Introduction to Robotics

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Sections of mechanics

Statics is concerned with the analysis of loads (force and torque, or "moment") acting on physical systems that do not experience an acceleration (a=0), but rather, are in static equilibrium with their environment.

Kinematics describes the motion of points, bodies (objects), and systems of bodies (groups of objects) without considering properties of objects (mass, density) or the forces that caused the motion.

Kinetics is concerned with the relationship between motion and its causes, specifically, forces and torques.

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Matrix operations

1. Addition
2. Scaling: $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot 3 = \begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix}$
3. Dot product (inner product): $(x_1 \dots x_n) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = (a_1)$
4. Multiplication -- consists of several dot product operations

5. Transposition: $A^T \,:\, a^T_{ij} = a_{ji},\ \forall i, j$

   Also, $(ABC)^T = C^T B^T A^T$
6. Inverse
7. Determinant
8. Power (only for square matrices)
9. Trace -- sum of elements on the diagonal

   $\mathrm{tr} AB = \mathrm{tr} BA$

   $\mathrm{tr}(A+B) = \mathrm{tr} A + \mathrm{tr} B$
10. etc (maybe)

Vector norm

A norm is a function $f \,:\, R^n \to R$ that satisfies 4 properties:

1. Non-negativity: $\forall x \in R^n \,:\, f(x) \geqslant 0$
2. Definiteness: $f(x) = 0 \Rightarrow x = 0$
3. Homogeneity: $\forall x \in R^n, t \in R \,:\, f(tx) = |t| f(x)$
4. Triangle inequality: $\forall x, y \in R^n \,:\, f(x+y) \leqslant f(x) + f(y)$

Euclidean vector norm (2-norm): $||x||_2 = \sqrt{\sum\limits_{i=1}^{n} x_i^2}$

$||x||_2 = \sqrt{x^T x}$

General p-norms, $p \geqslant 1$: $||x||_p = (\sum\limits_{i=1}^{n} |x_i|^p)^{\frac{1}{p}}$

$||x||_\infty = \max\limits_i |x_i|$

Matrix multiplication

- associative: $(AB)C = A(BC)$
- distributive: $A(B+C) = AB + AC$
- non-commutative: $AB \neq BA$

Determinant

Properties

- $\det AB = \det BA$
- $\det A^{-1} = \frac{1}{\det A}$
- $\det A^T = \det A$