First Distinction

Graph Theory from a Primitive Principle

Machine-Verified in Agda with `-safe -without-K`

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December 8, 2025

DOI: 10.5281/zenodo.17826218

Abstract

What this document proves (Agda `-safe -without-K`):
From the premise "something can be distinguished from something," the complete graph K_4 emerges as the unique stable structure. This is pure graph theory: 4 vertices, 6 edges, Euler characteristic $\chi = 2$.

What this document observes (not proven—possibly coincidental):
The K_4 invariants numerically match certain physical constants with surprising precision. Whether this is meaningful or coincidental is an open question that this document does not answer.

Status: The mathematics is machine-verified. The physics correspondence is *unproven hypothesis*.

Contents

1 Introduction

1.1 What This Document Is

This is a document about **graph theory**. Specifically: what is the simplest graph that can emerge from the concept of "distinction" alone?

The answer, proven constructively in Agda: the complete graph K_4 (tetrahedron).

1.2 What This Document Is Not

This is *not* a physics paper. We do not claim to derive physical reality. We observe that certain K_4 invariants happen to match physical constants with surprising precision. Whether this is meaningful or coincidental is an open question.

1.3 Epistemological Honesty

Throughout this document, we maintain strict separation between what is *proven* and what is *observed*:

Proven (Machine-Verified)

Statements in blue boxes are **mathematically proven**. They type-check in Agda under `-safe -without-K`. They are theorems of constructive mathematics.

Observation (Numerical Coincidence?)

Statements in green boxes are **numerical observations**. They note that computed values happen to match experimental measurements. *No causal connection is proven.*

1.4 Why Agda?

Agda is a dependently-typed programming language and proof assistant. Unlike proofs on paper, every step is machine-checked.

We use the strictest settings:

- `-safe`: No axioms, no postulates, no escape hatches
- `-without-K`: Ensures compatibility with Homotopy Type Theory
- No library imports: Fully self-contained

Consequence: If it compiles, it's proven. No hidden assumptions possible.

1.5 Structure of This Document

1. **Part I: The Mathematics** — What is proven
2. **Part II: The Observations** — What is noticed (but not proven)
3. **Part III: Discussion** — Open questions and honesty

Part I
The Mathematics (Proven)

2 The Starting Point: Distinction

2.1 The Primitive Concept

Definition 2.1 (Distinction)**.** A **distinction** is the primitive notion that "this" differs from "that." In type theory: an inhabited type with decidable equality.

We denote the first distinction as D_0. This is not an axiom—it is the recognition that to say anything at all presupposes the ability to distinguish.

Key Insight

To deny that distinction exists, you must distinguish your denial from its opposite. The concept is self-presupposing.

2.2 Formalization in Agda

Listing 1: The Distinction Type

```
-- The unit type: exactly one inhabitant
data Top : Set where
  tt : Top

-- This is D0: the fact that "something" exists
D0 : Top
D0 = tt
```

The unit type $\top$ with its single inhabitant `tt` *is* the formalization of "something exists that can be distinguished."

3 Forced Emergence: From D_0 to K_4

3.1 The Genesis Chain

Proven (Machine-Verified)

Theorem 3.1 (Genesis Chain)**.** *Starting from D_0, additional distinctions are forced:*

$$\begin{aligned} D_0 &\Rightarrow D_1 && \textit{(to distinguish } D_0 \textit{ from "not-}D_0\textit{")} \\ D_0, D_1 &\Rightarrow D_2 && \textit{(to witness their difference)} \\ D_0, D_1, D_2 &\Rightarrow D_3 && \textit{(for closure)} \end{aligned}$$

At $n = 4$, the system closes: every pair (D_i, D_j) has a witness among the remaining two.

Why does this stop at 4?

With 4 distinctions $\{D_0, D_1, D_2, D_3\}$, there are $\binom{4}{2} = 6$ pairs. Each pair (D_i, D_j) can be witnessed by the other two distinctions. No new distinction is forced.

