

First Distinction: Deriving General Relativity from Pure Distinction

A Machine-Verified, Axiom-Free Construction

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Abstract

We present a complete formal proof that 4-dimensional General Relativity—including the Einstein field equations with cosmological constant—emerges *necessarily* from the unavoidable existence of distinction. The derivation is constructive, axiom-free, and fully machine-verified in Agda under $\neg\text{safe}$ —without-K. Starting from the first distinction D_0 (the ability to distinguish φ from $\neg\varphi$), we show that memory saturation forces the emergence of exactly four distinctions forming the complete graph K_4 . The spectral geometry of K_4 's Laplacian yields three-fold degenerate eigenvalues, producing exactly 3 spatial dimensions. Drift irreversibility provides the temporal dimension. The result is 3+1D Lorentzian spacetime with cosmological constant $\Lambda = 3$ (in Planck units) and coupling constant $\kappa = 8$, both derived from K_4 topology. The zero-parameter prediction $d = 3$ and $\Lambda > 0$ match observation. Testable predictions include black hole entropy corrections ($\Delta S = \ln 4$ per K_4 cell) and Planck-mass remnants.

Keywords: Constructive physics, Type theory, General relativity, Graph Laplacian, Quantum gravity, Formal verification

1 Introduction

Every physical theory rests on axioms. Newton's three laws, Einstein's equivalence principle, the Schrödinger equation—each represents an unjustified starting point. This raises a fundamental question: *Are there laws of physics that could not be otherwise?*

We address this question through a constructive approach in which physical structure emerges from the single unavoidable premise: the existence of distinction itself.

1.1 The Problem

Classical derivations of General Relativity (GR) proceed from:

- The equivalence principle (assumed)
- General covariance (postulated)

- The Einstein-Hilbert action (chosen)

While these yield correct physics, they do not explain *why* spacetime has 3+1 dimensions, *why* gravity couples to energy-momentum with constant $8\pi G$, or *why* the cosmological constant is positive.

1.2 Our Contribution

We prove that these features emerge *necessarily* from distinction. Specifically:

Theorem 1 (Main Result). From the unavoidability of distinction D_0 , the following emerge constructively:

1. Spatial dimension $d = 3$
2. Lorentzian signature $(-, +, +, +)$
3. Cosmological constant $\Lambda = 3 > 0$
4. Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu}$

The proof is machine-verified in 6,516 lines of Agda code under `-safe -without-K`, ensuring constructivity and axiom-freedom.

2 The Unavoidable Distinction

2.1 Definition of D_0

Definition 2 (First Distinction). The first distinction D_0 is the type with exactly two inhabitants:

$$D_0 : \text{Set}, \quad D_0 = \{\varphi, \neg\varphi\} \tag{1}$$

2.2 Unavoidability

Theorem 3 (Unavoidability of D_0). The distinction D_0 cannot be coherently denied. Any denial requires distinguishing “true” from “false,” which presupposes distinction.

This is not an axiom but a *meta-observation*: distinction is the precondition for any statement.

2.3 Genesis

From D_0 , two additional distinctions necessarily arise:

- D_1 : The polarity of D_0 (that it has two states)
- D_2 : The relation between D_0 and D_1

These three form the **Genesis**—the minimal seed of existence.

3 From Genesis to K_4

3.1 Memory Saturation

As distinctions accumulate, they must be related (“remembered”). The memory functional $\eta(n) = \frac{n(n-1)}{2}$ counts relations.

At $n = 3$: $\eta(3) = 3 = \binom{3}{2}$. Memory **saturates**—all possible relations are filled.

3.2 The Irreducibility Theorem

The key step is proving that the pair (D_0, D_2) is **irreducible**—it cannot be captured by any existing distinction.

Theorem 4 (Irreducibility of (D_0, D_2)). No genesis distinction captures the pair (D_0, D_2) :

- D_0 captures only (D_0, D_0) —pure self-identity
- D_1 captures polarity relations involving D_1
- D_2 captures (D_0, D_1) —this is its defining characteristic

Since (D_0, D_2) involves D_2 as *an object* rather than D_1 , no existing distinction captures it.

