DISCRETE MATH 37181 HOMEWORK SHEET 1

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INSTRUCTIONS. Try these sometime after your workshop and before the next lecture. Set aside some time each week to keep up with the homework. Partial solutions at the end of the PDF.

1. Draw truth tables for the following statements.

   (a) $((p \to q) \land (q \to r)) \to (p \to r)$

   (b) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

   (c) $s \leftrightarrow (p \to ((\neg p) \lor s))$

   (d) $(p \to (r \to s)) \leftrightarrow ((p \to r) \to s)$

2. The statement
$$\neg (p \land (q \lor r))$$
   is logically equivalent to [1]

   A. $p \land (\neg q \land \neg r)$

   B. $\neg p \land (\neg q \lor \neg r)$

   C. $\neg p \lor (\neg q \land \neg r)$

   D. $\neg(p \lor \neg q) \land \neg r$

   E. none of the above.

Date: Week 1.

[1]To answer this, draw a truth table the questions and for each multiple choice answer, or ... is there an easier way to do this?

3. The statement
$$\forall x \in \mathbb{R} \; \forall y \in \mathbb{R} \left[(xy > 0) \lor (x = 0) \lor (y = 0)\right]$$

is [2]

A. true

B. false

C. only true for some values of x and y

D. not a statement

E. none of the above.

4. Consider the following pseudocode:

```
procedure (n positive integer)
while n>0
      n := floor(n/2)
      print n
```

On input $n = 50$ the output to the code is [3] [4]

A.   25   12   6   3   1   0

B.   0   0   0   0   0   0

C.   50   25   12   6   3   1

D.   25   13   6   3   1   0

E.   none of the above.

[5]

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[2]You think its true? Then *prove it* – show it is true for every x and y. You think its false? Give a *single example* of x, y for which it is not true. More on how to prove and disprove statements in maths next week. For now, can you read the symbols and understand the meaning?

[3]Question 4 is a preview of future topics, so if you never saw "pseudocode" before (*i.e.* you are a math major not a comp sci major) don't worry, can you try to guess how the procedure operates? What does `floor` mean?

[4]if you *are* a comp sci major, pseudocode is not a real programming language, its half-way between some programming language(s) and English (and mathematics).

[5]Questions 2,3,4 were on the 37181 final exam 2017

Brief solutions:

1 (a)

p	q	r	$((p \to q) \land (q \to r)) \quad \to \quad (p \to r)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

This is a tautology. It is called the *Law of Syllogism* in some textbooks.

(b)

p	q	$(p \to q)$	$\leftrightarrow$	$(\neg q$	$\to$	$\neg p)$
1	1	1	1	0	1	0
1	0	0	1	1	0	0
0	1	1	1	0	1	1
0	0	1	1	1	1	1

This is a tautology. It is called *contrapositive.*

(c)

p	s	s	$\leftrightarrow$	$(p \to ((\neg p) \lor s))$
1	1		1	
1	0		1	
0	1		1	
0	0		0	

(d)

p	r	s	$(p \to (r \to s)) \quad \leftrightarrow \quad ((p \to r) \to s)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

2 C

3 B

4 A