Werkcollege 7

You are allowed to answer in Dutch. Whenever an algorithm is required, it can be given in pseudocode or plain English (or Dutch), and its running time and correctness must always be justified (even informally, but in a clear way!).

7.1. Draw the binary search tree that results when you insert items with keys

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9 5 12 7 4 10 8 11 18
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in that order into an initially empty tree. Remove 9 and 5 and draw the resulting trees after each removal.

7.2. Each node in a binary search tree has tree attributes p, *left*, *right* to store pointers. Prove that, for each binary search tree with $n > 0$ nodes, the total number of attributes with value NIL equals $n+2$.

7.3. Prove that the procedure Tree-Insert (presented on slide 16 of the lecture on BST's and in section 12.3 of CLRS) runs in $O(h)$ time on a tree of height h. Either prove that Tree-Insert runs in time $\Omega(h)$ or present a counterexample.

7.4. Interestingly, there is no consensus on the precise definition of a binary search tree in the literature. The CLRS definition (which we adopt in this course) says that (1) all nodes stored in the left subtree of a node x whose key is K have keys less than or equal to K, and (2) all nodes stored in the right subtree of x have keys greater than or equal to K. Shaffer proposes a slightly different definition in *Data Structures & Algorithms in C++*: (1) all nodes stored in the left subtree of a node x whose key is K have keys less than K, and (2) all nodes stored in the right subtree of x have keys greater than or equal to K. Give at least one advantage of the CLRS definition, and at least one advantage of Shaffer's definition.

7.5. Consider an n-node complete binary tree T with n nodes (not necessarily a BST). Each node v of T is labeled with a real number x_v. You may assume that the real numbers labeling the nodes are all distinct. A node v of T is a *local minimum* if the label x_v is less than the label x_w of all nodes w that are joined to v by an edge. You are given such a complete binary tree T but the labeling is only specified in the following *implicit* way: for each node v, you can determine the value x_v by *probing* the node v. Show how to find a local minimum of T using only $O(\log n)$ probes to the nodes of T.