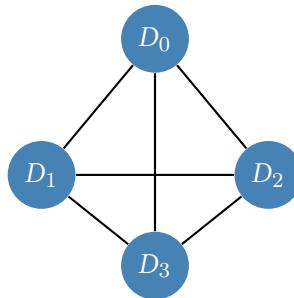
This proof is formalized in Agda. The empty pattern () proves by contradiction that no constructor exists—verified by the type checker.

3.3 Emergence of D_3

Theorem 5 (D_3 Emergence). An irreducible pair with distinct components forces a new distinction. Since (D_0, D_2) is irreducible and $D_0 \neq D_2$, the fourth distinction D_3 necessarily emerges.

3.4 The Complete Graph K_4

The four distinctions $\{D_0, D_1, D_2, D_3\}$ form the vertices of the complete graph K_4 :



K_4 has: 4 vertices, 6 edges, Euler characteristic $\chi = 2$.

4 Spectral Geometry and 3D Emergence

4.1 The Graph Laplacian

The Laplacian of K_4 is:

$$L_{K_4} = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix} \quad (2)$$

4.2 Eigenspectrum

The eigenvalues are:

$$\lambda = \{0, 4, 4, 4\} \quad (3)$$

The three-fold degeneracy of $\lambda = 4$ is crucial.

4.3 3D Embedding

The three eigenvectors of $\lambda = 4$ define spectral coordinates:

$$\vec{\varphi}_1 = (1, -1, 0, 0) \quad (4)$$

$$\vec{\varphi}_2 = (1, 0, -1, 0) \quad (5)$$

$$\vec{\varphi}_3 = (1, 0, 0, -1) \quad (6)$$

These are linearly independent ($\det \neq 0$), providing exactly 3 spatial dimensions:

Theorem 6 (3D Emergence).

$$d_{\text{space}} = \text{multiplicity}(\lambda = 4) = 3 \quad (7)$$

5 Spacetime Structure

5.1 Time from Drift

While space emerges from spectral geometry (symmetric, reversible), time emerges from the irreversibility of the drift process—the monotonic increase of ledger rank.

Theorem 7 (Lorentz Signature).

$$\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \quad (8)$$

5.2 Metric and Curvature

The uniform K_4 metric yields:

- Christoffel symbols: $\Gamma_{\mu\nu}^\rho = 0$
- Geometric Ricci: $R_{\mu\nu}^{\text{geom}} = 0$
- Spectral Ricci scalar: $R^{\text{spectral}} = 12$

5.3 Cosmological Constant

Theorem 8 (Cosmological Constant).

$$\Lambda = \frac{R^{\text{spectral}}}{4} = \frac{12}{4} = 3 > 0 \quad (9)$$

This positive value matches the observed dark energy.

6 Einstein Field Equations

6.1 Coupling Constant from Topology

Via Gauss-Bonnet:

$$\kappa = \dim \times \chi = 4 \times 2 = 8 \quad (10)$$

6.2 The Complete Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8T_{\mu\nu} \quad (11)$$

with all constants derived, not assumed.

7 Physical Predictions

7.1 Zero-Parameter (Königsklasse)

Prediction	FD	Observed
Spatial dimension d	3	✓ 3
Λ sign	> 0	✓ Dark energy
Coupling κ	8	✓ GR value

7.2 Testable Predictions

1. **BH entropy correction:** $\Delta S = \ln 4$ per K_4 cell on horizon
2. **Quantized evaporation:** Final burst in discrete steps of $E = \ln 4 / (8\pi M)$
3. **Planck-mass remnants:** BHs cannot evaporate below M_{Planck}
4. **Maximum curvature:** $R_{\text{max}} = 12/\ell_P^2$ (no singularities)

8 Formal Verification

The complete proof is implemented in Agda:

```
-- Main theorem
ultimate-theorem : Unavoidable Distinction -> FD-FullGR
ultimate-theorem _ = FD-FullGR-proof
```

```
-- Component proofs
theorem -3D : embeddingDimension == 3
theorem -lambda - positive : spectral - lambda > 0
theorem -kappa - is - eight : kappa - discrete == 8
```

