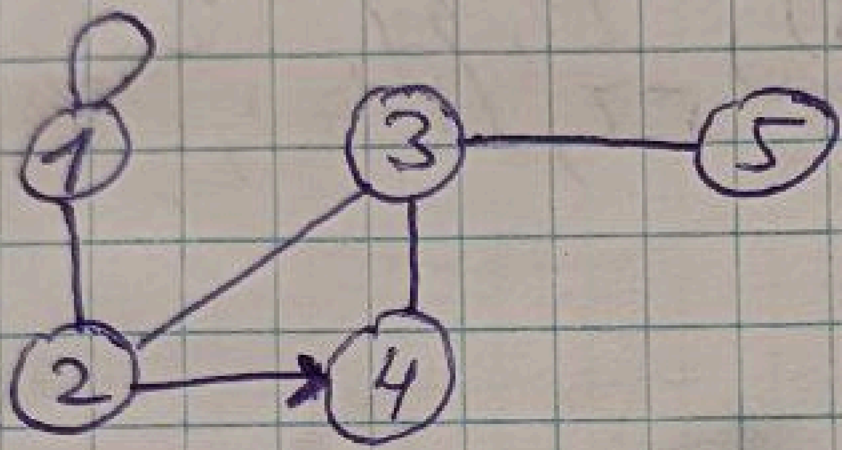
Графы

$G = \langle V, E \rangle$

$E \subset V^2$

Способы представления графов

1 Графический



2 Список дуг 2 и список вершин 1

1 $V = \{1, 2, 3, 4, 5\}$ ребро дуга

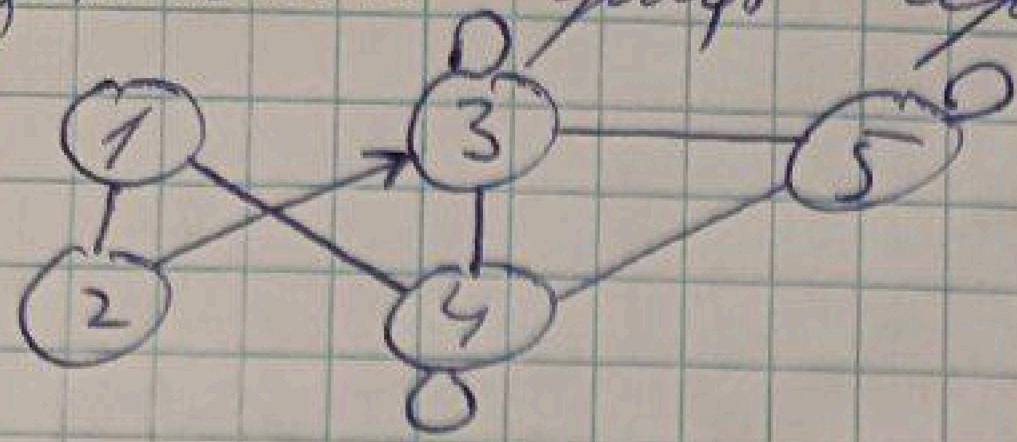
2 $E = \{(1, 1), [1, 2], [2, 3], (2, 4), [3, 4], [3, 5]\}$

3 Матрица смежности

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

1. Представить граф через списки и матрицы:

б)

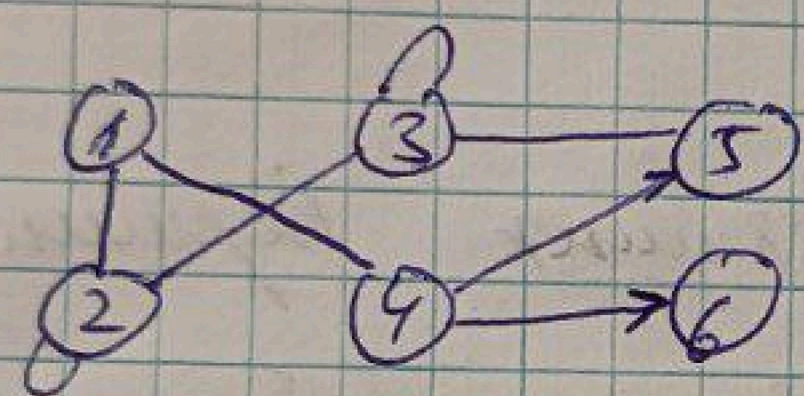


$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$V = \{1, 2, 3, 4, 5\}$

$E = \{ (1,1), [1,2], [1,4], (2,3), (3,3), [3,4], [3,5], (4,4), [4,5], (5,5) \}$

в)

