## 4. WERKCOLLEGE 4

You are allowed to answer in Dutch. Problem 4.1 is the most difficult exercise.

4.1.   A directed graph is *semiconnected* if, for all pairs of vertices u, v, there is either a path from u to v, or there is a path from v to u. Give an efficient algorithm to determine whether or not a directed graph is semiconnected. Prove that you algorithm is correct and analyze its running time.

4.2.   Run Dijkstra's algorithm on the weighted graph below, using vertex A as the source. Write the vertices in the order which they are marked and compute all distances at each step.
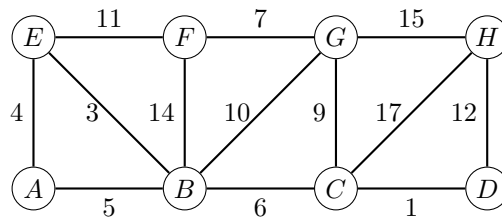
4.3.   Give an example of a directed graph for which Dijkstra's algorithm produces incorrect answers and explain why. The graph should have at most 4 vertices.

4.4.   Where in a max-heap might the smallest element reside?

4.5.   You are given a strongly connected directed graph with positive edge weights along with a particular node v_0. Give an efficient algorithm for finding shortest paths between *all pairs of nodes*, with the one restriction that that these paths must all pass through v_0.

4.6.   Consider the following algorithm for finding the shortest path from node s to node t in a directed graph with some negative edges: add a large constant to each edge weight so that all the weights become positive, then run Dijkstra's algorithm starting at node s, and return the shortest path found to node t.

Is this a valid method? Either prove that it works correctly, or give a counterexample.