Maximum-Likelihood Detection in DWT Domain Image Watermarking using Laplacian Modeling

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Outline

Motivation

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Imperceptible digital watermarks

- invisible to the human eye
- can disclose copyright infringements

in Discrete Wavelet domain

- robust to modification in spatial domain

using stochastic detection

- Neyman-Pearson Test yields optimal detection probability for given false alarm rate

Outline

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DWT watermarking

Maximum Likelihood detection

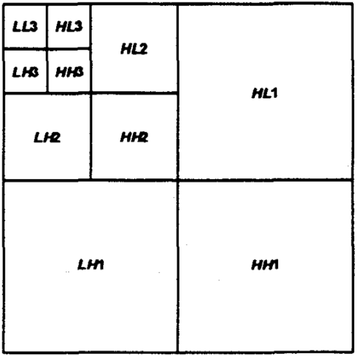
Decision Threshold

Experimental Results

Conclusion & Outlook

DWT watermarking
Original proposal by Kim, Kwon and Park

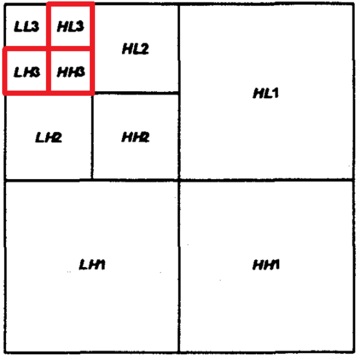
- three-level pyramid decomposition
- watermark embedding in all sub-bands except LH_1, HL_1, HH_1
  $\Rightarrow$ low energy in those bands
- multi-resolution for robustness
- embedding strength proportional to band energy



DWT decomposition (Kim et al. 1999)

DWT watermarking
Implemented method

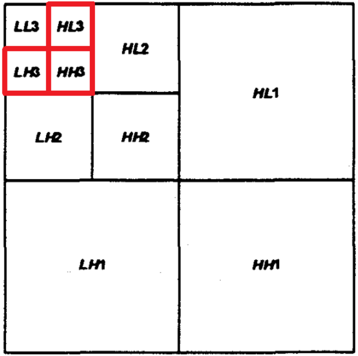
- Daubechies filter used for DWT
- three-level pyramid decomposition
- watermark embedding in high resolution sub-bands LH_3, HL_3, HH_3
- embedding strength α_i
  $\Rightarrow$ robustness vs. imperceptibility
- each sub-band B has $N_B = 4096$ coefficients
  $\Rightarrow$ watermark size $N = 12288$
- multiplicative embedding
  $\Rightarrow y_i = x_i(1 + \alpha_i w_i)$



DWT decomposition with used sub-bands

DWT watermarking
Implemented method

- each sub-band B has $N_B = 4096$ coefficients
  $\Rightarrow$ watermark size $N = 12288$
- Watermarks are i.i.d. $\mathcal{U}(-1, 1)$
- Original sub-bands i.i.d., assumed to be Laplacian distributed
- No correlation between sub-bands assumed



DWT decomposition with used sub-bands

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Maximum Likelihood detection

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- watermark $\mathbf{w}^*$ is detected if $L(\mathbf{y}) = \frac{f_{\mathbf{Y}}(\mathbf{y}|M_1)}{f_{\mathbf{Y}}(\mathbf{y}|M_0)} > \lambda$
- M_0 is the set of all unused watermarks including the null watermark. For imperceptible watermark, very small embedding strength α_i is used $\Rightarrow f_{\mathbf{Y}}(\mathbf{y}|M_0) \approx f_{\mathbf{Y}}(\mathbf{y}|\mathbf{0})$
- assuming statistically independent transform coefficients

$$L(\mathbf{y}) = \frac{\prod_{i=1}^{N} f_{Y_i}(y_i|w_i^*)}{\prod_{i=1}^{N} f_{Y_i}(y_i|0)} = \prod_{i=1}^{N} \frac{\frac{1}{1+\alpha_i w_i^*} f_{X_i}\left(\frac{y_i}{1+\alpha_i w_i^*}\right)}{f_{X_i}(y_i)} \underset{\mathbf{0}}{\overset{\mathbf{w}^*}{\gtrless}} \lambda$$

Maximum Likelihood detection

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$\Rightarrow$ taking the natural logarithm of the likelihood ratio, the decision rule becomes

$$
\begin{aligned}
l(\mathbf{y}) = \ln(L(\mathbf{y})) &= \ln\left(\prod_{i=1}^{N} \frac{\frac{1}{1+\alpha_i w_i^*} f_{X_i}\left(\frac{y_i}{1+\alpha_i w_i^*}\right)}{f_{X_i}(y_i)}\right) \\
&= \sum_{i=1}^{N}\left[\ln\left(f_{X_i}\left(\frac{y_i}{1+\alpha_i w_i^*}\right)\right) - \ln\left(f_{X_i}(y_i)\right)\right] \\
&\underset{\mathbf{0}}{\overset{\mathbf{w}^*}{\gtrless}} \underbrace{\ln(\lambda) + \sum_{i=1}^{N} \ln(1+\alpha_i w_i^*)}_{=\lambda'}
\end{aligned}
$$

Decision Threshold
Neyman-Pearson Criterion

$\Rightarrow$ NP Criterion used to find a λ' which minimizes P_{MD} for a fixed P_{FA}

$$P_{FA} = P(l(\mathbf{Y}) > \lambda' | M_0) \approx P(l(\mathbf{X}) > \lambda') = \int_{\lambda'}^{\infty} f_{l(\mathbf{x})}(l(\mathbf{x}))dl(\mathbf{x})$$

with $l(\mathbf{x}) = l(\mathbf{y})|_{\mathbf{y}=\mathbf{x}} = \sum_{i=1}^{N} \left[\ln \left(f_{X_i} \left(\frac{y_i}{1+\alpha_i w_i^*} \right) \right) - \ln \left(f_{X_i}(y_i) \right) \right]$