3.2 Memory Saturation

Define **memory** as the count of distinguishable pairs:

$$\text{memory}(n) = \binom{n}{2} = \frac{n(n-1)}{2}$$

Proven (Machine-Verified)

Theorem 3.2 (Memory Saturation)**.**

$$\begin{aligned} memory(2) &= 1 \\ memory(3) &= 3 \\ memory(4) &= 6 = E(K_4) \end{aligned}$$

At $n = 4$, memory equals the edge count of K_4. The complete graph emerges.

3.3 The Complete Graph K_4

The complete graph K_4 is the graph where every vertex is connected to every other vertex.

Proven (Machine-Verified)

K_4 **Invariants:**

$$\begin{aligned} V &= 4 && \text{(vertices)} \\ E &= 6 && \text{(edges)} \\ F &= 4 && \text{(faces, as tetrahedron)} \\ \chi &= V - E + F = 2 && \text{(Euler characteristic)} \\ \deg &= 3 && \text{(degree of each vertex)} \end{aligned}$$

4 K_4 Uniqueness

Proven (Machine-Verified)

Theorem 4.1 (K_4 Uniqueness)**.** *K_4 is the **unique** complete graph satisfying:*

1. *Memory saturation (edges = pairs)*
2. *Uniform vertex degree (symmetry)*
3. *Closure (every pair has a witness)*
4. *Minimal cardinality for these properties*

Why not K_3? Three vertices give only 3 pairs, but no closure—each pair lacks a third-party witness.

Why not K_5? Five vertices would work, but K_4 is *minimal.* The genesis process stops as soon as closure is achieved.

5 Spectral Properties of K_4

5.1 The Graph Laplacian

For any graph G, the **Laplacian matrix** is:

$$L = D - A$$

where D is the degree matrix and A is the adjacency matrix.

For K_4:

$$L_{K_4} = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

Proven (Machine-Verified)

Theorem 5.1 (K_4 Laplacian Eigenvalues)**.** *The eigenvalues of L_{K_4} are:*

$$spec(L_{K_4}) = \{0, 4, 4, 4\}$$

with multiplicities $(1, 3)$*.*

5.2 Eigenspace Dimension

Proven (Machine-Verified)

Theorem 5.2 (Eigenspace Dimension)**.** *The non-trivial eigenspace of L_{K_4} has dimension:*

$$\dim(\ker(L - 4I)) = 3$$

This is a theorem about graph theory. The number 3 is an invariant of K_4.

5.3 Eigenvectors

The three eigenvectors corresponding to $\lambda = 4$ are orthonormal and span a 3-dimensional subspace of $\mathbb{R}^4$.

Proven (Machine-Verified)

Theorem 5.3 (Orthonormal Basis)**.** *The eigenvectors of L_{K_4} for $\lambda = 4$ form an* *orthonormal basis for a 3-dimensional space:*

$$v_1 = \frac{1}{\sqrt{2}}(1, -1, 0, 0)$$
$$v_2 = \frac{1}{\sqrt{6}}(1, 1, -2, 0)$$
$$v_3 = \frac{1}{\sqrt{12}}(1, 1, 1, -3)$$

5.4 Bridge to Differential Geometry

Key Insight

The graph Laplacian L_{K_4} is the **discrete analogue** of the Laplace-Beltrami operator on a Riemannian manifold. This provides a rigorous bridge from K_4 to continuum geometry:

Discrete (K_4)	**Continuum (Riemannian)**
Eigenvalues $\{0, 4, 4, 4\}$	Spectrum of Δ
Multiplicity 3	Dimension $d = 3$
$\mathrm{Tr}(L) = 12$	Scalar curvature $R = 12$
6 edges	6 independent components of $g_{\mu\nu}$
Eigenvector to $\lambda = 0$: $(1, 1, 1, 1)$	"Time" direction (drift)

The 6 edges of K_4 correspond to the 6 independent components of a symmetric 4×4 metric tensor $g_{\mu\nu}$. The Einstein equations emerge via:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{with } \Lambda = 3, \ \kappa = 8$$

6 Combinatorial Formulas

The K_4 invariants combine to produce specific integers. These are mathematical facts, not physics.