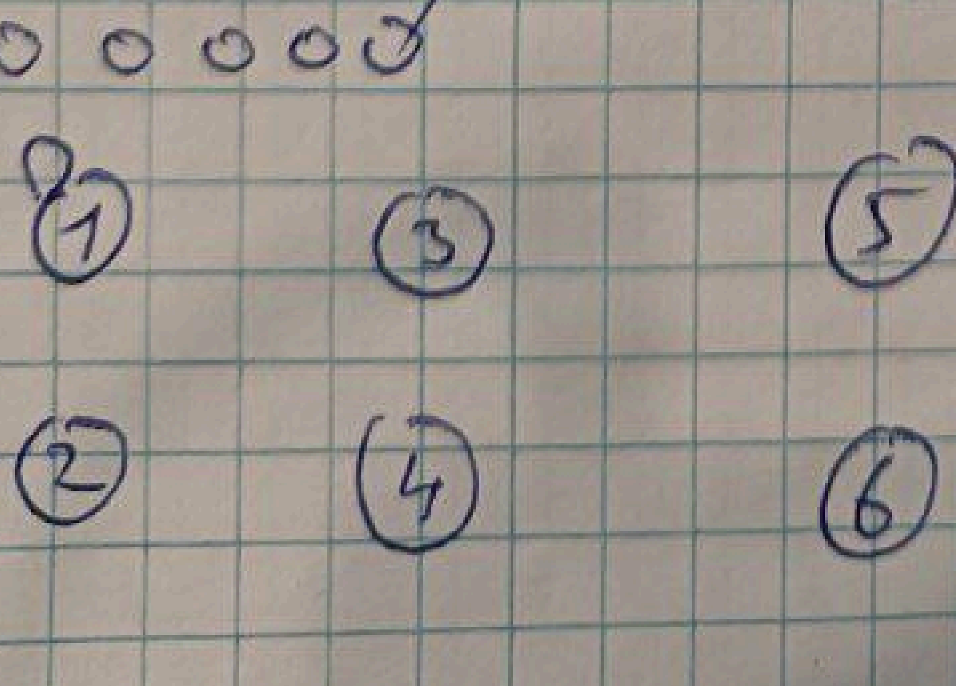


$V = \{1, 2, 3, 4, 5, 6\}$

$E = \{ [1,2], [1,4], (2,2), [2,3], (3,3), [3,5], (4,5), (4,6) \}$

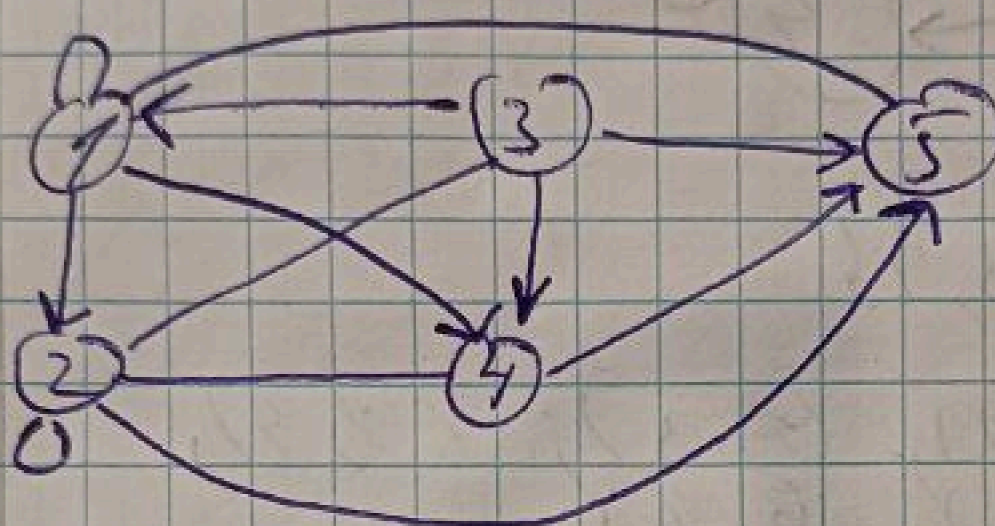
$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

