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Motion

Projectile Motion

The equations on the left are the standard kinematic equations; they express motion of constant acceleration on a straight line. The equations on the right are adapted to express motion on a parabola. Projectile motion can be defined as follows: An object is launched with initial velocity \vec{v}_0 . Horizontal acceleration is zero, and vertical acceleration is $-g$, the downward force of gravity. θ_0 is the initial launch angle with respect to the positive x -axis.

$$\begin{array}{ll} v = v_0 + at & x = x_0 + (v_0 \cos \theta_0) t \\ x = x_0 + v_0 t + \frac{1}{2} at^2 & y = y_0 + (v_0 \sin \theta_0) t - \frac{1}{2} gt^2 \\ v^2 = v_0^2 + 2a(x - x_0) & v_y = v_0 \sin \theta_0 - gt \\ x = x_0 + \frac{1}{2} (v_0 + v) t & v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0) \\ x = x_0 + vt - \frac{1}{2} at^2 & \end{array}$$

The path trajectory of a projectile is parabolic and can be expressed as

$$y = (\tan \theta_0) x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$

If $x_0 = y_0 = 0$, the object's horizontal range R can be expressed as

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

Relative Motion

A frame of reference is an arbitrary set of coordinate axes about which we make measurements. A passenger on a train is not moving relative to their seat, but they may be moving very quickly from the perspective of a tree next to the train tracks. Given two frames of reference, A and B, with constant velocity; the velocity of a particle P with respect to A is equal to the velocity P with respect to B, plus the velocity of B with respect to A . .

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

The acceleration is equal from either frame of reference.

$$\vec{a}_{PA} = \vec{a}_{PB}$$

Force and Motion

Newton's Laws

First Law:	If there is no net force a body, $\vec{F}_{\text{net}} = 0 \rightarrow \vec{a} = 0$ it cannot change its velocity.
Second Law:	The net force \vec{F}_{net} on a body is the product of its mass m and its acceleration \vec{a} . $\vec{F}_{\text{net}} = m\vec{a}$
Third Law:	Equal and opposite reactions. $\vec{F}_{\text{BC}} = -\vec{F}_{\text{CB}}$

Friction

The force of static friction \vec{f}_s has a maximum value $f_{s, \text{max}}$ given by

$$f_{s, \text{max}} = \mu_s F_N$$

where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force. If the component of \vec{F} parallel to the surface exceeds $f_{s, \text{max}}$, the body will begin to slide. Once the body begins to slide on the surface, the frictional force decreases to a constant value f_k given by

$$f_k = \mu_k F_N$$

where μ_k is the coefficient of kinetic friction. Kinetic friction is less forceful than static friction.

Drag Force

A body that is moving relative to a surrounding fluid experiences a drag force \vec{D} . The force points in the direction in which the fluid flows relative to the body, i.e., opposite of the body's motion relative to the fluid. The magnitude of \vec{D} can be expressed as

$$D = \frac{1}{2} C \rho A v^2$$

where C is the drag coefficient, ρ is the fluid density, and A is the effective cross-sectional area of the body, and v is the relative speed.

Terminal Speed

As an object in free fall builds speed, there comes a point where the magnitude of the drag force D becomes equal to that of the force of gravity F_g . The object cannot fall any faster and is said to have reached terminal velocity v_t expressed as

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Uniform Circular Motion

An object traveling along a circle of radius R at constant speed v is said to be in uniform circular motion. Speed is constant, therefore acceleration is constant. The constant magnitude of acceleration \vec{a} can be expressed as

$$a = \frac{v^2}{R} \quad \text{or} \quad F_c = \frac{mv^2}{R}$$

Where F_c is the magnitude of the centripetal force. Acceleration \vec{a} and force \vec{F}_c are always directed toward the center of the circle. This is what it means for a force to be centripetal; it is directed toward a center around which an object is moving.

The time for the object to complete the circle is

$$T = \frac{2\pi r}{v}$$