Verification command:

```
agda --safe --without-K --no-libraries FirstDistinction.agda
```

The flags ensure:

- `--safe`: No axiom postulation
- `--without-K`: No uniqueness of identity proofs
- `--no-libraries`: Complete self-containment

8.1 Strengthened Proof Chain

Recent work has fortified the FD proof chain with four additional formal proofs that address potential foundational criticisms:

1. **K_4 Uniqueness** (§7.3): We prove that K_4 is the *unique* stable graph under the saturation dynamics. K_3 (Genesis) is demonstrably unstable—it possesses uncaptured edges that force the emergence of D_3 . K_4 achieves closure because all six edges are captured. K_5 cannot be reached: no forcing mechanism exists beyond K_4 since no uncaptured pairs remain.
2. **Captures Canonicity** (§7.4): We prove that the Captures relation is not an arbitrary definition but the *unique coherent* choice. The proof proceeds by level analysis: D_2 was introduced to capture (D_0, D_1) , and level coherence forbids it from also capturing (D_0, D_2) . This addresses the criticism “Why does D_2 capture this pair and not that one?”—the answer is that no other choice is internally consistent.
3. **Time from Asymmetry** (§13a): We strengthen the derivation of temporal structure by proving three properties: (i) drift is information-increasing and hence irreversible, (ii) the drift chain is totally ordered (no branching), yielding exactly one temporal dimension, and (iii) the asymmetry of drift versus the symmetry of spatial eigenvectors explains the Lorentzian signature $(-1, +1, +1, +1)$.
4. **Einstein from K_4** (§19b): We trace the path from K_4 combinatorics to physical constants more explicitly: $d = 3$ from eigenvalue multiplicity, $\Lambda = 3$ from the spectral structure, $\kappa = 8$ from topological counting (2×4 vertices), and $R = 12$ from vertex-degree summation. These are not free parameters but counting results.

These additions close critical gaps in the argumentation chain. The reader asking “Why K_4 specifically?” or “Is the Captures relation arbitrary?” now has machine-verified answers.

9 Discussion

9.1 Relation to Prior Work

FD connects to:

- Spencer-Brown's *Laws of Form*: D_0 is his "mark"
- Regge calculus: Discrete spacetime geometry
- Loop quantum gravity: Combinatorial structures
- Causal set theory: Discrete causality

Unlike these, FD derives structure from pure construction without positing it.

9.2 Limitations

FD does not yet derive:

- Standard Model particle content
- Fine structure constant $\alpha \approx 1/137$
- Precise Λ magnitude (10^{-122} problem)

9.3 Philosophical Implications

If correct, FD implies the laws of physics are *necessary*, not contingent. The universe must be 3+1 dimensional with positive Λ because distinction must distinguish.

10 Conclusion

We have presented FD, a machine-verified proof that 4D General Relativity emerges from the unavoidable first distinction. The derivation is:

- **Constructive**: All objects are built, not assumed
- **Axiom-free**: No mathematical axioms postulated
- **Falsifiable**: Specific predictions about black holes
- **Machine-checked**: 6,367 lines verified by Agda

The main result—ultimate-theorem : Unavoidable Distinction → FD-FullGR—represents a new paradigm: physics not *from* first principles, but physics *as* first principles.

Code availability. The complete Agda proof is available at <https://github.com/de-johannes/FirstDifference>.

References

- [1] G. Spencer-Brown, *Laws of Form*, Julian Press, 1969.
- [2] P. Martin-Löf, *Intuitionistic Type Theory*, Bibliopolis, 1984.
- [3] U. Norell, “Towards a practical programming language based on dependent type theory,” PhD thesis, Chalmers, 2007.
- [4] T. Regge, “General relativity without coordinates,” *Nuovo Cimento* **19**, 558 (1961).
- [5] J. D. Bekenstein, “Black holes and entropy,” *Phys. Rev. D* **7**, 2333 (1973).
- [6] S. W. Hawking, “Particle creation by black holes,” *Commun. Math. Phys.* **43**, 199 (1975).