epsilon_0 (A/d)$$

**Capacitor**  $\kappa$  is called the **dielectric constant**.

$\sigma$  = charge density

The constant of proportionality is called **capacitance**,  $C$ , of a capacitor (F)

$$Q = C \Delta V; C = Q / \Delta V$$

$$\text{Energy} = \frac{1}{2} Q \Delta V$$

## Conductor

1: The electric field is zero everywhere inside the conductor

2: Any excess charge on an isolated conductor resides entirely on its surface

3: The electric field just outside a charged conductor is perpendicular to the conductor's surface

4: On an irregularly shaped conductor, the charge accumulates at sharp points

## Current

$n$  is the number of charge carriers per unit volume;  $q$  is the charge per carrier

$$\Delta Q = (n A \Delta x) q$$

$$\text{Current-drift speed relation } I = \frac{\Delta Q}{\Delta t} = n q v_d A$$

## Ohms law

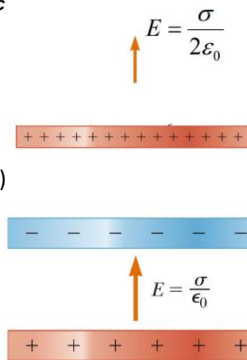
$$R = \frac{\Delta V}{I} \quad R = \rho \frac{\ell}{A}$$

$$\text{Power} = I^2 R$$

## 1.3.2 Magnetic force, field & motion

$$F = B q v \sin \theta$$

Right Hand Rule (positive charge) =>



## Magnetic Field (Tesla)

$$B = \frac{F}{qv \sin \theta}$$

$$T = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

## Force on a Charged Particle in a Magnetic Field

Causes uniform circular motion

$$F = qvB = \frac{mv^2}{r} \quad r = \frac{mv}{qB}$$

## Mass Spectrometer

The perpendicular component still follows the circular path  
The parallel component remains a constant, why?

$$v_{\text{para}} = \text{const} \tan t$$

Combined the two motional components, the particle follows a spiral

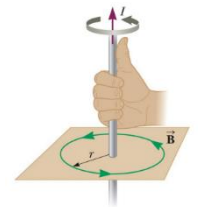
Two particles of small mass difference can have large radius difference by shooting them into a magnetic field with a high speed

## Magnitude of the Field of a Long Straight Wire

The magnitude of the field at a distance  $r$  from a wire carrying a current of  $I$  is

$$\mu_0 = 4 \pi \times 10^{-7} \text{ T m / A}$$

$$B = \frac{\mu_0 I}{2 \pi r} \quad \mu_0 \text{ is called the permeability of free space}$$



II

( Done by DDX-510 )

