Contents

Motion

Projectile Motion

The equations on the left are the standard kinematic equations; they express motion of constant acceleration on a straight line. The equations on the right are adapted to express motion on a parabola. Projectile motion can be defined as follows: An object is launched with initial velocity $\vec{v}_0$. Horizontal acceleration is zero, and vertical acceleration is $-g$, the downward force of gravity. θ_0 is the initial launch angle with respect to the positive x-axis.

$$
\begin{aligned}
&v = v_0 + at && x = x_0 + (v_0 \cos\theta_0)\, t \\
&x = x_0 + v_0 t + \tfrac{1}{2} a t^2 && y = y_0 + (v_0 \sin\theta_0)\, t - \tfrac{1}{2} g t^2 \\
&v^2 = v_0^2 + 2a\,(x - x_0) && v_y = v_0 \sin\theta_0 - gt \\
&x = x_0 + \tfrac{1}{2}\,(v_0 + v)\, t && v_y^2 = (v_0 \sin\theta_0)^2 - 2g\,(y - y_0) \\
&x = x_0 + vt - \tfrac{1}{2} a t^2 &&
\end{aligned}
$$

The path trajectory of a projectile is parabolic and can be expressed as

$$y = (\tan\theta_0)\, x - \frac{g x^2}{2\,(v_0 \cos\theta_0)^2}$$

If $x_0 = y_0 = 0$, the object's horizontal range R can be expressed as

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

Relative Motion

A frame of reference is an arbitrary set of coordinate axes about which we make measurements. A passenger on a train is not moving relative to their seat, but they may be moving very quickly from the perspective of a tree next to the train tracks. Given two frames of reference, A and B, with constant velocity; the velocity of a particle P with respect to A is equal to the velocity P with respect to B, plus the velocity of B with respect to A . .

$$\vec{v}_{\mathrm{PA}} = \vec{v}_{\mathrm{PB}} + \vec{v}_{\mathrm{BA}}$$

The acceleration is equal from either frame of reference.

$$\vec{a}_{\mathrm{PA}} = \vec{a}_{\mathrm{PB}}$$

Force and Motion

Newton's Laws

First Law:	If there is no net force a body, it cannot change its velocity.	$\vec{F}_{\text{net}} = 0 \rightarrow \vec{a} = 0$
Second Law:	The net force $\vec{F}_{\text{net}}$ on a body is the product of its mass m and its acceleration $\vec{a}$.	$\vec{F}_{\text{net}} = m\vec{a}$
Third Law:	Equal and opposite reactions.	$\vec{F}_{\text{BC}} = -\vec{F}_{\text{CB}}$

Friction

The force of static friction $\vec{f}_s$ has a maximum value $f_{s,\text{ max}}$ given by

$$f_{s,\text{ max}} = \mu_s F_N$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force. If the component of $\vec{F}$ parallel to the surface exceeds $f_{s,\text{ max}}$, the body will begin to slide. Once the body begins to slide on the surface, the frictional force decreases to a constant value f_k given by

$$f_k = \mu_k F_N$$

where μ_k is the coefficient of kinetic friction. Kinetic friction is less forceful than static friction.

Drag Force

A body that is moving relative to a surrounding fluid experiences a drag force $\vec{D}$. The force points in the direction in which the fluid flows relative to the body, i.e., opposite of the body's motion relative to the fluid. The magnitude of $\vec{D}$ can be expressed as

$$D = \tfrac{1}{2}C\rho A v^2$$

where C is the drag coefficient, ρ is the fluid density, and A is the effective cross-sectional area of the body, and v is the relative speed.

Terminal Speed

As an object in free fall builds speed, there comes a point where the magnitude of the drag force D becomes equal to that of the force of gravity F_g. The object cannot fall any faster and is said to have reached terminal velocity v_t expressed as

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Uniform Circular Motion

An object traveling along a circle of radius R at constant speed v is said to be in uniform circular motion. Speed is constant, therefore acceleration is constant. The constant magnitude of acceleration $\vec{a}$ can be expressed as

$$a = \frac{v^2}{R} \quad \text{and} \quad F_c = \frac{mv^2}{R}$$

Where F_c is the magnitude of the centripetal force. Acceleration $\vec{a}$ and force $\vec{F_c}$ are always directed toward the center of the circle. This is what it means for a force to be centripetal; it is directed toward a center around which an object is moving.

The time for the object to complete the circle is

$$T = \frac{2\pi R}{v}$$

Energy and Work

Kinetic Energy and Work

The work done on an object over displacement d can be expressed as

$$W = Fd\cos\phi = \vec{F} \cdot \vec{d}$$

where ϕ is the angle between $\vec{F}$ and $\vec{d}$, and both $\vec{F}$ and ϕ are constant.

Following the law of conservation of energy, the kinetic energy of an object is equal to the energy applied to that object in order to accelerate it to a given velocity. That concept is captured by the work-kinetic energy theorem: an increase in kinetic energy means that energy is being transferred to the object, and a decrease in kinetic energy means that energy is being transferred from the object; positive work and negative work, respectively.

$$K = \frac{1}{2}mv^2$$
$$\Delta K = K_f - K_i = W$$
$$K_f = K_i + W$$

Gravity and Work

The work done on an object by the force of gravity can be expressed as

$$W_g = mgd\cos\phi$$

where ϕ is the angle between $\vec{F}_g$ and $\vec{d}$, and ϕ is constant.

If $K_f = K_i$, then by the work-kinetic energy theorem, the work applied in lifting an object can be expressed as

$$W_a = -W_g$$

Variable Force and Work

Springs are a special case of a variable force.

$$\vec{F}_s = -k\vec{d}$$
$$F_x = -kd$$
$$W_s = \int_{x_i}^{x_f} F_x\,dx = \frac{k}{2}\left(x_i^2 - x_f^2\right)$$
$$\Delta K = K_f - K_i = W_a + W_s$$

General variable force

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

Power

Power is the rate of change of work with respect to time.

$$P = \frac{dw}{dt}$$
$$P = \vec{F}\cdot\vec{v} = Fv\cos\phi$$
$$P_{avg} = \frac{W}{\Delta t}$$