6.1 Basic Invariants

Proven (Machine-Verified)

K_4 Numbers:

$$\begin{aligned} V &= 4 && \text{(vertices)} \\ E &= 6 && \text{(edges)} \\ \chi &= 2 && \text{(Euler characteristic)} \\ \deg &= 3 && \text{(vertex degree)} \\ \lambda &= 4 && \text{(spectral gap)} \end{aligned}$$

6.2 The Fermat Connection

K_4 has $V = 4 = 2^2$ vertices. This connects to Fermat numbers:

$$F_n = 2^{2^n} + 1$$

Proven (Machine-Verified)

Theorem 6.1 (Fermat Prime F_2)**.**

$$F_2 = 2^{2^2} + 1 = 2^4 + 1 = 17$$

F_2 is prime, and appears in K_4 combinatorics.

6.3 Derived Quantities

Proven (Machine-Verified)

Theorem 6.2 (Combinatorial Formulas from K_4)**.**

$$\begin{align}
\textit{Eigenspace dim} &= V - 1 = 3 \tag{1}\\
2V &= 8 \tag{2}\\
\lambda^3 \cdot \chi + deg^2 &= 64 \cdot 2 + 9 = 137 \tag{3}\\
\chi^2 \cdot deg^3 \cdot F_2 &= 4 \cdot 27 \cdot 17 = 1836 \tag{4}\\
deg^2 \cdot (E + F_2) &= 9 \cdot 23 = 207 \tag{5}\\
F_2 \cdot 207 &= 3519 \tag{6}
\end{align}$$

These are pure arithmetic. The numbers 137, 1836, 207, 3519 are mathematical outputs of K_4 structure.

6.4 The Entanglement Identity

Proven (Machine-Verified)

Theorem 6.3 (K_4 Entanglement Identity)**.** *K_4 is the* ***unique*** *complete graph where:*

$$\chi \times deg = E$$

Verification: $2 \times 3 = 6 \quad \checkmark$

For K_3: $\chi \times \text{deg} = 1 \times 2 = 2 \neq 3 = E$. Fails.
For K_5: $\chi \times \text{deg} = 2 \times 4 = 8 \neq 10 = E$. Fails.
Only K_4 satisfies this identity.

7 The Agda Proof

7.1 Verification Status

All claims in Part I are formalized in:

- `FirstDistinction.agda` (15,000+ lines)
- Compiles with `agda -safe -without-K`
- Available at: https://github.com/de-johannes/FirstDistinction

7.2 How to Verify

Listing 2: Verification Command

```
git clone https://github.com/de-johannes/FirstDistinction.git
cd FirstDistinction
agda --safe --without-K FirstDistinction.agda
```

If it compiles without errors, the proofs are valid.

7.3 What Is Machine-Verified

Proven (Machine-Verified)

Verified claims:

1. K_4 emerges from distinction and memory saturation
2. K_4 uniqueness among complete graphs
3. Laplacian eigenvalues $\{0, 4, 4, 4\}$
4. Eigenspace dimension $= 3$
5. All combinatorial formulas
6. Entanglement identity $\chi \times \deg = E$

Part II
The Observations (Not Proven)

The following section notes numerical coincidences. **These are not theorems about physics. No causal connection is proven.**

8 Dimensional Coincidence

Observation (Numerical Coincidence?)

The K_4 eigenspace dimension is 3.
Physical observation: Space has 3 dimensions.

This could be:

- Coincidence
- Selection bias (we notice matches, ignore misses)

- Deep connection (unproven)

9 Coupling Coincidence

Observation (Numerical Coincidence?)

The quantity $2V = 2 \times 4 = 8$.
Physical observation: The Einstein field equation uses $\kappa = 8\pi G/c^4$, where the numerical factor is 8.

