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# Motion

## Projectile Motion

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The equations on the left are the standard kinematic equations; they express motion of constant acceleration on a straight line. The equations on the right are adapted to express motion on a parabola. Projectile motion can be defined as follows: An object is launched with initial velocity  $\vec{v}_0$ . Horizontal acceleration is zero, and vertical acceleration is  $-g$ , the downward force of gravity.  $\theta_0$  is the initial launch angle with respect to the positive  $x$ -axis.

$v = v_0 + at$	$x = x_0 + (v_0 \cos \theta_0) t$
$x = x_0 + v_0 t + \frac{1}{2} at^2$	$y = y_0 + (v_0 \sin \theta_0) t - \frac{1}{2} gt^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$v_y = v_0 \sin \theta_0 - gt$
$x = x_0 + \frac{1}{2} (v_0 + v) t$	$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$
$x = x_0 + vt - \frac{1}{2} at^2$	

The path trajectory of a projectile is parabolic and can be expressed as

$$y = (\tan \theta_0) x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

If  $x_0 = y_0 = 0$ , the object's horizontal range  $R$  can be expressed as

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

## Relative Motion

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A frame of reference is an arbitrary set of coordinate axes about which we make measurements. A passenger on a train is not moving relative to their seat, but they may be moving very quickly from the perspective of a tree next to the train tracks. Given two frames of reference, A and B, with constant velocity; the velocity of a particle P with respect to A is equal to the velocity P with respect to B, plus the velocity of B with respect to A . .

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

The acceleration is equal from either frame of reference.

$$\vec{a}_{PA} = \vec{a}_{PB}$$

## Force and Motion

### Newton's Laws

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First Law:	If there is no net force a body, $\vec{F}_{\text{net}} = 0 \rightarrow \vec{a} = 0$ it cannot change its velocity.
Second Law:	The net force $\vec{F}_{\text{net}}$ on a body is the product of its mass $m$ and its acceleration $\vec{a}$ . $\vec{F}_{\text{net}} = m\vec{a}$
Third Law:	Equal and opposite reactions. $\vec{F}_{\text{BC}} = -\vec{F}_{\text{CB}}$

### Friction

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The force of static friction  $\vec{f}_s$  has a maximum value  $f_{s, \text{max}}$  given by

$$f_{s, \text{max}} = \mu_s F_N$$

where  $\mu_s$  is the coefficient of static friction and  $F_N$  is the magnitude of the normal force. If the component of  $\vec{F}$  parallel to the surface exceeds  $f_{s, \text{max}}$ , the body will begin to slide. Once the body begins to slide on the surface, the frictional force decreases to a constant value  $f_k$  given by

$$f_k = \mu_k F_N$$

where  $\mu_k$  is the coefficient of kinetic friction. Kinetic friction is less forceful than static friction.

### Drag Force

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A body that is moving relative to a surrounding fluid experiences a drag force  $\vec{D}$ . The force points in the direction in which the fluid flows relative to the body, i.e., opposite of the body's motion relative to the fluid. The magnitude of  $\vec{D}$  can be expressed as

$$D = \frac{1}{2} C \rho A v^2$$

where  $C$  is the drag coefficient,  $\rho$  is the fluid density, and  $A$  is the effective cross-sectional area of the body, and  $v$  is the relative speed.

### Terminal Speed

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As an object in free fall builds speed, there comes a point where the magnitude of the drag force  $D$  becomes equal to that of the force of gravity  $F_g$ . The object cannot fall any faster and is said to have reached terminal velocity  $v_t$  expressed as

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

## Uniform Circular Motion

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An object traveling along a circle of radius  $R$  at constant speed  $v$  is said to be in uniform circular motion. Speed is constant, therefore acceleration is constant. The constant magnitude of acceleration  $\vec{a}$  can be expressed as

$$a = \frac{v^2}{R} \quad \text{and} \quad F_c = \frac{mv^2}{R}$$

Where  $F_c$  is the magnitude of the centripetal force. Acceleration  $\vec{a}$  and force  $\vec{F}_c$  are always directed toward the center of the circle. This is what it means for a force to be centripetal; it is directed toward a center around which an object is moving.

The time for the object to complete the circle is

$$T = \frac{2\pi R}{v}$$