Solution of Algebraic & Transcental Equations

① Algebraic terms

$2x$; $3x$, $9x^2+5x+6$ etc.

② Algebraic Equations

$2x=0$, $3x=0$, $9x^2+5x+6=0$ ... etc.

↓

this is an algebraic equation with degree 2.

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_nx^0 = 0$

↓

this is a algebraic equation with degree n.

③ Transcental Equations

An equation which contains algebraic term and also contains logarithmic, exponential or trigonometric functions are known as Transcendental equations.

Eg - $\cos x + 2x = 0$, $9\log x + e^{\sin x} = 0$

④ Need to Study Numerical Analysis?

We need Numerical Analysis to solve equation which cent be solved using normal formula or for equation which doesn't have a formula to get solved like we can solve x^2-5x+6, but to find the roots of $x^{1000} + 998x^{990} + 66x^{50} + 23 = 0$, we need methods like Numerical Analysis.

Another reason why we need to study the Numerical Analysis is to find out the approximate roots of the equations whose exact roots cent be found.

Numerical analysis gives an approximate value similar to the root value.

⑤ Different Numerical Approach to Solve an Equation

① Iteration Method
② Newton Raphson Method
③ Bisection Method
④ Regula falsi Method
⑤ Secant Method.

} → in syllabus.

* Iteration Method

Let us assume that we are given a function $f(x) = 0$ in x. We will find 2 values $x = a$ & $x = b$ & their corresponding $f(a)$ & $f(b)$. The values of a & b for which

$$f(a) \cdot f(b) < 0$$

then we can say that there exists at least one root of the equation $f(x) = 0$ within $x = a$ & $x = b$.

Let us say $x = \alpha \in [a, b]$

then, $f(x) = 0$.

∴ we need to find an interval where α lies.

Questions

Question 1 find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places using iteration method.

Solution 1 Given equation : $\cos x = 3x - 1$

$$\Rightarrow \cos x - 3x + 1 = 0$$

$\Rightarrow f(x) = 0$, where $f(x) = \cos x - 3x + 1$

let $x = 0$, then

$$f(0) = \cos(0) - 3(0) + 1$$
$$= 1$$

let $x = \frac{\pi}{4}$, then

$$f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) - 3 \cdot \frac{\pi}{4} + 1$$

$$= \frac{1}{\sqrt{2}} - \frac{3\pi}{4} + 1 = -0.65$$

as $f(0) \times f\left(\frac{\pi}{4}\right) < 0$

$\Rightarrow$ One root lies b/w $x = 0$ & $x = \frac{\pi}{4}$

Also, the given eqn $f(x) = 0$ can be written as

$$x = \frac{1}{3}(1 + \cos x) = \phi(x) \text{ (say)}$$

$$\therefore \phi'(x) = \frac{1}{3}(-\sin x)$$

$$\Rightarrow |\phi'(x)| = \frac{1}{3}|\sin x| < 1 \quad \forall \quad x \in \left(0, \frac{\pi}{4}\right)$$

$\Rightarrow$ The problem can be solved by taking $\phi(x) = \frac{1}{3}(\cos x + 1)$ in the interval $\left(0, \frac{\pi}{4}\right)$

Let the initial iteration is $x = 0$

$$\therefore x_1 = \phi(x_0) = \phi(0) = \frac{1}{3}(\cos 0 + 1) = 0.666667$$

$$x_2 = \phi(x_1) = \frac{1}{3}(\cos(0.666667) + 1) = 0.595295$$

$$x_3 = \phi(x_2) = \frac{1}{3}(\cos(0.595295) + 1) = 0.609328$$

$$x_4 = \phi(x_3) = \frac{1}{3}(\cos(0.609328) + 1) = 0.606678$$

$$x_5 = \phi(x_4) = \frac{1}{3}(\cos(0.606678) + 1) = 0.607182$$

$$x_6 = \phi(x_5) = \frac{1}{3}(\cos(0.607182) + 1) = 0.607086$$

$$x_7 = \phi(x_6) = \frac{1}{3}(\cos(0.607086) + 1) = 0.6071045$$

$\therefore$ The root is $x = 0.607$, correct to 3 decimal places.