г)



2. а) $V = \{1, 2, 3, 4, 5\}$

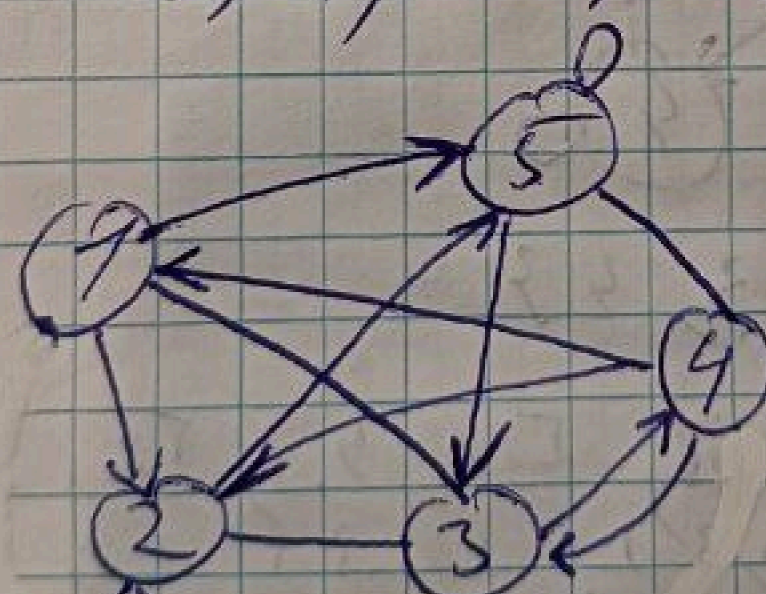
$E = \{(1, 1), (1, 2), (1, 4), [1, 5], (2, 2), [2, 3], [2, 4], (2, 5), (3, 1), (3, 4), (3, 5), (4, 5)\}$