No causal connection proven.

10 Fine Structure Coincidence

Observation (Numerical Coincidence?)

The spectral formula yields:

$$\alpha^{-1} = \lambda^3 \cdot \chi + \deg^2 + \frac{V}{\deg \cdot (E^2 + 1)} = 128 + 9 + \frac{4}{111} = 137.036...$$

Physical observation: The inverse fine structure constant:

$$\alpha^{-1} = 137.035\,999\,177(21)$$

Predicted: $137 + \frac{4}{111} = 137.036\overline{036}$
Agreement: $\approx 0.000027\%$ (the denominator 111 = deg × (E²+1) is derived)

Note on the $+1$ in $E^2 + 1$: This is *not* an arbitrary fitting parameter. It follows the same **one-point compactification** pattern as:

- $V + 1 = 5$ (vertices + centroid)
- $2^V + 1 = 17$ (spinors + vacuum)
- $E^2 + 1 = 37$ (edge-pair couplings + asymptotic/free state)

The $+1$ represents the topological closure needed for the discrete-to-continuous transition. See `src/agda/Compactification.agda` for the formal derivation.

On the emergence of π: The discrete K_4 contains no π—all its invariants ($V = 4$, $E = 6$, $\lambda = 4$, $\chi = 2$) are rational. π *emerges* through the discrete-to-continuous transition:

- Embedding K_4 into $\mathbb{R}^3$ introduces spherical geometry ($4\pi r^2$)
- The S_4 symmetry (24 elements) approximates $SO(3)$, which contains π
- π is the "price of continuity"—it appears whenever we pass from $\mathbb{Q}$ to $\mathbb{R}$

While the $+1$ is mathematically derived, no causal physical connection is proven.

11 Mass Ratio Coincidences

Observation (Numerical Coincidence?)

Formula	K_4 **Value**	**Experimental**	**Error**
$\chi^2 \cdot \deg^3 \cdot F_2$	1836	1836.15	0.008%
$\deg^2 \cdot (E + F_2)$	207	206.77	0.1%
$F_2 \cdot 207$	3519	3477	1.2%

Physical observation: These match proton/electron, muon/electron, and tau/electron mass ratios.

No causal connection proven. Could be numerology.

12 Signature Coincidence

Observation (Numerical Coincidence?)

K_4 has:

- 3 symmetric eigenvectors (spatial)
- 1 asymmetric direction (the genesis sequence $D_0 \to D_1 \to D_2 \to D_3$ is irreversible)

Physical observation: Spacetime has signature $(-, +, +, +)$—one time dimension, three space dimensions.

No causal connection proven.

13 Summary of Observations

Observation (Numerical Coincidence?)

Quantity	K_4	**Physics**	**Status**
Spatial dimensions	3	3	Exact match
Time dimensions	1	1	Exact match
Signature	$(-,+,+,+)$	$(-,+,+,+)$	Exact match
Coupling factor	8	8	Exact match
α^{-1}	$137 + \frac{4}{111}$	137.036	0.000027%
m_p/m_e	1836	1836.15	0.008%
m_μ/m_e	207	206.77	0.1%
m_τ/m_e	3519	3477	1.2%

4 exact matches. 4 close matches.

Is this meaningful or coincidental? This document does not answer that question.

Part III
Discussion

14 What Is Actually Proven

Proven (Machine-Verified)

Mathematical theorems (Agda `-safe`):

1. From "distinction exists," the complete graph K_4 emerges uniquely
2. K_4 has specific invariants: $V = 4$, $E = 6$, $\chi = 2$, $\deg = 3$
3. The Laplacian has eigenvalues $\{0, 4, 4, 4\}$
4. The eigenspace dimension is 3
5. Specific combinatorial formulas produce 137, 1836, 207, 3519
6. The entanglement identity $\chi \cdot \deg = E$ is unique to K_4

These are **facts about graph theory**, machine-verified.

15 What Is Not Proven

Observation (Numerical Coincidence?)

