Fuzzy Identification of Systems and Its Applications to Modeling and Control

Abstract— **This work is reproduction of the work done by Michio Sugeno and Tomohiro Takagi [1]. It presents a mathematical technique for creating a fuzzy model of a system with fuzzy implications and reasoning. The description of a fuzzy subspace of inputs is the premise of an implication. As result, the input-output relationship is linear. The identifying method the output-input data of a system is then displayed. There are two applications. The method's applicability to industrial operations is further discussed: a body of water in the steel-making process, there is a cleaning process and a converter.**

I. Introduction

The primary goal of this study [1] is to propose a mathematical technique for creating a fuzzy system model. In earlier studies, the attempt was made to form a fuzzy implication containing fuzzy variable function in unimodal form. These variables were called linguistic variables as they were linguistically understood. But there was the difficulty in modeling using these linguistic variables as we may have to use multiple variables.

In this study, we tried to reduce these linguistic variables. In order to do that we must consider multidimensional thinking as we are dealing with a multi-variable system. We know that in general, building a model using input-output data must have two factors: technique for identification of the system and the mathematical tool for expressing it. As a result, we proposed a simple yet extensive mathematical tool as well as fuzzy implications based on fuzzy space partition. Also the method for identification of the system using its input-output data is shown.

This paper is divided into 6 sections. Section II describes format of if…then rule. Section III describes the algorithms. Section IV describes the mathematical background of before mentioned algorithm. The algorithm is explained with the help of few examples given in section V. After that, the two applications of the method: water cleaning process, steel-making process is discussed and simulated in section VI.

II. Format of Fuzzy Implication

A. Format of Fuzzy variables

The membership function of a fuzzy set A can be denoted as A(x), ($x \in X$). The fuzzy variables are assumed to be linear with minimum value of '0' and maximum value of '1'. As a result, the truth table for the statement "x is A and y is B" may be reduced to – A(x) ^ B. (y).

here,

^ : min operation.

X : universe of discourse

B. Format of Implications

We have defined implication R as follows:

R : If x_1 is A_1 and ……. and x_k is A_k

Or $f(\, x_1 \text{ is } A_1, \cdots, x_k \text{ is } A_k)$

then $y = g(x_1, \cdots, x_k)$

Or $y = p_0^1 + p_1^1 x_1 + \cdots + p_k^1 x_k$

here,

y : Consequence variable

$x_1 - x_k$: Premise variables

A_1 - A_k : Membership function

f : Logical function

g : Output function of y with $x_1 - x_k$

$p_0, \cdots, p_k$: Consequence parameters

III. Algorithms

A. Algorithm of Reasoning

Let's assume we have 'n' implications R (I = 1, … ,n) as shown in above and also, we have ($x_1 = x_1^0, \dots, x_1 = x_k^0$).

In the procedures that follows, the value of y is deduced:

- For every R^i, y is given by

$$y^i = g^i(x_1^0, \dots, x_k^0) = p_0^i + p_1^i x_1^i + \cdots + p_k^i x_k^i$$

- Now, $y = y^i$ is given by

$$\left|y = y^i\right| = \left|x_1^0 \text{ is } A_1^i \text{ and } \dots\ x_k^0 \text{ is } A_k^i\right| \wedge |R^i|$$

$$\left|y = y^i\right| = (A_1^i(x_1^0) \wedge \dots \wedge A_k^i(x_k^0)) \wedge \left|R^i\right|$$

- Let $\left|R^i\right| = 1$ for simplicity. Thus, the final output y is calculated by finding weighted mean as shown below

$$y = \frac{\sum|y = y^i| \times y^i}{\sum|y = y^i|}$$

This algorithm is explained using example 1 in section V.