$$\begin{matrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{matrix}$$

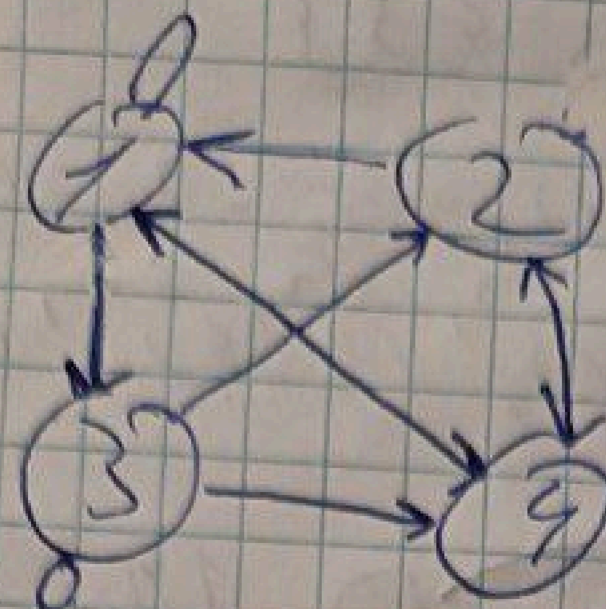
б) $V = \{1, 2, 3, 4, 5\}$

$E = \{(1, 2), [1, 3], (1, 5), (2, 2), [2, 3], (2, 5), (3, 4), (4, 1), (4, 2), (4, 3), [4, 5], (5, 3), (5, 5)\}$



$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

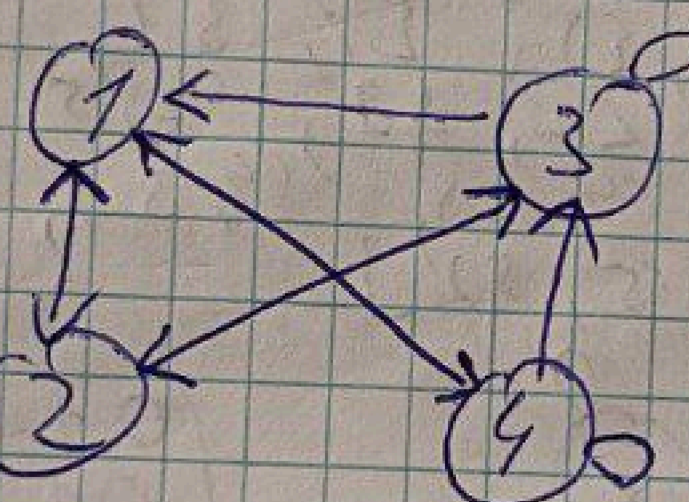
3. а)



$V = \{1, 2, 3, 4\}$

$E = \{(1, 1), (1, 3), (1, 4), (2, 1), (2, 4), (3, 3), (3, 2), (3, 4), (4, 1), (4, 2)\}$

б)