Question 2 find a real root of $2x - \log x = 7$, correct to 4 decimal places using iteration method

Solution Given Equation : $2x - \log x = 7$

$\Rightarrow$ ~~f(x)~~ $2x - \log x - 7 = 0$

$\Rightarrow f(x) = 0$, where $f(x) = 2x - \log x - 7$

Let $X = 3$, then

$$f(3) = 2\times 3 - \log 3 - 7$$

$$= -1.4712$$

Let $X = 4$, then

$$f(4) = 2\times 4 - \log 4 - 7$$

$$= 0.3979$$

as $f(3) \times f(4) < 0$

$\Rightarrow$ One of root lies b/w $X = 3$ & $X = 4$

Also the given equation $f(x)$ can be written as

$$X = \frac{7 + \log x}{2} = \phi(x) \text{ (say)}$$

$$\phi'(x) = \frac{1}{2x} \cdot \log x.$$

$$\Rightarrow \left|\phi'(x)\right| = \frac{1}{2x} < 1 \quad \forall\ x \in (3,4)$$

$\Rightarrow$ The problem can be solved by taking $\phi(x) = \frac{7+\log x}{2}$ in the interval $(3,4)$

Let the initial iteration is $x_0 = 3$

$$\therefore \phi(x_0) = \phi(3) = \frac{7 + \log 3}{2} = 3.738560$$

$$x_2 = \phi(x_1) = \frac{7 + \log(3.738560)}{2} = 3.786352$$

$$x_3 = \phi(x_2) = \frac{7 + \log(3.786352)}{2} = 3.789110$$

$$x_4 = \phi(x_3) = \frac{7 + \log(3.789110)}{2} = 3.789269$$

$$x_5 = \phi(x_4) = \frac{7 + \log(3.789269)}{2} = 3.789278$$

∴ The root is $x = 3.7892$, correct to upto 4 decimal places.

Question 3 find the positive roots of $x^3 - 9x + 1 = 0$ correct to 4 significant figures.

Solution 3 Given equation : $x^3 - 9x + 1 = 0$

$\Rightarrow$ $f(x) = 0$ where $f(x) = x^3 - 9x + 1$

let $x = 1$

$f(1) = 1 - 9 + 1 = -7$

let $x = 2$

$f(2) = 8 - 18 + 1 = -9$

let $x = 3$

$f(3) = 27 - 27 + 1 = 1$

let $x = 0$

$f(0) = 1$

$\Rightarrow$ as $f(0) \times f(1) < 0$ & $f(2) \times f(3) < 0$

$\Rightarrow$ one root lies between $x = 0$ & $x = 1$ & one root lies between $x = 2$ & $x = 3$.

Also, the given equation $f(x) = 0$ can be written as

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Newton - Raphson's Method

Soln

Find the, $f(x) = 0$ —— (1)

Let x_0 be the initial sol of (1)

Let h, be a small increment

If $x_1 = x_0 + h$, be the root of the eqn, then

$$f(x_1) = 0$$

$$\Rightarrow f(x_0 + h) = 0$$

$$f(x_0) + h f'(x_0) + \frac{h^2}{\lfloor 2} f''(x_0) + \frac{h^3}{\lfloor 3} f'''(x_0) + \dots = 0$$

(using Taylor's series expansion)

$h = 0$.

$$f(x_0) + h f'(x_0) = 0$$

$$h = \frac{-f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

. . . . .

Generally

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \quad n = 1, 2, 3 \dots$$

Eg① find the +ve root of $x^4 - x = 10$, correct upto 3 decimal places, using Newton - Raphson's method.

Sol^n Let $f(x) = x^4 - x - 10$

We are to find the +ve root of $f(x) = 0$

Here $f(0) = -10$, $f(1) = -10$, $f(2) = 16 - 2 - 10 = 4$

$\therefore f(1)\, f(2) < 0 \Rightarrow$ are +ve roots of the eq^n $f(x) = 0$ lies b/w $x = 1$ & $x = 2$.