B. Algorithm of Identification

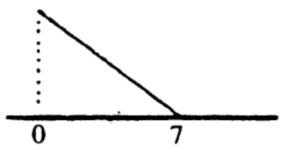
This algorithm is divided into three parts:

1. Choice of Premise Variables: Out of given possible inputs premise variables are chosen then optimum consequence parameters are found using it. We then calculate errors in the output and find performance index, which is root mean square value of errors. Thus, premise variables are improved in each step choosing least performance index among different choices. To choose premise variables a heuristic search method is used. This method is further explained in section IV.

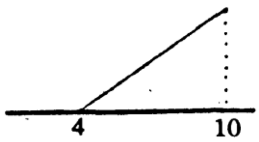
2. Premise Parameter Identification: In this step optimum premise parameters are found with respect to the above premise variables. This is done by dividing fuzzy variables into different fuzzy sub-spaces like "small, big etc."
3. Consequence Parameter Identification: Now using these parameters consequences are calculated.

Now, let us see the algorithm step by step:

Step 1: A premise parameter is chosen and is divided into subspaces like "small, big etc." Let's assume x_1 is chosen and is divided into two fuzzy subspaces as follows:

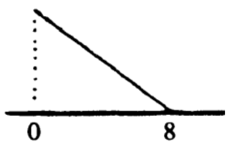


x_1 is small

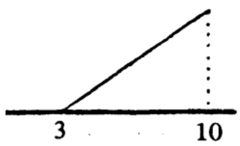


x_1 is big

Let us call this model 1-1. Similarly, for x_k we can have a model 1-k, which is,



x_k is small



x_k is big

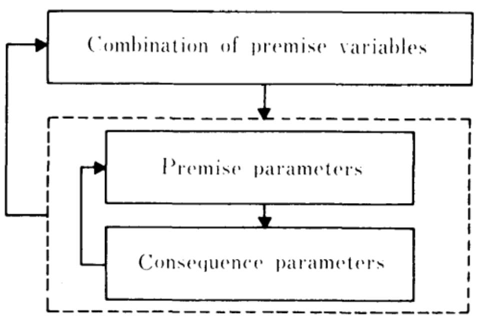


Fig 1: Algorithm outline

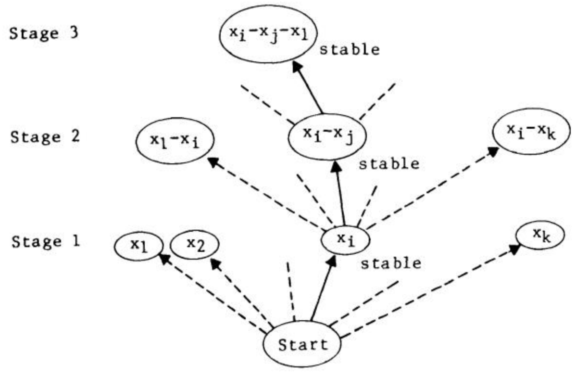


Fig 2: Choice of parameter variable

Step 2: Now we will find consequence parameters for every combination of subspaces calculated in step 1. This is done using the mathematical equations as stated in section IV. The process is further explained in example. Now for each rule or combination a performance index is found, which is root mean square value of the error.

Step 3: Now the model with least performance index is chosen and rest are discarded. This model is called as stable form.

Step 4: After finding stable model we will move to further divide variables in model 2-k, model 3-k etc. Let's now have two inputs in the input premise arguments. Then all the $x_i - x_j$ combination will be covered under the step and for each variable of x_i and x_j the subspace will be divided in two fuzzy subspaces which are generally 'big' and 'small'. For model x_i - x_i the ranges will be divided into four subspaces namely 'small', 'big', 'medium small' and 'medium big'. So, we for each implication we will get model 2-j. Then we will again go with the model having least performance index. The process is shown in fig 2.

Step 5: Now, we will repeat the process starting from step 2 until one of the following conditions are fulfilled.

- The final performance index value is less than desired value.
- Number of premise variables exceeds the maximum permissible number of rules.

