IGL Project Plan

Homotopy, Modular Forms, and the Spectrum $\mathbb{Q}(\ell)$

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1 Goals

As a part of the spectral sequences arising from the computation of the stable homotopy groups of spheres, the computation of certain endomorphisms on $R = \mathbb{Z}_3[\![j]\!][q_2, q_4]$ are of particular interest. This ring, while arising from homotopy theory, has intimate connections with modular forms which in turn can be studied via elliptic curves. To (1) motivate the study of endomorphisms of R, in particular the maps arising from Sai's work, and to (2) demonstrate novel computations for the kernels and cokernels of these endomorphisms. This will give information on the homotopy groups of spheres at place 3 and height 2. Compared to the previous semester, this semesters focus will be more heavily on the connective spectra and their associated rings which are not completed or localized. We hope that information gained by these investigations can give us more information about what happens in the non-connective case.

1.1 Understanding The Theory

A major part in figuring out how to compute the (co)kernels of the map of interest is to determine where the maps come from (to an extent) and to understand what they do. To do this, we will look over resources on the subject starting with Silverman's *The Arithmetic Of Elliptic Curves* and Mark Behrens' paper "A modular description of the $K(2)$-local sphere at the prime 3". Another reference of interest if *A First Course in Modular Forms* by Diamond and Shurman. In terms of which references/methods of approach we will focus on, we've decided upon the following focus:

1. Focus on Silverman/Algebraic Aspects
   (a) Garrett
   (b) Elias
2. Focus on Diamond and Shurman/Analytic Aspects
   (a) Dimitrios
   (b) Rishi

Along with gaining an understanding of the baseline theory needed to effectively work with our objects of interest, we will also learn the rudimentary elements of the homotopy theory related to this project.

1.2 Computation

We will use CAS such as MAGMA to explicitly compute the (co)kernels of interest. This will involve both learning the capabilities of MAGMA as a software as well as discovering the methods by which we can effectively compute the (co)kernels. We will also use MAGMA and possibly other Computer Algebra Systems to work with representations of modular forms in order to get intuition as to what the homotopy maps are doing at the level of modular form endomorphisms.

1.3 Specific Goals

1. Understanding the origin of the ring and the maps in question.
2. Discovering properties of the endomorphism that aid in computation.
3. Learning how to work with MAGMA.
4. Developing routines to compute the kernels and cokernels of the endomorphism map.
5. Attempting to understand the analytic meaning of maps like ψ when the base ring is $\mathbb{C}$ to gain intuition about what happens over other base rings.
6. Producing a paper at the end to communicate both the theory and results in a novel manner.

2 Responsibilities

All undergraduates will be expected to familiarize themselves with what readings are suggested, understand the exercises provided, and develop a good base for understanding the more complicated parts of the project. In terms of the novel aspect of the project, there will be two main responsibilities for them that should be performed in tandem. The first is to try and understand the theory of what is going on. This will involve learning from Sai and the sources above what exactly is going on with both the geometric and algebraic aspects of the relevant computations. From there we will work on discovering properties of the rings and endomorphism maps that allow for easier computations. Then we will all work on learning the mechanisms of MAGMA in order to develop the methods to compute the kernels and cokernels. At least at the beginning, Professor Stojanoksa and Sai will provide the undergraduates with goals for the upcoming week and plans for what we should work on.

3 Meeting Times

The entire team meets on Mondays from 4-5 p.m. The undergraduates will also meet with each other on Saturday from 4-5 p.m.