

Constants

$c = \lambda\nu = 2.997\,925 \times 10^8 \text{ m s}^{-1}$	(Speed of Light)
$e = q_e = q_p = 1.602\,177 \times 10^{-19} \text{ C} \implies 1 \text{ eV} = 1.602\,177 \times 10^{-19} \text{ J}$	(Unit Charge)
$k_e = 8.987\,551 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$	(Coulomb Constant)
$\epsilon_0 = 8.854\,188 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$	(Permittivity of Free Space)
$m_e = 9.109\,384 \times 10^{-31} \text{ kg}$	(Electron Mass)
$m_p = 1.672\,622 \times 10^{-27} \text{ kg}$	(Proton Mass)
$h = 6.626\,070 \times 10^{-34} \text{ Js} = 4.135\,668 \times 10^{-15} \text{ eV s}$	(Planck Constant)
$\hbar = h/2\pi = 1.054\,572 \times 10^{-34} \text{ Js} = 6.582\,120 \times 10^{-16} \text{ eV s}$	(Reduced Planck Constant)
$R_H = 1.097\,373 \times 10^{-7} \text{ m}^{-1}$	(Rydberg Constant of an H Atom)
$hc = 1.986\,446 \times 10^{-25} \text{ J m} = 1.239\,842 \times 10^{-6} \text{ eV m}$	(Product of h and c)
$E_1 = 13.598\,44 \text{ eV}$	(Total Mechanical Energy of a Ground-State Electron of a H atom)
$\lambda_C = h/m_e c = 2.426\,310 \times 10^{-12} \text{ m}$	(Compton Wavelength)

Formulae and Equations

0.1 Relativity

$\beta := v/c$	(Definition of Fraction of Speed of Light)
$\gamma := 1/\sqrt{1 - (v/c)^2} = 1/\sqrt{1 - \beta^2}$	(Definition of Lorentz Factor)
$s^2 = (c\Delta t)^2 - (\Delta \mathbf{x})^2 \implies dS = (cdt)^2 - (d\mathbf{x})^2$	(Spacetime Interval)
$L_{\text{obs}} = L_{\text{real}}/\gamma$	(Length Contraction)
$t_{\text{obs}} = t_{\text{real}}\gamma$	(Time Dilation)

0.2 Particle Variables

$E = h\nu = hc/\lambda$	(Planck relation)
$\lambda = h/p = h/mv \implies p = E/c = h/\lambda$	(De Broglie Wavelength)

0.2.1 Quantized Particle Model

$E = nh\nu = nhc/\lambda$	(Quantized Planck relation)
$\Delta\lambda = h/m_e c (1 - \cos\theta) = \lambda_C (1 - \cos\theta)$	(Compton Scattering)

0.2.2 Electrons

Properties of photoelectrons from metal with work function W :

$$E_\gamma = W + K_{e^-} \text{ (max)} \implies K_{e^-} \text{ (max)} = h\nu - W = h(\nu - \nu_0) \quad (\text{Conservation of Energy})$$

$$eV_S := K_{e^-} \text{ (max)} \implies V_S = h/e(\nu - \nu_0) \quad (\text{Definition of Stopping Potential})$$

Properties of the electron in a Z -hydrogenic atom of principal quantum number n :

$$2\pi r_n = \lambda n \implies L_n = m_e v_n r_n = nh \quad (\text{Bohr Model; Angular Momentum})$$

$$F_c = \frac{mv^2}{r} = \frac{m_e v_n^2}{r_n} \quad (\text{Centripetal Force})$$

$$F_e = k_e \frac{q_1 q_2}{r^2} = k_e \frac{(e)(Ze)}{r_n^2} = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n^2} = k_e \frac{Ze^2}{r_n^2} \quad (\text{Electrostatic Force})$$

$$F_e = F_c \implies v_n^2 r_n = k_e \frac{Ze^2}{m_e} \quad (\text{Electrostatic Force functioning as Centripetal Force})$$

$$v_n = \left(m_e \cdot v_n^2 r_n \right) / (m_e v_n r_n) = \frac{k_e Ze^2}{nh} \quad (\text{Orbital Velocity})$$