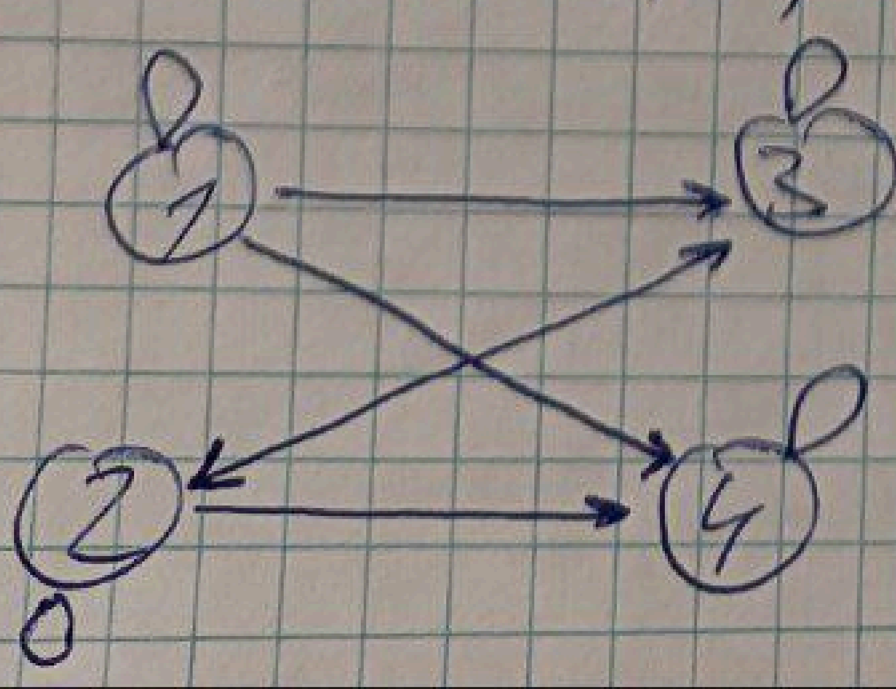


$V = \{1, 2, 3, 4\}$

$E = \{[1, 2], [1, 4], [2, 3], (3, 1), (3, 3), (4, 3), (4, 4)\}$

в) $V = \{1, 2, 3, 4\}$

$E = \{(1, 1), (1, 3), (1, 4), (2, 2), [2, 3], (2, 4), (3, 3), (4, 4)\}$



Операции с графами

доб вер a $\langle V \cup \{a\}, E \rangle$

доб дугу (a, b) $\langle V \cup \{a, b\}, E \cup \{(a, b)\} \rangle$

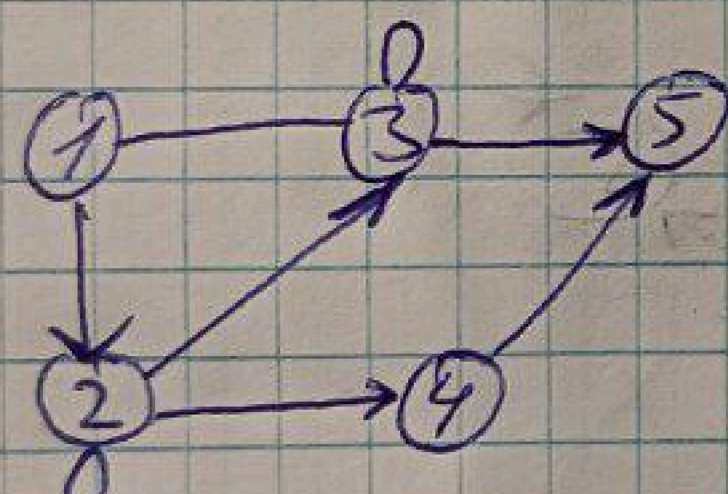
удл дугу (a, b) $\langle V, E \setminus \{(a, b)\} \rangle$

удл верш a $\langle V \setminus a, E \setminus \{(b, c) \mid a \in \{b, c\}\} \rangle$

скл (отож) a, b

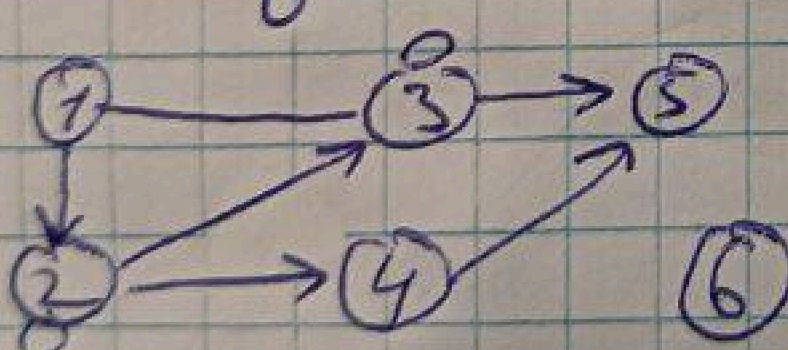
$$\langle (V \setminus \{a, b\}) \cup \{ab\}, E \setminus \{(x, y) \mid \{x, y\} \cap \{a, b\} \neq \varnothing\} \cup \{(ab, x) \mid (a, x) \in E \vee (b, x) \in E\} \cup \{(x, ab) \mid (x, a) \in E \vee (x, b) \in E\} \rangle$$

4. а)

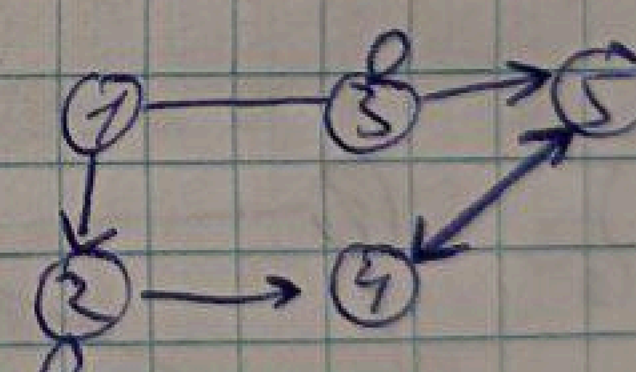


1) доб 6
2) доб (5, 4)
3) уд 1
4) отож 1, 3

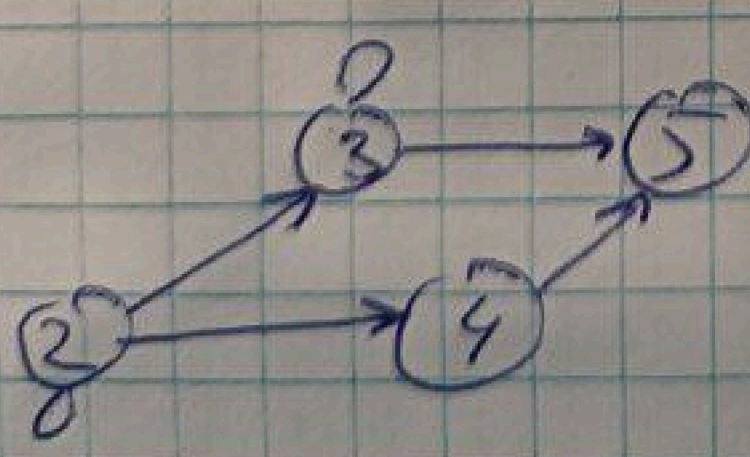
1)



