Graph Drawing Contest 2020
Crossing Minimization with Randomness

Sebastian Benner

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Abstract

Stuff

1 Introduction

The annual Graph Drawing Contest[1] is an open challenge to design an algorithm for optimized graph drawing. The exact criteria for such a drawing are changed every couple of years, the current ones remain the same as last years challenge. The Live Challenge will contain between five to ten acyclic directed graphs with up to a few thousands nodes each. All resulting layouts must be submitted within one hour of the graphs being handed out.

The main criteria this time around are crossing which ought to be minimal in the resulting drawing. In itself this already poses a NP-hard problem. Additional constraints placed on the drawing are:

- Each edge must be a straight upward facing line, meaning the source of each directed edge must be lower than the target.
- Each node must be placed upon a grid of given size.
- Crossings between a node and an edge are not permitted, as well as overlapping nodes.

After all graphs are collected, for each of the original graphs a best drawing is determined with all the other graphs receiving a weighted score based on the difference in crossings. The highest overall score wins the contest. Each team has to bring its own hardware to run their respective algorithm, meaning there is no limitation in terms of tools used and the given time to solve the task can be counterbalanced by more powerful hardware.

During the last couple of years most if not all of the top scoring contestants based their algorithm at least to some part on randomness which will be the basis of this work. The goal of is to evaluate different base drawings and to find a balance between lightweight calculations for random steps and a more directed approach to randomness.

[1] http://mozart.diei.unipg.it/gdcontest/contest2020/challenge.html

2 Foundation

A *graph* G is defined as an ordered pair (V, E) of *vertices* V and *edges* E. Edges are unordered pairs of two vertices $\{x, y\}$, said to *join* them, and therefore E is a subset of $V^{(2)}$. In the special case where edges are ordered pairs (x, y) with x as *source* and y as *target* the graph D is called *directed* graph or *digraph*. We call the number of vertices the *order* of G and the number of edges the *size* of G. $V(G)$ and $E(G)$ are the sets of vertices and edges of G respectively, $x \in V(G)$ with vertex x can be written as $x \in G$ while $\{x, y\} \in E(G)$ with unordered edge $\{x, y\}$ is written as $\{x, y\} \in G$.

For a more comprehensive explanation I refer to the book *Modern Graph Theory* [2] upon which this notation is based on.

3 Prior Work

A Heuristic Approach towards Drawings of Graphs with High Crossing Resolution [1] While not aiming at the same goal, this algorithm served as basis for the last years winner. The aim was to create a drawing with largest minimum angle, called the resolution of G, possible. To achieve this the algorithm build and maintained two sets: all nodes and only such nodes that are deemed critical for the current resolution of G. Critical are all nodes connected with edges involved in minimal angles. In each iteration either a node from the set of all critical nodes is chosen uniformly or inverse proportionally to its proximity to a critical nodes from the set of all nodes. To determine the new position of a node rays are used which are cast out uniformly distributed in all directions with the best result becoming the new position. Combined with a energy-based base drawing and some tweaks to avoid local minima, the algorithm proved itself also won its respective year.

4 Sugiyama Framework

5 Crossing Minimization

6 Results

References

[1] Michael A. Bekos et al. “A Heuristic Approach towards Drawings of Graphs with High Crossing Resolution.” In: *CoRR* abs/1808.10519 (2018). URL: `http://dblp.uni-trier.de/db/journals/corr/corr1808.html#abs-1808-10519`.

[2] Bélla Bollobás. *Modern Graph Theory*. 1998. DOI: `10.1007/978-1-4612-0619-4`.