Contents

Basics

Charge, Current, Voltage, Power, and Energy

Current is rate of the flow of charge through an element in a circuit.

$$i = \frac{dq}{dt} \qquad Q = \int_{t_0}^{t} i \, dt$$

Voltage, or electric potential difference, is analogous to pressure. The difference in electric potential between two points is what causes current.

$$v = \frac{dw}{dq}$$

Power is the rate of change of work with respect to time, measured in watts (W). Power expended or absorbed between time t_0 and t is work.

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \quad \Longleftrightarrow \quad w = \int_{t_0}^{t} p \, dt = \int_{t_0}^{t} vi \, dt$$

Passive sign convention dictates that when current flows into the positive terminal of an element $p = +vi$ and when current flows into the negative terminal of an element $p = -vi$. An element with positive power is absorbing power from the circuit; an element with negative power is supplying power to the circuit.

Problem-Solving

1. Understand the fundamentals of the problem.
2. Detail all relevant information.
3. Create alternative paths forward, and assess each for their viability.
4. Attempt a solution, then review and check for accuracy.

Fundamental Laws

Ohm's Law

Ohm's law states that the current flowing through a conductor is directly proportional to the voltage across it. Resistance is that constant of proportionality. Resistance is a material's opposition to electric current.

$$v = iR$$

The conductance G of an element is the reciprocal of its resistance:

$$G = \frac{1}{R}$$

Using Ohm's law, we now have additional ways to express power. The power dissipated by a resistor can be expressed as

$$p = vi = i^2 R = \frac{v^2}{R} = v^2 G = \frac{i^2}{G}$$

Fundamental Theorem of Network Topology

A *branch* is a single element in an electric circuit. A *node* is the point of connection between two or more branches. A *loop* is a closed path in a circuit. The number of b branches, n nodes, and l independent loops are related by the fundamental theorem of network topology.

$$b = l + n - 1$$

Kirchhoff's Laws

Kirchhoff's current law (KCL) states that the currents at any node algebraically sums to zero; the sum of currents entering a node equals the sum of currents leaving the node. Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed loop is zero; the sum of voltage drops equals the sum of voltage rises. KCL and KVL can be expressed as

$$\sum_{n=1}^{N} i_n = 0 \qquad \sum_{m=1}^{M} v_m = 0$$

Series Resistors - Voltage Division

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances. For N resistors in series

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

Given a voltage divider with N resistors in series with a voltage source v, the nth resistor (R_n) will have a voltage

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Parallel Resistors - Current Division

The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

$$G_{eq} = G_1 + G_1 + \cdots + G_N \implies R_{eq} = (G_1 + G_2 + \cdots + G_N)^{-1}$$

Given a current divider with N conductors in parallel with a current source i, the nth conductor (G_n) will have a current

$$i_n = \frac{G_n}{G_1 + G_2 + \ \ldots \ G_N} i$$

Wye-Delta Transformations

Each resistor in the Y network is the product of the two adjacent Δ resistors, divided by the sum of the Δ resistors. Each resistor in the Δ network is the sum of the products of the Y resistors, taken two at a time, over the opposite Y resistor.

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

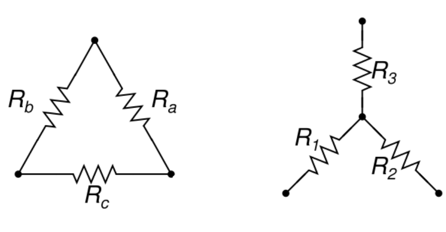

Figure 1: Wye-Delta Transformation. Source: Wikipedia (CC BY-SA 3.0).

Nodal and Mesh Analysis

Nodal Analysis

Steps to determine node voltages:

1. Select a reference node and assign voltage variables to the remaining nodes.
2. If there is a voltage source between the reference node and another node, the non-reference node's voltage is equal to that of the voltage source.
3. If there is a voltage source between two non-reference nodes, those two nodes form a supernode, so we and apply both KCL and KVL.
4. Apply KCL to each node with an unknown voltage.
5. Use Ohm's law to express the branch currents in terms of voltage. In a resistor, current flows from a higher potential to a lower potential.

$$i = \frac{v_{\text{ higher}} - v_{\text{ lower}}}{R}$$

6. Solve the resulting system of equations.

Mesh Analysis

A loop is a path in which no node is passed more than once. A mesh is a loop that does not contain any other loops within it. Mesh analysis only works for circuits that are planer, i.e., they can be drawn on a plane without crossing branches.

1. Assign mesh currents to each mesh.
2. When a current source is only in one mesh it defines the mesh current.
3. When a current source is between two meshes, create a supermesh by excluding the current source and any elements connected in series with it. We apply both KVL and KCL to the supermesh.
4. Apply KVL to each of the meshes, using Ohm's law to express the voltages in terms of mesh currents.
5. Solve the resulting system of equations.