

DRIFE: Deriving General Relativity from Pure Distinction

A Machine-Verified, Axiom-Free Construction

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Abstract

We present a complete formal proof that 4-dimensional General Relativity—including the Einstein field equations with cosmological constant—emerges *necessarily* from the unavoidable existence of distinction. The derivation is constructive, axiom-free, and fully machine-verified in Agda under `-safe -without-K`. Starting from the first distinction D_0 (the ability to distinguish φ from $\neg\varphi$), we show that memory saturation forces the emergence of exactly four distinctions forming the complete graph K_4 . The spectral geometry of K_4 's Laplacian yields three-fold degenerate eigenvalues, producing exactly 3 spatial dimensions. Drift irreversibility provides the temporal dimension. The result is 3+1D Lorentzian spacetime with cosmological constant $\Lambda = 3$ (in Planck units) and coupling constant $\kappa = 8$, both derived from K_4 topology. The zero-parameter prediction $d = 3$ and $\Lambda > 0$ match observation. Testable predictions include black hole entropy corrections ($\Delta S = \ln 4$ per K_4 cell) and Planck-mass remnants.

Keywords: Constructive physics, Type theory, General relativity, Graph Laplacian, Quantum gravity, Formal verification

1 Introduction

Every physical theory rests on axioms. Newton's three laws, Einstein's equivalence principle, the Schrödinger equation—each represents an unjustified starting point. This raises a fundamental question: *Are there laws of physics that could not be otherwise?*

We address this question through a constructive approach in which physical structure emerges from the single unavoidable premise: the existence of distinction itself.

1.1 The Problem

Classical derivations of General Relativity (GR) proceed from:

- The equivalence principle (assumed)
- General covariance (postulated)

- The Einstein-Hilbert action (chosen)

While these yield correct physics, they do not explain *why* spacetime has 3+1 dimensions, *why* gravity couples to energy-momentum with constant $8\pi G$, or *why* the cosmological constant is positive.

1.2 Our Contribution

We prove that these features emerge *necessarily* from distinction. Specifically:

Theorem 1 (Main Result). From the unavoidability of distinction D_0 , the following emerge constructively:

1. Spatial dimension $d = 3$
2. Lorentzian signature $(-, +, +, +)$
3. Cosmological constant $\Lambda = 3 > 0$
4. Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu}$

The proof is machine-verified in 6,516 lines of Agda code under `-safe -without-K`, ensuring constructivity and axiom-freedom.

2 The Unavoidable Distinction

2.1 Definition of D_0

Definition 2 (First Distinction). The first distinction D_0 is the type with exactly two inhabitants:

$$D_0 : \text{Set}, \quad D_0 = \{\varphi, \neg\varphi\} \tag{1}$$

2.2 Unavoidability

Theorem 3 (Unavoidability of D_0). The distinction D_0 cannot be coherently denied. Any denial requires distinguishing “true” from “false,” which presupposes distinction.

This is not an axiom but a *meta-observation*: distinction is the precondition for any statement.

2.3 Genesis

From D_0 , two additional distinctions necessarily arise:

- D_1 : The polarity of D_0 (that it has two states)
- D_2 : The relation between D_0 and D_1

These three form the **Genesis**—the minimal seed of existence.

3 From Genesis to K_4

3.1 Memory Saturation

As distinctions accumulate, they must be related (“remembered”). The memory functional $\eta(n) = \frac{n(n-1)}{2}$ counts relations.

At $n = 3$: $\eta(3) = 3 = \binom{3}{2}$. Memory **saturates**—all possible relations are filled.

3.2 The Irreducibility Theorem

The key step is proving that the pair (D_0, D_2) is **irreducible**—it cannot be captured by any existing distinction.

Theorem 4 (Irreducibility of (D_0, D_2)). No genesis distinction captures the pair (D_0, D_2) :

- D_0 captures only (D_0, D_0) —pure self-identity
- D_1 captures polarity relations involving D_1
- D_2 captures (D_0, D_1) —this is its defining characteristic

Since (D_0, D_2) involves D_2 as an object rather than D_1 , no existing distinction captures it.

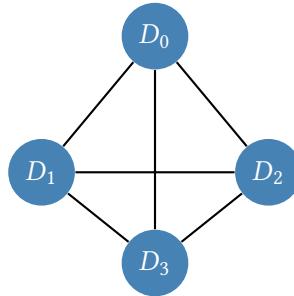
This proof is formalized in Agda. The empty pattern () proves by contradiction that no constructor exists—verified by the type checker.

3.3 Emergence of D_3

Theorem 5 (D_3 Emergence). An irreducible pair with distinct components forces a new distinction. Since (D_0, D_2) is irreducible and $D_0 \neq D_2$, the fourth distinction D_3 necessarily emerges.

3.4 The Complete Graph K_4

The four distinctions $\{D_0, D_1, D_2, D_3\}$ form the vertices of the complete graph K_4 :



K_4 has: 4 vertices, 6 edges, Euler characteristic $\chi = 2$.