IV. Mathematics

Let the system be defined by:

R^1: If x_1 is A_1^1 and ……. and x_k is A_k
then $y = p_0^1 + p_1^1.x_1 + \cdots + p_k^1.x_k$

⋮ ⋮ ⋮

⋮ ⋮ ⋮

R^n: If x_1 is A_1^n and ……. and x_k is A_k
then $y = p_0^n + p_1^n.x_1 + \cdots + p_k^n.x_k$

Therefore, output can be written as

$$y = \frac{\sum_{i=1}^{n}(A_1^i(x_1)\wedge \dots \wedge A_k^i(x_k)).(p_0^i + p_1^i.x_1 + \dots + p_k^i.x_k)}{\sum_{i=1}^{n}(A_1^i(x_1)\wedge \dots \wedge A_k^i(x_k))}$$

Thus,

$$y = \sum_{i=1}^{n}(p_0^i.\beta_i + p_1^i.x_1.\beta_i + \dots + p_k^i.x_k.\beta_i)$$

here,

$$\beta = \frac{A_1^i(x_1)\wedge \dots \wedge A_n^i(x_n)}{\sum_{i=1}^{n}(A_1^i(x_1)\wedge \dots \wedge A_n^i(x_n))}$$

Let,

$$X = \begin{bmatrix} \beta_{11}, \cdots, \beta_{n1,} & x_{11}\beta_{11}, \cdots, x_{11}\beta_{n1} & \cdots & x_{k1}\beta_{11}, \cdots, x_{k1}\beta_{n1} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1m}, \cdots, \beta_{nm,} & x_{1m}\beta_{1m}, \cdots, x_{1m}\beta_{nm} & \cdots & x_{km}\beta_{1m}, \cdots, x_{km}\beta_{nm} \end{bmatrix}$$

here,

$$\beta_{ij} = \frac{A_{i1}(x_{1j})\wedge \dots \wedge A_{ik}(x_{kj})}{\sum_{i=1}^{n} A_{i1}(x_{1j})\wedge \dots \wedge A_{ik}(x_{kj})}$$

If

$Y = [y_1, \cdots, y_m]^T$ and

$P = [p_0^1, \cdots, p_0^n, p_1^1, \cdots, p_1^n, \cdots, p_k^1, \cdots, p_k^n]^T$ such that,

$y = X \cdot P$

Then,

$$\boldsymbol{P} = (\boldsymbol{X^T} \cdot \boldsymbol{X})^{-1} \cdot \boldsymbol{X^T} \cdot \boldsymbol{Y}$$

V. Examples

A. Example 1

Let us have following three relations:

R^1: If x_1 is small and x_2 is small then $y = x_1 + x_2$

R^1: If x_1 is big then $y = 2 \times x_1$

R^1: If x_1 is big then $y = 3 \times x_2$

Fig 3 shows reasoning process of each relation when $x_1 = 12$, $x_2 = 5$.

$$\left|y = y^i\right| = |x_1^0 \text{ is } small| \wedge |x_2^0 \text{ is } small|$$
$$= small(x_1^0) \wedge small(x_2^0)$$
$$= 0.25$$

Now,

$$y = \frac{0.25 \times 17 + 0.2 \times 24 + 0.375 \times 15}{0.25 + 0.2 + 0.375}$$

