Team Name:	
Team member 1:	Student ID:
Team member 2:	Student ID:
Team member 3:	Student ID:

37181 DISCRETE MATHEMATICS TEAM ASSIGNMENT 2019

INSTRUCTIONS. – Work on these problems together as a team of 3. Each team member should attempt all problems, and then work together to combine all solutions into the optimal one. Effort should be approximately one third each.

- Staple your assignment with all details on the front page, and with questions in the correct order. Use this sheet as a cover page. Do not submit any plastic folders etc.
- Start each question on a new page.
- For some questions you may need to prove some preliminary results to use in your main proof. Prove these separately first, then refer to them as Lemma 1, Lemma 2 in your main proof.
- Arrange times to meet and work together, including the Lecture time on 4 Sept 4-6pm.
- This assignment is worth 20% of your final grade. Total points available: 30.

1. (3 marks) Attach a photo of your team working together (eg. at a whiteboard). Under the photo, write a brief paragraph describing each team member's contribution.

2. (3 marks)
   (a) Construct a truth table for the logical connective "unless" [*].

   (b) Write a statement logically equivalent to "p unless q" that uses only $\vee, \wedge, \neg, \rightarrow$.

   (c) Write the negation of the statement "I will fail this subject unless I pay Murray \$1000". Final answer in English, but go via propositional logic.

3. (3 marks) Let X, Y be sets and p, q, r statements. Prove or disprove:

   (a) $(X \cup Y) \cap \overline{Y} \subseteq X$

   (b) $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$ are logically equivalent.

4. (3 marks) Let $f : X \rightarrow Y, g : Y \rightarrow Z$ be functions where X, Y, Z are sets. Recall that $g \circ f : X \rightarrow Z$ is the function defined by $g \circ f(x) = g(f(x))$ for all $x \in X$. Prove that if f, g are bijections then so is $g \circ f$. [†]

Date: Due at the start of the lecture, 4pm Wednesday 18 September 2019.

[*] *i.e.* p unless q. Discuss together and check online.

[†] Hint: prove two things: if f, g are 1-1 then so is $g \circ f$; if f, g are onto then so is $g \circ f$.

5. (3 marks) Prove that $\sqrt[3]{3}$ is not rational. ‡

6. (3 marks) Consider the following pseudocode program:

```
program(int n)
     if n == 1
          return 0
     else
          return 1+program(floor(n/2))
```

   (a) Prove that on input $n \in \mathbb{N}_+$ the output is $\lfloor \log_2 n \rfloor$. §

   (b) On input n, (roughly) how many times is the line starting with `return` $1 +$ `program`(... executed?

7. (3 marks)
   Let A be a set¶ and $\mathscr{P}(A)$ the power set of A. Prove that there is no bijection from A to $\mathscr{P}(A)$. ‖

8. (3 marks) Prove that for all $n \in \mathbb{N}$, $(3n+1)7^n - 1$ is divisible by 9.

9. (3 marks)
   (a) Prove that the WOP is equivalent to PMI. **

   (b) Is the WOP true?

   (c) Consider the set $\mathbb{N} \times \mathbb{N}$ with an order $\preceq$ defined by $(a, b) \preceq (c, d)$ if $a < c$ or ($a = c$ and $b \leq d$) ††. Assuming WOP is true, prove that every non-empty subset of $\mathbb{N} \times \mathbb{N}$ has a first element with respect to $\preceq$.

10. (3 marks) Find the 2018 37181 Final Exam from the UTS Library website. Answer with your team these questions: ‡‡ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17, 18, 21, 22, 23, 24, 25, 26.

END OF ASSIGNMENT

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‡Hint: copy the $\sqrt{2}$ proof from Worksheet 2, but now everything has 3's in it. Prove a lemma about if x^3 is divisible by 3 then so is x.

§Hint: Let a_n be the output of `program(n)`, and show that $a_n = \lfloor \log_2 n \rfloor$. Use induction (strong form). Break into cases: when the input is an exact power of 2, and when the input is $2^m + r$ for $0 < r < 2^m$.

¶finite or infinite

‖Hint: suppose (for contradiction) $f : A \to \mathscr{P}(A)$ is a bijection, and consider a set $B = \{x \in A \mid x \notin f(x)\}$.

**Prove two things: WOP implies PMI, PMI implies WOP

††this is called *lexicographic ordering*

‡‡Discuss and prove each one, but only submit your final answers as 1. A 2. B 3. C etc.