$\Rightarrow$ central limit theorem: PDF of $l(\mathbf{X})$ assumed to be Gaussian

$$P_{FA} = \int_{\lambda'}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{l(\mathbf{x})}^2}} \exp\left(-\frac{l(\mathbf{x}) - \mu_{l(\mathbf{x})}}{2\sigma_{l(\mathbf{x})}^2} \right) = \frac{1}{2}\mathrm{erfc}\left(\frac{\lambda' - \mu_{l(\mathbf{x})}}{\sqrt{2\sigma_{l(\mathbf{x})}^2}} \right)$$

$$\Rightarrow \lambda' = \mathrm{erfc}^{-1}(2P_{FA})\sqrt{2\sigma_{l(\mathbf{x})}^2} + \mu_{l(\mathbf{x})}$$

Decision Threshold
Laplacian Model

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⇒ DWT coefficient X_i is modeled by a Laplacian PDF

$$f_{X_i}(x_i) = 0.5 b_i e^{-b_i |x_i - \mu_i|}$$

⇒ decision rule is obtained by substituting Laplacian PDF in $l(\mathbf{y})$

$$l(\mathbf{y}) = \sum_{i_1}^{N} b_i \left[|y_i - \mu_i| - |1 + \alpha_i w_i^*|^{-1} |y_i - \mu_i - \mu_i \alpha_i w_i^*| \right]$$

$$\underset{\mathbf{0}}{\overset{\mathbf{w}^*}{\gtrless}} \lambda' = \mathrm{erfc}^{-1}(2P_{FA}) \sqrt{2\sigma^2_{l(\mathbf{X})}} + \mu_{l(\mathbf{X})}$$

Decision Threshold
Unknowns

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$\mu_{I(\mathbf{x})}$ and $\sigma^2_{I(\mathbf{x})}$ are derived as

$$\mu_{I(\mathbf{x})} = \sum_{i=1}^{N} 1 - \frac{\bar{b} + e^{-\bar{b}}}{|p|}$$

$$\sigma^2_{I(\mathbf{x})} = \sum_{i=1}^{N} 1 - \frac{2e^{-\bar{b}}(1+\bar{b})}{|p|} + \frac{2 - e^{-2\bar{b}} - 2\bar{b}e^{-\bar{b}}}{|p|^2}$$

where $\bar{b} = b_i|\mu_i \alpha_i w_i^*|$ and $p = 1 + \alpha_i w_i^*$.

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Experimental Results

- blind detection is used
- estimation of μ_i and σ_i from watermarked image:

$$\hat{\mu}_i = \frac{1}{N_B} \sum_{y \in B} y$$

$$\hat{\sigma}_i^2 = \frac{1}{N_B - 1} \sum_{y \in B} (y - \hat{\mu}_i)^2$$

where y is the corresponding DWT coefficient of the watermarked image

Peppers

Lena

Harbour

F16

Experimental Results

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Three tested image processing operations:

1. JPEG compression with 50% quality factor
2. blurred, using 4×4 spatial filter
3. corrupted by Gaussian noise with $\mu = 0$ and $\sigma^2 = 0.5$

$\Rightarrow$ results are based on 10000 trials
$\Rightarrow$ in each trial, embedded watermark $\mathbf{w}^*$ is chosen from a set W of 100 randomly generated watermarks
$\Rightarrow$ successful detection, if threshold is exceeded for this watermark, but not for any other watermark in W

Experimental Results

TABLE I
PERCENTAGE OF SUCCESSFUL DETECTIONS UNDER JPEG COMPRESSION

Image	Laplacian Model	Gaussian Model
Peppers	96.23	81.95
Lena	91.41	83.61
Harbour	95.63	85.89
F16	98.11	91.43

TABLE II
PERCENTAGE OF SUCCESSFUL DETECTIONS UNDER BLURRING

Image	Laplacian Model	Gaussian Model
Peppers	93.17	85.69
Lena	89.38	86.35
Harbour	92.62	90.16
F16	97.21	84.63

TABLE III
PERCENTAGE OF SUCCESSFUL DETECTIONS UNDER GAUSSIAN NOISE

Image	Laplacian Model	Gaussian Model
Peppers	89.26	87.46
Lena	91.45	90.67
Harbour	97.10	87.31
F16	96.41	91.18

Outline

Conclusion & Outlook

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Conclusion:

- formulation of an ML detection scheme based on modeling the distribution of the DWT coefficients using Laplacian pdf
- derivation of decision threshold for Neyman-Pearson criterion
- comparison of Laplacian model with Gaussian model
- Laplacian model yields a better watermark detection than Gaussian model

Critique:

- μ_i and σ_i are estimated as sample mean and unbiased sample variance
  $\Rightarrow$ inconsistent with the Laplacian assumption:
  $\hat{\mu}_{i,MLE} = \text{median}(y \in B)$ and $\hat{\sigma}^2_{i,MLE} = 2/N_B \sum_{y \in B}(y - \hat{\mu}_i)^2$

Outlook:

- evaluation of robustness under other standard image processing operations

References

Kim, Young-Sik et al. (1999). "Wavelet based watermarking method for digital images using the human visual system". In: *Circuits and Systems, 1999. ISCAS '99. Proceedings of the 1999 IEEE International Symposium on*. Vol. 4, 80–83 vol.4.

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