y = 17.8

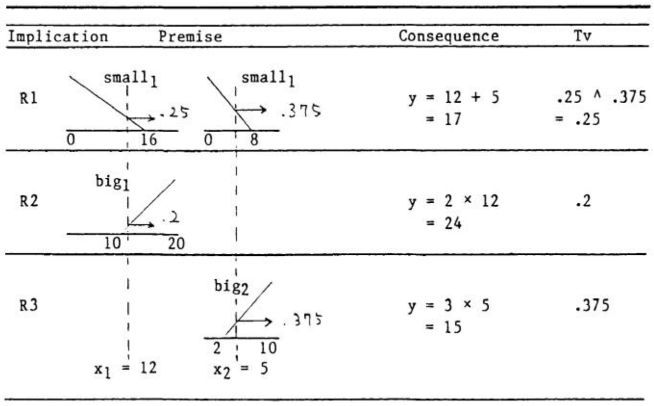


Fig 3: Tabulated data

VI. Simulation Results

Let's assume we have 'n' implications

A. Water Cleaning Process

In the process of cleaning water turbidity, PH etc. are analyzed then substances like PAC and chlorine are introduced to murky river water and blended with it in a mixing tank. After that, the combined water is pushed into a sedimentation tank, where the turbid portion is blended with PAC and settles to the bottom. After sedimentation, the purified water is sent into a filter tank, where it is cleaned. The process is shown in the fig 4

