Modern Physics Reference Sheet — Ch. 38,39

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Constants

$c = \lambda\nu = 2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (Speed of Light in Free Space)

$e = q_e = q_p = 1.602\,177 \times 10^{-19}\,\mathrm{C} \implies 1\,\mathrm{eV} = 1.602\,177 \times 10^{-19}\,\mathrm{J}$ (Unit Charge)

$k_\mathrm{e} = 8.987\,551 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$ (Coulomb Constant)

$\varepsilon_0 = 8.854\,188 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ (Permittivity of Free Space)

$m_e = 9.109\,384 \times 10^{-31}\,\mathrm{kg}$ (Electron Mass)

$m_p = 1.672\,622 \times 10^{-27}\,\mathrm{kg}$ (Proton Mass)

$h = 6.626\,070 \times 10^{-34}\,\mathrm{J\,s} = 4.135\,668 \times 10^{-15}\,\mathrm{eV\,s}$ (Planck Constant)

$\hbar = h/2\pi = 1.054\,572 \times 10^{-34}\,\mathrm{J\,s} = 6.582\,120 \times 10^{-16}\,\mathrm{eV\,s}$ (Reduced Planck Constant)

$R_\mathrm{H} = 1.097\,373 \times 10^{7}\,\mathrm{m^{-1}}$ (Rydberg Constant of an H Atom)

$hc = 1.986\,446 \times 10^{-25}\,\mathrm{J\,m} = 1.239\,842 \times 10^{-6}\,\mathrm{eV\,m}$ (Product of h and c)

$E_1 = 13.598\,44\,\mathrm{eV}$ (Total Mechanical Energy of a Ground-State Electron of a H atom)

$\lambda_\mathrm{C} = h/m_e c = 2.426\,310 \times 10^{-12}\,\mathrm{m}$ (Compton Wavelength)

Formulae and Equations

0.1 Relativity

$\beta \coloneqq v/c$ (Definition of Fraction of Speed of Light)

$\gamma \coloneqq 1/\sqrt{1-(v/c)^2} = 1/\sqrt{1-\beta^2}$ (Definition of Lorentz Factor)

$s^2 = (c\Delta t)^2 - \|\Delta\mathbf{x}\|^2 \implies (\mathrm{d}S)^2 = (c\mathrm{d}t)^2 - \|\mathrm{d}\mathbf{x}\|^2$ (Spacetime Interval)

$L_\mathrm{obs} = L_\mathrm{real}/\gamma$ (Length Contraction)

$t_\mathrm{obs} = t_\mathrm{real}\gamma$ (Time Dilation)

0.2 Particle Variables

$E = h\nu = hc/\lambda$ (Planck relation)

$\lambda = h/p = h/mv \implies p = E/c = h/\lambda$ (De Broglie Wavelength)

0.2.1 Quantized Particle Model

$E = nh\nu = nhc/\lambda$ (Quantized Planck relation)

$\Delta\lambda = h/m_e c\,(1-\cos\theta) = \lambda_\mathrm{C}\,(1-\cos\theta)$ (Compton Scattering)

0.2.2 Electrons

Properties of photoelectrons from metal with work function W:

$E_\gamma = W + K_{e^-(\max)} \implies K_{e^-(\max)} = h\nu - W = h(\nu - \nu_0)$ (Conservation of Energy)

$eV_S \coloneqq K_{e^-(\max)} \implies V_S = h/e\,(\nu - \nu_0)$ (Definition of Stopping Potential)

Properties of the electron in a Z-hydrogenic atom of principal quantum number n:

$2\pi r_n = \lambda n \implies L_n = m_e v_n r_n = n\hbar$ (Bohr Model; Angular Momentum)

$$F_c = \frac{mv^2}{r} = \frac{m_e v_n^2}{r_n} \quad \text{(Centripetal Force)}$$

$$F_\mathrm{e} = k_\mathrm{e}\frac{q_1 q_2}{r^2} = k_\mathrm{e}\frac{(e)(Ze)}{r_n^2} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n^2} = k_\mathrm{e}\frac{Ze^2}{r_n^2} \quad \text{(Electrostatic Force)}$$

$$F_\mathrm{e} = F_c \implies v_n^2 r_n = k_\mathrm{e}\frac{Ze^2}{m_e} \quad \text{(Electrostatic Force functioning as Centripetal Force)}$$

$$v_n = \left(m_e \cdot v_n^2 r_n\right)/(m_e v_n r_n) = \frac{k_\mathrm{e} Z e^2}{n\hbar} \quad \text{(Orbital Velocity)}$$

