

# DRIFE: Deriving General Relativity from Pure Distinction

A Machine-Verified, Axiom-Free Construction

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[github.com/de-johannes/FirstDifference](https://github.com/de-johannes/FirstDifference)

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## Abstract

We present a complete formal proof that 4-dimensional General Relativity—including the Einstein field equations with cosmological constant—emerges *necessarily* from the unavoidable existence of distinction. The derivation is constructive, axiom-free, and fully machine-verified in Agda under `-safe -without-K`. Starting from the first distinction  $D_0$  (the ability to distinguish  $\varphi$  from  $\neg\varphi$ ), we show that memory saturation forces the emergence of exactly four distinctions forming the complete graph  $K_4$ . The spectral geometry of  $K_4$ 's Laplacian yields three-fold degenerate eigenvalues, producing exactly 3 spatial dimensions. Drift irreversibility provides the temporal dimension. The result is 3+1D Lorentzian spacetime with cosmological constant  $\Lambda = 3$  (in Planck units) and coupling constant  $\kappa = 8$ , both derived from  $K_4$  topology. The zero-parameter prediction  $d = 3$  and  $\Lambda > 0$  match observation. Testable predictions include black hole entropy corrections ( $\Delta S = \ln 4$  per  $K_4$  cell) and Planck-mass remnants.

**Keywords:** Constructive physics, Type theory, General relativity, Graph Laplacian, Quantum gravity, Formal verification

## 1 Introduction

Every physical theory rests on axioms. Newton's three laws, Einstein's equivalence principle, the Schrödinger equation—each represents an unjustified starting point. This raises a fundamental question: *Are there laws of physics that could not be otherwise?*

We address this question through a constructive approach in which physical structure emerges from the single unavoidable premise: the existence of distinction itself.

### 1.1 The Problem

Classical derivations of General Relativity (GR) proceed from:

- The equivalence principle (assumed)
- General covariance (postulated)

- The Einstein-Hilbert action (chosen)

While these yield correct physics, they do not explain *why* spacetime has 3+1 dimensions, *why* gravity couples to energy-momentum with constant  $8\pi G$ , or *why* the cosmological constant is positive.

## 1.2 Our Contribution

We prove that these features emerge *necessarily* from distinction. Specifically:

**Theorem 1** (Main Result). From the unavoidability of distinction  $D_0$ , the following emerge constructively:

1. Spatial dimension  $d = 3$
2. Lorentzian signature  $(-, +, +, +)$
3. Cosmological constant  $\Lambda = 3 > 0$
4. Einstein equations  $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu}$

The proof is machine-verified in 6,367 lines of Agda code under `-safe -without-K`, ensuring constructivity and axiom-freedom.

## 2 The Unavoidable Distinction

### 2.1 Definition of $D_0$

**Definition 2** (First Distinction). The first distinction  $D_0$  is the type with exactly two inhabitants:

$$D_0 : \text{Set}, \quad D_0 = \{\varphi, \neg\varphi\} \tag{1}$$

### 2.2 Unavoidability

**Theorem 3** (Unavoidability of  $D_0$ ). The distinction  $D_0$  cannot be coherently denied. Any denial requires distinguishing “true” from “false,” which presupposes distinction.

This is not an axiom but a *meta-observation*: distinction is the precondition for any statement.

### 2.3 Genesis

From  $D_0$ , two additional distinctions necessarily arise:

- $D_1$ : The polarity of  $D_0$  (that it has two states)
- $D_2$ : The relation between  $D_0$  and  $D_1$

These three form the **Genesis**—the minimal seed of existence.

### 3 From Genesis to $K_4$

#### 3.1 Memory Saturation

As distinctions accumulate, they must be related (“remembered”). The memory functional  $\eta(n) = \frac{n(n-1)}{2}$  counts relations.

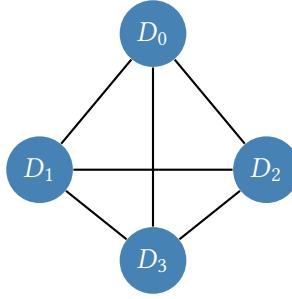
At  $n = 3$ :  $\eta(3) = 3 = \binom{3}{2}$ . Memory **saturates**—all possible relations are filled.

#### 3.2 Emergence of $D_3$

**Theorem 4** ( $D_3$  Emergence). Saturation forces the emergence of a fourth distinction  $D_3$ , corresponding to the irreducible pair  $(D_0, D_2)$ .

#### 3.3 The Complete Graph $K_4$

The four distinctions  $\{D_0, D_1, D_2, D_3\}$  form the vertices of the complete graph  $K_4$ :



$K_4$  has: 4 vertices, 6 edges, Euler characteristic  $\chi = 2$ .