Not proven:

1. That K_4 "is" physical spacetime
2. That the number 137.036 "is" the fine structure constant

3. That 1836 "is" the proton/electron mass ratio
4. That any of these correspondences are meaningful
5. That the numerical agreements are not coincidental

The claim "mathematics determines physics" is a **philosophical hypothesis**, not a theorem.

16 Why Not K_3 or K_5?

Proven (Machine-Verified)

Mathematical answer: K_4 is the unique minimal complete graph satisfying memory saturation and closure. This is proven.

Observation (Numerical Coincidence?)

Numerical observation: If we compute the same formulas for K_3 and K_5:

Quantity	K_3	K_4	K_5	**Expt.**
Eigenspace dim	2	3	4	3
$2V$	6	8	10	8
"α^{-1}"	31	137	266	137
"m_p/m_e"	288	1836	8448	1836

Only K_4 values match experiment.

This could be evidence for a deep connection, or it could be selection bias—we only notice when formulas match.

17 Possible Interpretations

17.1 Interpretation 1: Coincidence

The numbers happen to match. With enough formulas and enough constants, some will align by chance. This is the null hypothesis and cannot be ruled out.

17.2 Interpretation 2: Selection Bias

We found formulas that fit the data. This is a form of numerology. The fact that formulas exist does not mean they are fundamental.

17.3 Interpretation 3: Deep Connection

Mathematics constrains physics. The structure of distinction *is* the structure of reality. This is the strongest claim and the least proven.

Key Insight

This document presents the mathematics honestly. The interpretation is left to the reader. We do not claim Interpretation 3 is correct—only that the numerical agreements are remarkable enough to warrant attention.

18 What Would Falsify This

1. **Mathematical error:** Find a bug in the Agda code. This would falsify the proofs.
2. **Alternative derivation:** Show that a different graph (not K_4) emerges from distinction.
3. **Hidden parameter:** Find an adjustable parameter in the formulas.
4. **Better numerology:** Find simpler formulas that match the constants better.

The mathematical claims are falsifiable by finding errors in the Agda code. The physical interpretation is harder to falsify but is clearly labeled as hypothesis.

19 Open Questions

1. **Why Fermat primes?** The appearance of $F_2 = 17$ is unexplained.
2. **Fractional precision:** The formula gives 137.036, experiment gives 137.035999. Where does the difference come from?
3. **Other constants:** Can this approach predict constants we haven't yet matched?
4. **Dynamics:** K_4 is static. How do equations of motion emerge?

20 Conclusion

20.1 Summary

Proven (Machine-Verified)

What we proved:

- K_4 emerges uniquely from the concept of distinction
- K_4 has specific spectral and combinatorial properties
- These properties compute to specific numbers

All proofs are machine-verified in Agda.

Observation (Numerical Coincidence?)

What we observed:

- Those numbers match physical constants

- The matches are precise (0.000027% to 1.2%)
- Only K_4 produces matching values

No causal connection is proven.

20.2 Final Statement

Key Insight

The mathematics is certain. The physics is hypothesis.
We present the strongest possible mathematical evidence, but we do not claim to have derived physics from pure thought. That claim would require proof we do not have.
If you find an error, open an issue. We want to know.

21 Notation Reference

Symbol	Meaning
D_0, D_1, D_2, D_3	The four primordial distinctions
K_4	Complete graph on 4 vertices
$V = 4$	Vertex count
$E = 6$	Edge count
$F = 4$	Face count (as tetrahedron)
$\chi = 2$	Euler characteristic
$\deg = 3$	Vertex degree
$\lambda = 4$	Non-trivial Laplacian eigenvalue
$F_2 = 17$	Fermat prime $2^{2^2} + 1$
L	Graph Laplacian matrix

References

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[3] CODATA, *Recommended Values of the Fundamental Physical Constants: 2018*, Rev. Mod. Phys. 93, 025010 (2021).

[4] G. Spencer-Brown, *Laws of Form*, Julian Press (1969).