$$r_n = (m_e v_n r_n)/(m_e \cdot v_n) = \frac{n^2\hbar^2}{m_e k_\mathrm{e} Z e^2} = a_0\frac{n^2}{Z} \quad \text{(Orbital)}$$

$$a_0 \coloneqq \frac{\hbar^2}{m_e k_\mathrm{e} e^2} = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{\varepsilon_0 h^2}{\pi m_e e^2} \quad \text{(Definition of Bohr Radius)}$$

$$U_n = -k_\mathrm{e}\frac{q_1 q_2}{r} = -k_\mathrm{e}\frac{(e)(Ze)}{r_n} = -\frac{k_\mathrm{e} Z e^2}{r_n} = -\frac{k_\mathrm{e}^2 Z^2 m_e e^4}{n^2\hbar^2} \quad \text{(Potential Energy)}$$

$$K_n = \frac{1}{2}mv^2 = \frac{1}{2}m_e v_n^2 = \frac{k_\mathrm{e}^2 Z^2 m_e e^4}{2n^2\hbar^2} \implies 2K_n + U_n = 0 \quad \text{(Kinetic Energy)}$$

$$E_n = K_n + U_n = -\frac{k_\mathrm{e}^2 Z^2 m_e e^4}{2n^2\hbar^2} = -E_1\frac{Z^2}{n^2} \quad \text{(Total Mechanical Energy Magnitude)}$$

$$E_1 \coloneqq \frac{k_\mathrm{e}^2 m_e e^4}{2\hbar^2} = \frac{m_e e^4}{8\varepsilon_0 h^2} \quad \text{(Total Mechanical Energy Magnitude for } n=1 \text{ H Electron)}$$

$$\frac{1}{\lambda} = R_\mathrm{H}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{(Rydberg Formula for a H electron)}$$

$$R_\mathrm{H} \coloneqq -\frac{m_e e^4}{8h^3\varepsilon_0^3 c} \quad \text{(Rydberg Constant for a H electron)}$$

Note: a_0 also equals the modal distance between the nucleus and electron of a H atom.
Note: Velocities are assumed to be non-relativistic ($v \ll c$).

0.3 Sinusoidal Waves

T is to ω as λ is to k. Note: φ, phase shift, will vary between different expressions of D where initial conditions like x_0 and t_0 are set. Note: **SWD** refers to the formula of sinusoidal wave spatial–temporal displacement, $D(x,t)$.

$f \coloneqq 1/T \implies T = 1/f,\, fT = 1$ (Definition of Frequency)

$\omega \coloneqq 2\pi/T \implies \omega = 2\pi f,\, T = 2\pi/\omega,\, f = \omega/2\pi$ (Defintion of Angular Frequency)

$\lambda \coloneqq v/f \implies v = \lambda f = \lambda/T,\, vT = \lambda$ (Definition of Wavelength)

$k \coloneqq 2\pi/\lambda \implies v = \omega/k$ (Definition of Wavenumber)

$$D(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \varphi\right) = A\sin(kx - \omega t + \varphi) \quad \text{(SWD)}$$

$$D(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt) + \varphi\right) \quad \text{(SWD parametrized to } x - vt\text{)}$$

$$D(x,t_0) = A\sin\left(2\pi\frac{x}{\lambda} + \varphi'\right) = A\sin\left(kx + \varphi'\right) \quad \text{(SWD in a Snapshot Graph)}$$

$$D(x_0,t) = A\sin\left(-2\pi f t + \varphi'\right) = A\sin\left(-\omega t + \varphi'\right) \quad \text{(SWD in a History Graph)}$$

$$D(x,t) = D(x + n\lambda, t + mT)\ \forall\, n, m \in \mathbb{Z} \quad \text{(SWD spatial-temporal periodicity)}$$

$$V(x,t) = \frac{\partial D(x,t)}{\partial t} = -\omega A\cos(kx - \omega t + \varphi) \quad \text{(SWD particle velocity)}$$

$$V(x_0,t) = \left.\frac{\partial D(x,t)}{\partial t}\right|_{x=x_0} = -\omega A\cos\left(\omega t + \varphi'\right) \quad \text{(SWD particle velocity history graph)}$$

0.4 Quantum Mechanics

0.4.1 Schrödinger's Equation

$$\hat{H}\langle\Psi(t)| = i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\langle\Psi(t)| \implies -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\mathbf{x})\Psi = i\hbar\frac{\partial\Psi}{\partial t} \quad \text{(Time-dependent)}$$

$$\hat{H}\langle\Psi| = E\langle\Psi| \implies -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\mathbf{x})\Psi = E\Psi \quad \text{(Time-independent)}$$