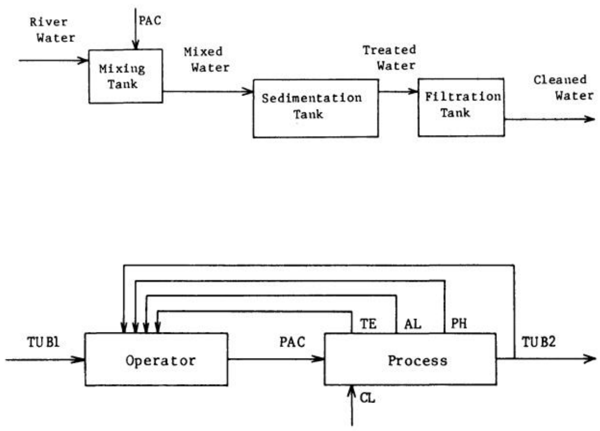


Fig 4: Water Cleaning Process

- MATLAB Code

```
close all; clear; clc;

      % TUB1 PH TE AL PAC TUB2
data = [10 7.1 18.8 53 1300 1;
        17 7 18.6 50 1300 1;
        22 7.3 19.4 46 1400 2;
        50 7.1 19.5 40 1400 1;
        9 7.3 23.3 48 900 4;
        11 7.1 20.7 50 900 1;
        12 7.2 21.3 50 900 3;
        14 7.2 23.6 53 900 4;
        35 7 17.8 35 1200 1;
        20 7 16.6 40 1100 1;
        20 6.9 17.8 42 1100 1;
        18 7.1 17.3 40 1100 1;
        12 7.2 18.8 55 900 3;
        8 7.2 18 50 1000 1.5;
        11 7.1 19.2 49 1000 2;
        50 7 18 37 1200 1.5;
        35 7 17.7 42 1200 1.5;
        30 7 17.3 41 1100 1.5;
        16 7.1 19.3 42 1100 3];

input = [data(:,1:4) data(:,6)];
output = data(:,5);
PH = input(:,2);
AL = input(:,4);
TE = input(:,3);

k = size(data,2) - 1;    % Number of
variables
m = size(data,1);        % Number of
data points
n = 8;                   % Maximum
number of rules

TUB1min = min(data(:,1)); TUB1max =
max(data(:,1));
PHmin = min(data(:,2)); PHmax =
max(data(:,2));
TEmin = min(data(:,3)); TEmax =
max(data(:,3));
ALmin = min(data(:,4)); ALmax =
max(data(:,4));
PACmin = min(data(:,5)); PACmax =
max(data(:,5));
TUB2min = min(data(:,6)); TUB2max =
max(data(:,6));

mPH = 1/(PHmin-PHmax); cPH =
PHmin/(PHmin-PHmax); cPH1 =
PHmax/(PHmax-PHmin);
mAL = 1/(ALmin-ALmax); cAL =
ALmin/(ALmin-ALmax); cAL1 =
ALmax/(ALmax-ALmin);
mTE = 1/(TEmin-TEmax); cTE =
TEmin/(TEmin-TEmax); cTE1 =
TEmax/(TEmax-TEmin);

%% Model 3-k
n = 8;
beta = zeros(m,n);
b = beta;

    for i = 1:n
        for j = 1:m
            smallPH(j) = -
mPH*PH(j)+cPH1;
            smallAL(j) = -
mAL*AL(j)+cAL1;
            smallTE(j) = -
mPH*TE(j)+cTE1;
            bigPH(j) = mPH*PH(j)+cPH;
            bigAL(j) = mAL*AL(j)+cAL;
            bigTE(j) = mTE*TE(j)+cTE;
            switch i
                case 1
                    b(j,i) =
min([smallPH(j) smallAL(j)
smallTE(j)]);
                case 2
                    b(j,i) =
min([smallPH(j) smallAL(j) bigTE(j)]);
                case 3
                    b(j,i) =
min([smallPH(j) bigAL(j) smallTE(j)]);
                case 4
                    b(j,i) =
min([smallPH(j) bigAL(j) bigTE(j)]);
                case 5
                    b(j,i) =
min([bigPH(j) smallAL(j) smallTE(j)]);
                case 6
                    b(j,i) =
min([bigPH(j) smallAL(j) bigTE(j)]);
                case 7
                    b(j,i) =
min([bigPH(j) bigAL(j) smallTE(j)]);
                case 8
                    b(j,i) =
min([bigPH(j) bigAL(j) bigTE(j)]);
            end
        end
        beta(:,i) =
b(:,i)./sum(b(:,i));
    end

    X = beta;
    for i = 1:n
        X = [X
repmat(beta(:,i),1,k).*input];
    end

    P = pinv(X'*X)*X'*output;
```

```
    p = P(1:n)';
    for i = 1:k
        p = [p; P(n*i+1:n*(i+1))'];
    end

    x = [ones(m,1) input];
    y = x*p;
    y = b.*y;
    Y = [];
    for i = 1:m
        Y = [Y;
sum(y(i,:))./sum(b(i,:))];
    end
    Y = X*P;
    PI = sqrt(sum((Y - output).^2)/m)

    % Statistical model
    % PAC = 9.11*sqrt(TB1) - 79.8*PH +
12.7*CL + 1255.6
    Ystat = [994.7; 995.9; 1119.6;
1151.1; 1409.4; 1066.4; 1068.9;
1012.3; 1286.8; 1246.8; 1151.4;
1199.5; 1159.4; 985.7; 1009.3; 1038.2;
1398.3; 1290.6; 1038.5];
    PIstat = sqrt(sum((Ystat -
output).^2)/m)

    %Plot
    figure(1);
    scatter(output,Y);
    grid on;
    hold on;
    plot([500 1500],[500 1500]);
    xlabel('Operator');
    ylabel('Model');
```

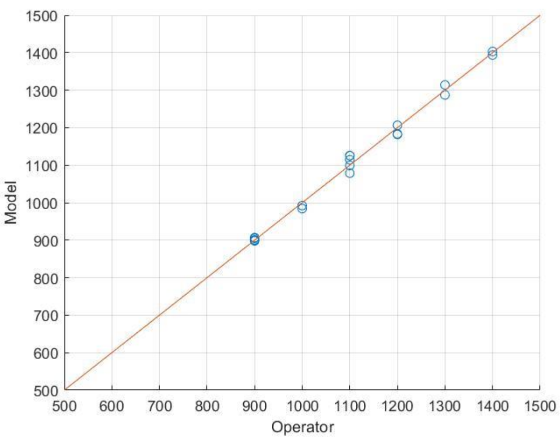


Fig 5: Variation of model from ideal values

```
PI =

   13.3474


PIstat =

  213.9103
```

Fig 6: Results of water cleaning process

B. *Steel making Process*

Depending on the components used, a steel-making facility produces several types of steel. A variety of adjustments must be made to the manganese ratio in the goods. The controller to determine the amount of manganese alloy to produce desired type of steel is shown in the fig7.