Now, the Newton Raphson's Method states that

$$X_n = X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})}, \quad n = 1, 2, 3, \ldots$$

Let $X_0 = 1.5$

$$\therefore X_1 = X_0 - \frac{f(X_0)}{f'(X_0)} = 1.5 - \frac{f(1.5)}{f'(1.5)}$$

$$= 1.5 - \frac{(-6.4375)}{12.5}$$

$$= 1.5 + 0.515$$

$$= 2.015$$

$$X_2 = 2.015 - \frac{f(2.015)}{f'(2.015)} = 2.015 - \frac{4.47}{31.725}$$

$$= 2.015 - 0.1408$$

$$= 1.8741$$

$$X_3 = 1.8741 - \frac{f(1.8741)}{f'(1.8741)} = 1.8741 - \frac{0.461805}{25.3492}$$

$$= 1.8559$$

$$X_4 = 1.8559 - \frac{f(1.8559)}{f'(1.8559)} = 1.855475$$

$$X_5 = 1.855475 - \frac{f(1.855475)}{f'(1.855475)} = 1.85558$$

$$X_6 = 1.85558 - \frac{f(1.85558)}{f'(1.85558)} = 1.85558$$

∴ One of the +ve root of the eqn $f(x)=0$ is $x = 1.856$, correct to the 3 decimal places.

Eg (2) find by NRM, the real root of the eqn $3x = \cos x + 1$, correct to 4 decimal places.

$f'(x) = 3 + \sin x$

Soln Let $f(x) = 3x - \cos x - 1 = 0$

We are to find the +ve root of $f(x) = 0$

Here, $f(0) = -2$, $f(1) = 1.45$

∴ $f(0)\,f(1) < 0$ ⇒ are +ve roots of the eqn $f(x)=0$ lies b/w $x=0$ & $x=1$

Now, the Newton Raphson's Method states that

$$X_n = X_{n-1} - \frac{f(X_{n-1})}{f'(X_{n-1})}, \quad n = 1, 2, 3$$

Let $X_0 = 0.5$

$$\therefore X_1 = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.5 + \frac{0.377582}{3.479}$$

$$= 0.5 + 0.108518 = 0.608518$$

$$X_2 = 0.608518 - \frac{f(0.608518)}{f'(0.608518)} = 0.60710$$

$x_3^0 = 0.60710 - \frac{f(0.60710)}{f'(0.60710)} = 0.60710 - \frac{0.0000050}{3.5704800}$

$= 0.6071038$

HW Use NRM, find the real root of $x \log_{10} x = 1.2$, correct to 5 decimal places. Ans - 2.74065

Eg-3 The bacteria concentration in a water resource varies $C = 4e^{-2t} + e^{-0.1t}$. Use NRM, calculate the time req for the bacteria concentration to be 0.5.

Sol^n Let $f(t) = 0$ $\quad C = 0.5$ (given)

$\Rightarrow 4e^{-2t} + e^{-0.1t} - C = 0$

$\Rightarrow 4e^{-2t} + e^{-0.1t} - 0.5 = 0$ $\quad$ 6.889

We are to find the +ve root of $f(t) = 0$.

Here, $f(6) = 0.0488$ $\quad f'(t) =$

$f(7) = -0.003411$

$\therefore f(6)\, f(7) < 0$ are +ve roots of the eq $f(t) = 0$ lies b/w $t=6$ & $t=7$.

Now, the Raphson's method states that

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

let $X_0 = 6.5$

$$X_1 = 6.5 - \frac{f(6.5)}{f'(6.5)}$$

HW The current i in an electric circuit is given by $i = 10e^{-t}\sin 2\pi t$ Where t is in seconds. Use the NRM, find the value of t correct to 3 decimal places for $i = 2A$

Answer → 0.033s

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English

Types of communication
- Channel
  - Verbal
    - Written
    - Vocal
  - Visual → Designs (Eg- Traffic signal)
  - Non-Verbal
    - Signs
    - gesture
- Purpose
  - General (usual communication)
  - Technical

Polarity

→ Differences b/w general communication & Technical communication.

GC	TC
① Not topic specific	① Topic specific
② Amt of Content not specified	② Amt of content specified
③ No instructions are followed	③ Instructions are followed.

Process of communication ?

→ Sender → person who is conveying their msg.