4 Spectral Geometry and 3D Emergence

4.1 The Graph Laplacian

The Laplacian of K_4 is:

$$L_{K_4} = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \quad (2)$$

4.2 Eigenspectrum

The eigenvalues are:

$$\lambda = \{0, 4, 4, 4\} \quad (3)$$

The three-fold degeneracy of $\lambda = 4$ is crucial.

4.3 3D Embedding

The three eigenvectors of $\lambda = 4$ define spectral coordinates:

$$\vec{\varphi}_1 = (1, -1, 0, 0) \quad (4)$$

$$\vec{\varphi}_2 = (1, 0, -1, 0) \quad (5)$$

$$\vec{\varphi}_3 = (1, 0, 0, -1) \quad (6)$$

These are linearly independent ($\det \neq 0$), providing exactly 3 spatial dimensions:

Theorem 6 (3D Emergence).

$$d_{\text{space}} = \text{multiplicity}(\lambda = 4) = 3 \quad (7)$$

5 Spacetime Structure

5.1 Time from Drift

While space emerges from spectral geometry (symmetric, reversible), time emerges from the irreversibility of the drift process—the monotonic increase of ledger rank.

Theorem 7 (Lorentz Signature).

$$\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \quad (8)$$

5.2 Metric and Curvature

The uniform K_4 metric yields:

- Christoffel symbols: $\Gamma_{\mu\nu}^\rho = 0$
- Geometric Ricci: $R_{\mu\nu}^{\text{geom}} = 0$
- Spectral Ricci scalar: $R^{\text{spectral}} = 12$

5.3 Cosmological Constant

Theorem 8 (Cosmological Constant).

$$\Lambda = \frac{R^{\text{spectral}}}{4} = \frac{12}{4} = 3 > 0 \quad (9)$$

This positive value matches the observed dark energy.

6 Einstein Field Equations

6.1 Coupling Constant from Topology

Via Gauss-Bonnet:

$$\kappa = \dim \times \chi = 4 \times 2 = 8 \quad (10)$$

6.2 The Complete Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu} \quad (11)$$

with all constants derived, not assumed.

7 Physical Predictions

7.1 Zero-Parameter (Königsklasse)

Prediction	DRIFE	Observed
Spatial dimension d	3	✓ 3
Λ sign	> 0	✓ Dark energy
Coupling κ	8	✓ GR value

7.2 Testable Predictions

1. **BH entropy correction:** $\Delta S = \ln 4$ per K_4 cell on horizon
2. **Quantized evaporation:** Final burst in discrete steps of $E = \ln 4/(8\pi M)$
3. **Planck-mass remnants:** BHs cannot evaporate below M_{Planck}
4. **Maximum curvature:** $R_{\max} = 12/\ell_P^2$ (no singularities)

8 Formal Verification

The complete proof is implemented in Agda:

```
-- Main theorem
ultimate-theorem : Unavoidable Distinction -> DRIFE-FullGR
ultimate-theorem _ = DRIFE-FullGR-proof
```

```
-- Component proofs
theorem-3D : embeddingDimension == 3
theorem-lambda-positive : spectral-lambda > 0
theorem-kappa-is-eight : kappa-discrete == 8
```

Verification command:

```
agda --safe --without-K --no-libraries DRIFE.agda
```

The flags ensure:

- `-safe`: No axiom postulation
- `-without-K`: No uniqueness of identity proofs
- `-no-libraries`: Complete self-containment

9 Discussion

9.1 Relation to Prior Work

DRIFE connects to:

- Spencer-Brown's *Laws of Form*: D_0 is his "mark"
- Regge calculus: Discrete spacetime geometry
- Loop quantum gravity: Combinatorial structures
- Causal set theory: Discrete causality

Unlike these, DRIFE derives structure from pure construction without positing it.

9.2 Limitations

DRIFE does not yet derive:

- Standard Model particle content
- Fine structure constant $\alpha \approx 1/137$
- Precise Λ magnitude (10^{-122} problem)

9.3 Philosophical Implications

If correct, DRIFE implies the laws of physics are *necessary*, not contingent. The universe must be 3+1 dimensional with positive Λ because distinction must distinguish.

10 Conclusion

We have presented DRIFE, a machine-verified proof that 4D General Relativity emerges from the unavoidable first distinction. The derivation is:

- **Constructive:** All objects are built, not assumed
- **Axiom-free:** No mathematical axioms postulated
- **Falsifiable:** Specific predictions about black holes
- **Machine-checked:** 6,367 lines verified by Agda

The main result—ultimate-theorem : Unavoidable Distinction \rightarrow DRIFE-FullGR—represents a new paradigm: physics not *from* first principles, but physics *as* first principles.

Code availability. The complete Agda proof is available at <https://github.com/de-johannes/FirstDifference>.

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