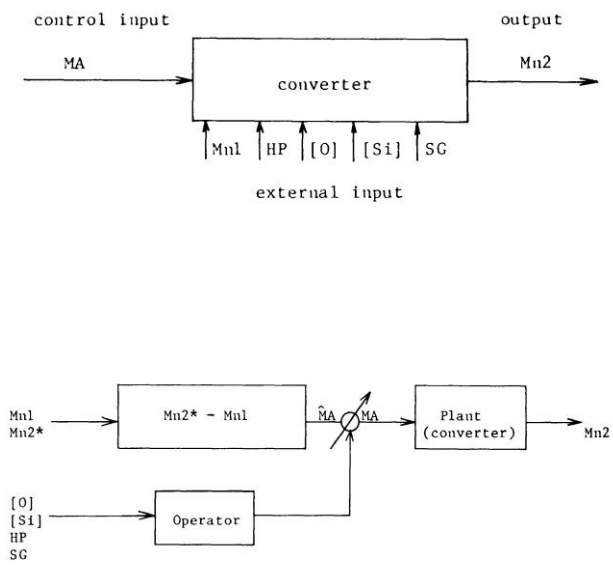


Fig 7: Steel Making Process

- MATLAB Code

```
close all; clear; clc;

    %   HP   Mn   SG  MA
data = [93.9 1.11 0.3 14;
        94 14.52 -0.08 135;
        93.1 5.22 -0.05 54;
        93.6 5.36 -0.43 53;
        85.6 13.56 0.11 129;
        85.8 11.55 0.12 106;
        86.4 11.88 -0.06 113;
        93.1 13.85 0.16 119;
        88.5 12.27 -0.02 129;
        93 11.69 -0.03 127;
        93 9.86 -0.01 98;
        95 4.96 -0.13 46;
        94.7 2.19 0.01 25;
        85 1.89 -0.1 23;
        87.5 14.26 -0.09 121;
        90.5 11.52 -0.1 112;
        90.1 12.59 -0.06 116;
        90.2 5.67 -0.28 60];

input = [data(:,1) data(:,3:4)];
output = data(:,2);

HP = input(:,1);
SG = input(:,2);
MA = input(:,3);

k = size(data,2) - 1;   % Number of
variables
m = size(data,1);       % Number of
data points
n = 4;                  % Maximum
number of rules

HPmin = min(data(:,1)); HPmax =
max(data(:,1));
Mnmin = min(data(:,2)); Mnmax =
max(data(:,2));
SGmin = min(data(:,3)); SGmax =
max(data(:,3));
MAmin = min(data(:,4)); MAmax =
max(data(:,4));

mHP = 1/(HPmin-HPmax); cHP =
HPmin/(HPmin-HPmax); cHP1 =
HPmax/(HPmax-HPmin);
mSG = 1/(SGmin-SGmax); cSG =
SGmin/(SGmin-SGmax); cSG1 =
SGmax/(SGmax-SGmin);
mMA = 1/(MAmin-MAmax); cMA =
MAmin/(MAmin-MAmax); cMA1 =
MAmax/(MAmax-MAmin);


%% MODEL 2-1
beta = zeros(m,n);
b = beta;

    for i = 1:n
        for j = 1:m
            smallHP(j) = -
mHP*HP(j)+cHP1;
            smallSG(j) = -
mSG*SG(j)+cSG1;
            smallMA(j) = -
mMA*MA(j)+cMA1;
            bigHP(j) = mHP*HP(j)+cHP;
            bigSG(j) = mSG*SG(j)+cSG;
            bigMA(j) = mMA*MA(j)+cMA;
            switch i
                case 1
                    b(j,i) =
min([smallHP(j) smallSG(j)]);
                case 2
                    b(j,i) =
min([smallHP(j) smallSG(j)]);
                case 3
                    b(j,i) =
min([smallHP(j) bigSG(j)]);
                case 4
                    b(j,i) =
min([smallHP(j) bigSG(j)]);
            end
        end
        beta(:,i) =
b(:,i)./sum(b(:,i));
    end

    X = beta;
    for i = 1:n
        X = [X
repmat(beta(:,i),1,k).*input];
    end

    P = pinv(X'*X)*X'*output;

    p = P(1:n)';
    for i = 1:k
        p = [p; P(n*i+1:n*(i+1))'];
    end

    x = [ones(m,1) input];
    y = x*p;
    y = b.*y;
    Y = [];
    for i = 1:m
        Y = [Y;
sum(y(i,:))./sum(b(i,:))];
    end
    Y = X*P;
    PI1 = sqrt(sum((Y -
output).^2)/m);

    %% MODEL 2-2

    beta = zeros(m,n);
```

```
b = beta;

    for i = 1:n
        for j = 1:m
            smallHP(j) = -mHP*HP(j)+cHP1;
            smallSG(j) = -mSG*SG(j)+cSG1;
            smallMA(j) = -mMA*MA(j)+cMA1;
            bigHP(j) = mHP*HP(j)+cHP;
            bigSG(j) = mSG*SG(j)+cSG;
            bigMA(j) = mMA*MA(j)+cMA;
            switch i
                case 1
                    b(j,i) = min([smallSG(j) smallMA(j)]);
                case 2
                    b(j,i) = min([smallSG(j) bigMA(j)]);
                case 3
                    b(j,i) = min([bigSG(j) smallMA(j)]);
                case 4
                    b(j,i) = min([bigSG(j) bigMA(j)]);
            end
        end
        beta(:,i) = b(:,i)./sum(b(:,i));
    end

    X = beta;
    for i = 1:n
        X = [X repmat(beta(:,i),1,k).*input];
    end

    P = pinv(X'*X)*X'*output;

    p = P(1:n)';
    for i = 1:k
        p = [p; P(n*i+1:n*(i+1))'];
    end

    x = [ones(m,1) input];
    y = x*p;
    y = b.*y;
    Y = [];
    for i = 1:m
        Y = [Y; sum(y(i,:))./sum(b(i,:))];
    end
    Y = X*P;
    PI2 = sqrt(sum((Y - output).^2)/m);

    %% OUTPUT

    if PI1 < PI2
        fprintf('Stable: Model 2-2\nVariables: HP & SG\n');
    else
        fprintf('Stable: Model 2-2\nVariables: SG & MA\n');
    end

    %Plot
    figure(1);
    scatter(output,Y);
    grid on;
    hold on;
    plot([0 20],[0 20]);
    xlabel('Operator');
    ylabel('Model');
```

