DISCRETE MATH 37181 HOMEWORK 5

MURRAY ELDER

INSTRUCTIONS. Try these sometime after the lecture and before your workshop.

1. Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined by the recursive definition

$$\begin{array}{rcll} f(0) & = & 1 & \\ f(n) & = & nf(n-1) & \quad n > 0. \end{array}$$

The value of $f(7)$ is

A. 2520

B. 28

C. 5040

D. 420

E. none of the above.

2. What is another name for the function defined in Question 1?

3. Define a function $A : \mathbb{N}^2 \to \mathbb{N}$ using the following *recursive* definition.

$$\begin{array}{rcll} A(0,n) & = & n+1 & \quad n \geqslant 0, \\ A(m,0) & = & A(m-1,1) & \quad m > 0, \\ A(m,n) & = & A(m-1, A(m,n-1)) & \quad m,n > 0. \end{array}$$

Then $A(2,1)$ is equal to

A. 5

B. 4

C. 100

D. 6

E. none of the above.

Date: Week 5 (after the lecture 21 August) 2018.

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$.
   (a) Give an example of a one-to-one function from A to B.[1]
   (b) Give an example of an onto function from A to B.
   (c) How many different functions are there from A to B?
   (d) Give an example of a relation from A to B that is not a function.

5. Let $A = \mathbb{N}$ and $\mathscr{R}$ be the relation defined by "$a \mathscr{R} b$ if $a < b$ or $5 \mid (b - a)$". So for example $(1, 6) \in \mathscr{R}$ and $(6, 1) \in \mathscr{R}$.
   (a) Is $\mathscr{R}$ reflexive?
   (b) Is $\mathscr{R}$ symmetric?
   (c) Is $\mathscr{R}$ antisymmetric?
   (d) Is $\mathscr{R}$ transitive?

6. If A is a set, the notation $|A|$ means the number of elements in A. Let $|A| = 4$ and $|B| = 3$.
   (a) What is $|A \times B|$?
   (b) How many functions are there from A to B?
   (c) How many relations are there from A to B?
   (d) How many one-to-one functions are there from A to B?
   (e) How many one-to-one functions are there from B to A?

[1]Hint: give an example means write $f = \{(1, a), (2, b), (2, c)$ etc

Brief solutions:

1. **C** 5040

2. $n!$

3. **A** 5

$$A(1,1) = A(0, A(1,0)) = A(0, A(0,1)) = A(0,2) = 3$$
$$A(1,2) = A(0, A(1,1)) = A(1,1) + 1 = 4$$
$$A(1,3) = A(0, A(1,2)) = A(1,2) + 1 = 5$$
$$A(2,1) = A(1, A(2,0)) = A(1, A(1,1)) = A(1,3) = 5$$

4. (a) Does not exist. Not enough elements in B. Need $|A| \leqslant |B|$.

   (b) $f = \{(1,a), (2,b), (3,c)(4,a)\}$.

   (c) $3^4 = 81$. 3 choices for $f(1)$, 3 choices for $f(2)$, etc.

   (d) $\mathscr{R} = \{(1,a), (1,b)\}$.

5. (a) Yes since 5 divides $(a - a) = 0$ for all $a \in \mathbb{N}$.

   (b) Not symmetric, $(1,2) \in \mathscr{R}$ because $1 < 2$, but $(1-2) = -1$ is not divisible by 5 and $2 \not< 1$ so $(2,1) \notin \mathscr{R}$.

   (c) Not antisymmetric. $(0,5)$ and $(5,0)$ are both in $\mathscr{R}$ but $0 \neq 5$.

   (d) Not transitive. $(4,7)$ and $(7,2)$ are in $\mathscr{R}$ but $(4,2)$ is not.

6. (a) 12.

   (b) $3^4 = 81$ (same as Question 4(c)).

   (c) A relation can be any subset of $\mathscr{P}(A \times B)$, and the size of the power set of a set of size 12 is $2^{12} = 4096$ (induction problem Lecture 4).

   (d) There are none.

   (e) For $f(a)$ there are 4 possible choices. Then for $f(b)$ we only have 3 numbers to choose from, and then for $f(c)$ only 2 numbers left, so $4 \times 3 \times 2 = 24$.