Contents

Boolean Algebra

Basics

Basic	$X + 0 = 0$	$X + 1 = 1$	$X \cdot 0 = 0$	$X \cdot 1 = X$
	$X + X = X$	$X \cdot X = X$		
	$(X')' = X$	$X + X' = 1$	$X \cdot X' = 0$	

DeMorgan's Laws	$(X + Y)' = X'Y'$	$(XY)' = X' + Y'$

Algebraic Simplification

1. Combine terms using the uniting theorem.
2. Absorb terms using the absorption theorem.
3. Eliminate literals using the elimination theorem.
4. Add redundant terms, if needed, to enable further simplification.

Use the sum-of-products (SOP) theorems to get a SOP result, and the product-of-sums (POS) theorems to get a POS result.

	SOP	POS
Uniting	$XY + XY' = X$	$(X + Y)(X + Y') = X$
Absorption	$X + XY = X$	$X(X + Y) = X$
Elimination	$X + X'Y = X + Y$	$X(X' + Y) = XY$

Expanding and Factoring Boolean Expressions

Starting with a POS expression, the SOP expression can be found by using the distributive laws to expand.

Ordinary distributive law: AND distributes over OR
Special distributive law: OR distributes over AND

$$X(Y+Z) = XY + XZ \qquad \text{(ordinary)}$$
$$(X+Y)(X+Z) = X + YZ \qquad \text{(special)}$$

It is also useful to note that

$$(X+Y)(X'+Z) = XZ + X'Y$$

The same theorems can be used to convert a SOP expression into a POS expression; just go the other way.

Exclusive-OR and Equivalence Operations

Exclusive OR ($\oplus$) can be expressed in familiar terms:

$$X \oplus Y = X'Y + XY'$$

That is, $X \oplus Y = 1$ iff one of the terms is true, but not both. Hence, "exclusive." Exclusive OR is commutative, associative, and distributive.

The equivalence operation ($\equiv$) is the complement of exclusive OR.

$$(X \equiv Y) = XY + X'Y'$$

Equivalence is commutative and associative.

In manipulating expressions that contain these operations, it is usually helpful to first rewrite them in terms of AND, OR using the definitions above. It is also useful to note that

$$(X'Y + XY')' = XY + X'Y'$$

The Consensus Theorem

Given the terms $X'Y$ and XZ, the consensus term is YZ. The consensus term is the product of all literals from the two factors, excluding the literal whose compliment is in the opposite term. For example, the consensus of $A'BC$ and ACD is BCD. The consensus of $AB'CD$ and $BFGH'$ is $ACDFGH'$. The significance of the consensus term is that it is redundant – it does not change the behaviour of the expression, so we can remove it. The consensus theorem and the dual form of the consensus theorem:

$$XY + X'Z + YZ = XY + X'Z$$
$$(X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)$$

Applying Boolean Algebra

Steps to Design Logic Circuit

1. Set up a truth table.
2. Derive Boolean equations from the truth table.
3. Simplify the Boolean equation.
4. Draw a circuit from the Boolean equation.

Minterm and Maxterm Expansions

A minterm is the product of n literals in which each of the literals appears only once. A maxterm is the sum of n literals in which each of the literals appears only once. When writing minterms primes correspond to 0's; the opposite applies to maxterms. Following DeMorgan's Laws, a minterm and a maxterm are each other's compliments.

ABC	Minterms	Maxterms
0 0 0 0	$A'B'C'D' = m_0$	$A + B + C + D = M_0$
0 0 0 1	$A'B'C'D = m_1$	$A + B + C + D' = M_1$
0 0 1 0	$A'B'CD' = m_2$	$A + B + C' + D = M_2$
$\vdots$	$\vdots$	$\vdots$
1 1 0 1	$ABC'D = m13$	$A' + B' + C + D' = M_{13}$
1 1 1 0	$ABCD' = m_{14}$	$A' + B' + C' + D = M_{14}$
1 1 1 1	$ABCD = m_{15}$	$A' + B' + C' + D' = M_{15}$

A function f, defined below, can be expressed in disjunctive normal form, i.e., as a minterm expansion. Note that, because a minterm and maxterm are each other's compliment, the equivalent conjunctive normal form, i.e., maxterm expansion, contains only the terms absent from the minterm expansion, and vice versa.

$$f = A'B'C'D' + A'B'C'D + A'BC'D' + A'BCD + AB'C'D' + AB'C'D + ABCD'$$

$$f = \Sigma\, m(0, 1, 4, 7, 8, 9, 14) = m_0 + m_1 + m_4 + m_7 + m_8 + m_9 + m_{14}$$

$$f = \Pi\, M(2, 3, 5, 6, 10, 11, 12, 13, 15) = M_2 M_3 M_5 M_6 M_{10} M_{11} M_{12} M_{13} M_{15}$$

Truth Tables

NAND

X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

$$Z = (XY)'$$

NOR

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	0

$$Z = (X + Y)'$$

XOR

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

$$Z = X'Y + XY'$$

XNOR

X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	1

$$Z = XY + X'Y'$$