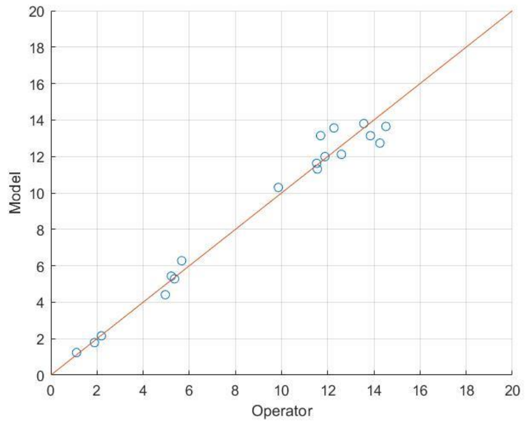


Fig 8: Variation of model from ideal values

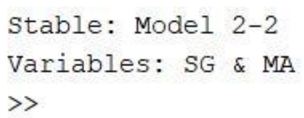


Fig 9: Results of steel making process

VII. Conclusion

We have fuzzy models of the two real-world systems that has been successfully constructed, and the mathematical approach has been used to evaluate the fuzzy models.

We have shown variation of model from ideal scenario in both of the cases an also least PI is used while calculating the fuzzy variables.

REFERENCE

[1] 'Fuzzy identification of Systems and it's applications to modeling and control' by Tomohiro Takagi and Michio Sugeno.