

## WERKCOLLEGE 6

You are allowed to answer in Dutch. Whenever an algorithm is required, it can be given in pseudocode or plain English (or Dutch), and its running time and correctness must always be justified (even informally, but in a clear way!).

6.1. Consider a set of mobile computing *clients* in a certain town who each need to be connected to one of several possible *base stations*. There are  $n$  clients, with the position of each client specified by its  $(x, y)$  coordinates in the plane. There are also  $k$  base stations; the position of each of these is specified by  $(x, y)$  coordinates as well.

We wish to connect each client to exactly one of the base stations. Our choice of connection is constrained in the following ways. There is a *range parameter*  $r$  — a client can only be connected to a base station that is within distance  $r$ . There is also a *load parameter*  $L$  — no more than  $L$  clients can be connected to any single station.

Design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters, decide whether every client can be connected simultaneously to a base station, subject to the conditions from the previous paragraph.

6.2. Suppose that, in addition to edge capacities, a flow network has *vertex capacities*. That is each vertex  $v$  has a limit  $l(v)$  on how much flow can pass through  $v$ . Show how to transform a flow network  $G = (V, E)$  with vertex capacities into an equivalent flow network  $G' = (V', E')$  without vertex capacities, such that a maximum flow in  $G'$  has the same value as a maximum flow in  $G$ .

6.3. An edge of a network is called a bottleneck edge if increasing its capacity results in an increase in the maximum flow. Give an efficient algorithm to identify all bottleneck edges in a flow network.

**Hints:** Start by running the usual network flow algorithm, and then examine the residual graph.

6.4. Suppose you are organizing a conference where researchers present articles they have written. Researchers who want to present an article send a paper to the conference organizers. The conference organizers have access to a set  $A$  of reviewers who are each willing to read up to  $m_A$  articles. Let  $B$  the set of papers to review: each gets reviewed by up to  $m_B$  reviewers. Moreover, each submission has a particular topic and each reviewer has a specialization for a set of topics, so papers on

a given topic only get reviewed by those reviewers who are experts on that topic. The conference organizers need to decide which reviewers will review each article (or equivalently, which articles will be reviewed by which reviewers). Explain how this problem can be solved using a flow network.

6.5. The managers of a popular website have identified  $k$  distinct *demographic groups*  $G_1, G_2, \dots, G_k$ . These groups may overlap; for example  $G_1$  can be equal to all residents of Gelderland, and  $G_2$  can be equal to all people with a degree in computer science. The site has contracts with  $m$  different *advertisers*, to show a certain number of copies of their ads to users of the site. Here's what the contract with the  $i^{\text{th}}$  advertiser looks like:

- For a subset  $X_i \subseteq \{G_1, \dots, G_k\}$  of the demographic groups, advertiser  $i$  wants its ads shown only to users who belong to at least one of the groups in the set  $X_i$ .
- Advertiser  $i$  wants its ads shown to at least  $r_i$  users each minute, for some number  $r_i$ .

Now consider the problem of designing a good *advertising policy* — a way to show a single ad to each user of the site. Suppose at a given minute, there are  $n$  users visiting the site. Because we have registration information on each of these users, we know that user  $j$  belongs to a subset  $U_j \subseteq \{G_1, \dots, G_k\}$  of the demographic groups. Is there a way to show a single ad to each user so that the site's contracts with each of the  $m$  advertisers is satisfied for this minute?

Give an efficient algorithm to decide if this is possible, and if so, to actually choose an ad to show to each user.