DISCRETE MATH 37181 HOMEWORK 3

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INSTRUCTIONS. Try these sometime after your workshop and before the next lecture. Set aside some time each week to keep up with the homework. Partial solutions at the end of the PDF.

1. Let $A, B \in \mathscr{U}$ be sets. The *complement* of the set
$$\emptyset \cup \overline{A \cap (\overline{B} \cup A)}$$
is equal to

A. $\emptyset$

B. $A \cup (\overline{A} \cap B)$

C. $\overline{B} \cap A$

D. A

E. none of the above.

2. Compute $\gcd(32, 124)$ using the Euclidean algorithm.

3. Prove that for all real numbers x, y, if $x + y \geq 100$ then $x \geq 50$ or $y \geq 50$. [1]

4. Prove or disprove: [2]

(a) If $A \subseteq B$ and $C \subseteq D$ then $A \cap C \subseteq B \cap D$. [3]

(b) $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$. [4]

(c) $\mathscr{P}(A \cup B) = \mathscr{P}(A) \cup \mathscr{P}(B)$.

(d) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.

Date: Week 3.

[1]Hint: try contrapositive

[2]remember a Venn diagram is not a proof. It may help you find a counterexample though if its false

[3]Hint: start with, let $x \in A \cap C$. Then ...

[4]if and only if means you have two proofs to do – one for each direction

Brief solutions:

1 **D**. A. Proof:

$$\emptyset \cup \overline{A \cap (\overline{B} \cup A)} = \overline{A \cap (\overline{B} \cup A)}$$

so its complement is

$$\begin{aligned} & A \cap (\overline{B} \cup A) & \\ = \;& (A \cap \overline{B}) \cup (A \cap A) & \text{distributive} \\ = \;& (A \cap \overline{B}) \cup A & \text{idempotent} \\ = \;& A & \text{adsorption} \end{aligned}$$

2.

$$\begin{aligned} 124 &= 3 \cdot 32 + 28 \\ 32 &= 1 \cdot 28 + 4 \\ 28 &= 7 \cdot 4 + 0 \end{aligned}$$

so $\gcd(32, 124) = 4$.

3. *Proof.* Contrapositive. If $x < 50$ and $y < 50$ then $x + y < 50 + 50 = 100$. □

Note: $\neg(x \geq 50$ or $y \geq 50)$ is the same as $(\neg(x \geq 50)$ and $\neg(y \geq 50))$ is the same as $(x < 50$ and $y < 50)$.

4. (a) *Proof.* Let $x \in A \cap C$. Then $x \in A$ and $x \in C$. Since $A \subseteq B$, we have $x \in B$, and since $C \subseteq D$, we have $x \in D$, so $x \in C \cap D$. □

(b) *Proof.* Suppose $A \cap \overline{B} \neq \emptyset$, so $\exists x \in A \cap \overline{B}$. Then $x \in A$ and $x \notin B$ which means $A \not\subseteq B$. This proves (contrapositive) the direction $A \subseteq B$ implies $A \cap \overline{B} = \emptyset$.

Now suppose $A \not\subseteq B$. This means there is some element of A that is not also in B, so this element (call is $a \in A$) lives in $\overline{B}$. Thus $a \in A \cap \overline{B}$ so this set is not empty. This proves (contrapositive) the direction $A \subseteq B$ implies $A \cap \overline{B} = \emptyset$. □

(c) This is false. Say $A = \{1\}$ and $B = \{2\}$, then $\{1, 2\} \in \mathscr{P}(A \cup B)$ but $\mathscr{P}(A) \cup \mathscr{P}(B) = \{\emptyset, \{1\}, \{2\}\}$ only.

(d) This is false. Let $\mathscr{U} = \{1, 2\}$ and $A = \{1\}$, $B = \{2\}$. Then $A \cap B = \emptyset$ so $\overline{A \cap B} = \{1, 2\}$ but $\overline{A} = \{2\}, \overline{B} = \{1\}$ so $\overline{A} \cap \overline{B} = \emptyset$.