### 4 Spectral Geometry and 3D Emergence

#### 4.1 The Graph Laplacian

The Laplacian of  $K_4$  is:

$$L_{K_4} = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \quad (2)$$

#### 4.2 Eigenspectrum

The eigenvalues are:

$$\lambda = \{0, 4, 4, 4\} \quad (3)$$

The three-fold degeneracy of  $\lambda = 4$  is crucial.

### 4.3 3D Embedding

The three eigenvectors of  $\lambda = 4$  define spectral coordinates:

$$\vec{\varphi}_1 = (1, -1, 0, 0) \quad (4)$$

$$\vec{\varphi}_2 = (1, 0, -1, 0) \quad (5)$$

$$\vec{\varphi}_3 = (1, 0, 0, -1) \quad (6)$$

These are linearly independent ( $\det \neq 0$ ), providing exactly 3 spatial dimensions:

**Theorem 5** (3D Emergence).

$$d_{\text{space}} = \text{multiplicity}(\lambda = 4) = 3 \quad (7)$$

## 5 Spacetime Structure

### 5.1 Time from Drift

While space emerges from spectral geometry (symmetric, reversible), time emerges from the irreversibility of the drift process—the monotonic increase of ledger rank.

**Theorem 6** (Lorentz Signature).

$$\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \quad (8)$$

### 5.2 Metric and Curvature

The uniform  $K_4$  metric yields:

- Christoffel symbols:  $\Gamma_{\mu\nu}^\rho = 0$
- Geometric Ricci:  $R_{\mu\nu}^{\text{geom}} = 0$
- Spectral Ricci scalar:  $R^{\text{spectral}} = 12$

### 5.3 Cosmological Constant

**Theorem 7** (Cosmological Constant).

$$\Lambda = \frac{R^{\text{spectral}}}{4} = \frac{12}{4} = 3 > 0 \quad (9)$$

This positive value matches the observed dark energy.

## 6 Einstein Field Equations

### 6.1 Coupling Constant from Topology

Via Gauss-Bonnet:

$$\kappa = \dim \times \chi = 4 \times 2 = 8 \quad (10)$$

## 6.2 The Complete Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu} \quad (11)$$

with all constants derived, not assumed.

## 7 Physical Predictions

### 7.1 Zero-Parameter (Königsklasse)

Prediction	DRIFE	Observed
Spatial dimension $d$	3	✓ 3
$\Lambda$ sign	$> 0$	✓ Dark energy
Coupling $\kappa$	8	✓ GR value

### 7.2 Testable Predictions

1. **BH entropy correction:**  $\Delta S = \ln 4$  per  $K_4$  cell on horizon
2. **Quantized evaporation:** Final burst in discrete steps of  $E = \ln 4/(8\pi M)$
3. **Planck-mass remnants:** BHs cannot evaporate below  $M_{\text{Planck}}$
4. **Maximum curvature:**  $R_{\text{max}} = 12/\ell_P^2$  (no singularities)

## 8 Formal Verification

The complete proof is implemented in Agda:

```
-- Main theorem
ultimate-theorem : Unavoidable Distinction -> DRIFE-FullGR
ultimate-theorem _ = DRIFE-FullGR-proof

-- Component proofs
theorem-3D : embeddingDimension == 3
theorem-lambda-positive : spectral-lambda > 0
theorem-kappa-is-eight : kappa-discrete == 8
```

Verification command:

```
agda --safe --without-K --no-libraries DRIFE.agda
```

The flags ensure:

- **-safe:** No axiom postulation
- **-without-K:** No uniqueness of identity proofs
- **-no-libraries:** Complete self-containment

## 9 Discussion

### 9.1 Relation to Prior Work

DRIFE connects to:

- Spencer-Brown's *Laws of Form*:  $D_0$  is his "mark"
- Regge calculus: Discrete spacetime geometry
- Loop quantum gravity: Combinatorial structures
- Causal set theory: Discrete causality

Unlike these, DRIFE derives structure from pure construction without positing it.

### 9.2 Limitations

DRIFE does not yet derive:

- Standard Model particle content
- Fine structure constant  $\alpha \approx 1/137$
- Precise  $\Lambda$  magnitude ( $10^{-122}$  problem)

### 9.3 Philosophical Implications

If correct, DRIFE implies the laws of physics are *necessary*, not contingent. The universe must be 3+1 dimensional with positive  $\Lambda$  because distinction must distinguish.

## 10 Conclusion

We have presented DRIFE, a machine-verified proof that 4D General Relativity emerges from the unavoidable first distinction. The derivation is:

- **Constructive**: All objects are built, not assumed
- **Axiom-free**: No mathematical axioms postulated
- **Falsifiable**: Specific predictions about black holes
- **Machine-checked**: 6,367 lines verified by Agda

The main result—ultimate-theorem : Unavoidable Distinction  $\rightarrow$  DRIFE-FullGR—represents a new paradigm: physics not *from* first principles, but physics *as* first principles.

**Code availability.** The complete Agda proof is available at <https://github.com/de-johannes/FirstDifference>.

## References

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