$$r_n = (m_e v_n r_n) / (m_e \cdot v_n) = \frac{n^2 \hbar^2}{m_e k_e Ze^2} = a_0 \frac{n^2}{Z} \quad (\text{Orbital})$$

$$a_0 := \frac{\hbar^2}{m_e k_e e^2} = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{\epsilon_0 \hbar^2}{\pi m_e e^2} \quad (\text{Definition of Bohr Radius})$$

$$U_n = -k_e \frac{q_1 q_2}{r} = -k_e \frac{(e)(Ze)}{r_n} = -\frac{k_e Ze^2}{r_n} = -\frac{k_e^2 Z^2 m_e e^4}{n^2 \hbar^2} \quad (\text{Potential Energy})$$

$$K_n = \frac{1}{2} m v^2 = \frac{1}{2} m_e v_n^2 = \frac{k_e^2 Z^2 m_e e^4}{2n^2 \hbar^2} \implies 2K_n + U_n = 0 \quad (\text{Kinetic Energy})$$

$$E_n = K_n + U_n = -\frac{k_e^2 Z^2 m_e e^4}{2n^2 \hbar^2} = -E_1 \frac{Z^2}{n^2} \quad (\text{Total Mechanical Energy Magnitude})$$

$$E_1 := \frac{k_e^2 m_e e^4}{2\hbar^2} = \frac{m_e e^4}{8\epsilon_0 \hbar^2} \quad (\text{Total Mechanical Energy Magnitude for } n=1 \text{ H Electron})$$

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (\text{Rydberg Formula})$$

$$R_H := -\frac{m_e e^4}{8\hbar^3 \epsilon_0^3 c} \quad (\text{Rydberg Formulae for a H Electron})$$

Note: a_0 also equals the modal distance between the nucleus and electron of a H atom.

Note: Velocities are assumed to be non-relativistic ($v \ll c$).

0.3 Sinusoidal Waves

T is to ω as λ is to k . Note: φ , phase shift, will vary between different expressions of D where initial conditions like x_0 and t_0 are set. Note: SWD refers to the formula of

$\sinusoidal \text{ wave spatial-temporal displacement, } D(x, t).$	
$f := 1/T \implies T = 1/f, fT = 1$	(Definition of Frequency)
$\omega := 2\pi/T \implies \omega = 2\pi f, T = 2\pi/\omega, f = \omega/2\pi$	(Definition of Angular Frequency)
$\lambda := v/f \implies v = \lambda f = \lambda/T, vT = \lambda$	(Definition of Wavelength)
$k := 2\pi/\lambda \implies v = \omega/k$	(Definition of Wavenumber)
$D(x, t) = A \sin \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) + \varphi \right) = A \sin(kx - \omega t + \varphi)$	(SWD)
$D(x, t) = A \sin \left(\frac{2\pi}{\lambda} (x - vt) + \varphi \right)$	(SWD parametrized to $x - vt$)
$D(x, t_0) = A \sin \left(2\pi \frac{x}{\lambda} + \varphi' \right) = A \sin(kx + \varphi')$	(SWD in a Snapshot Graph)
$D(x_0, t) = A \sin \left(-2\pi ft + \varphi' \right) = A \sin(-\omega t + \varphi')$	(SWD in a History Graph)
$D(x, t) = D(x + n\lambda, t + mT) \forall n, m \in \mathbb{Z}$	(SWD spatial-temporal periodicity)
$V(x, t) = \frac{\partial D(x, t)}{\partial t} = -\omega A \cos(kx - \omega t + \varphi)$	(SWD particle velocity)
$V(x_0, t) = \frac{\partial D(x, t)}{\partial t} \Big _{x=x_0} = -\omega A \cos(\omega t + \varphi')$	(SWD particle velocity history graph)

0.4 Quantum Mechanics

0.4.1 Schrödinger's Equation

$$\hat{H}|\Psi(t)\rangle = i\hbar \frac{d}{dt}|\Psi(t)\rangle \implies -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\mathbf{x})\Psi = i\hbar \frac{\partial \Psi}{\partial t} \quad (\text{Time-dependent})$$

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \implies -\frac{\hbar^2}{2m} \nabla^2 \Psi + V(\mathbf{x})\Psi = E\Psi \quad (\text{Time-independent})$$