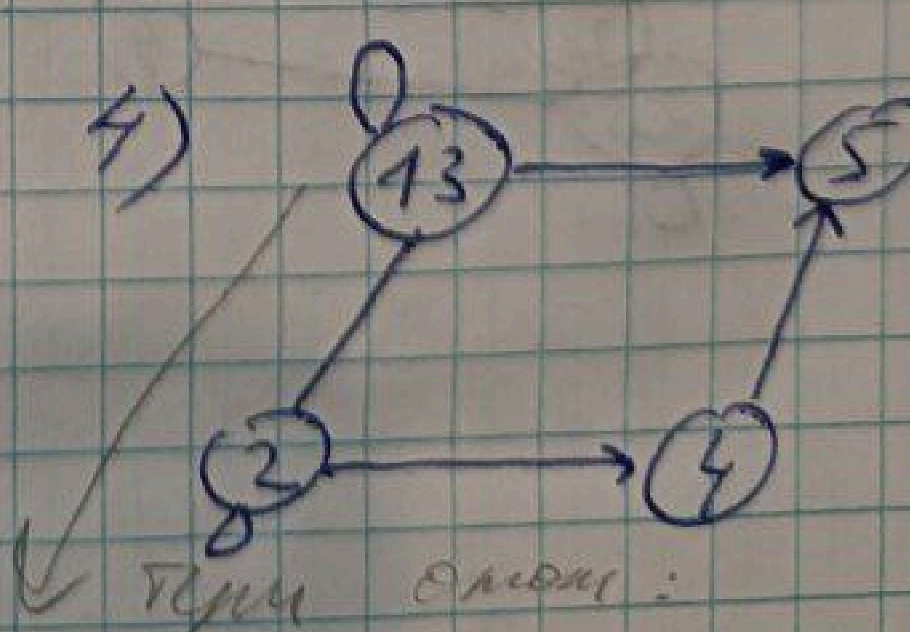
2)



3)

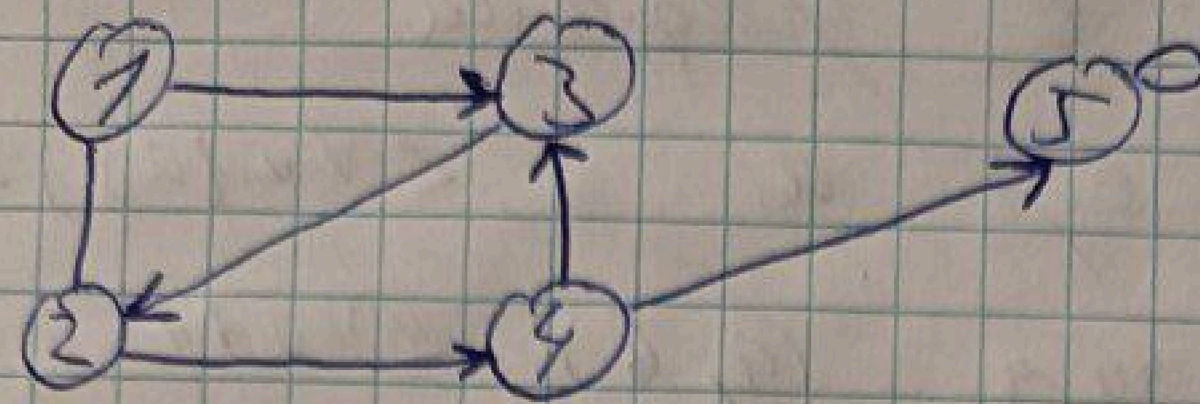


4)



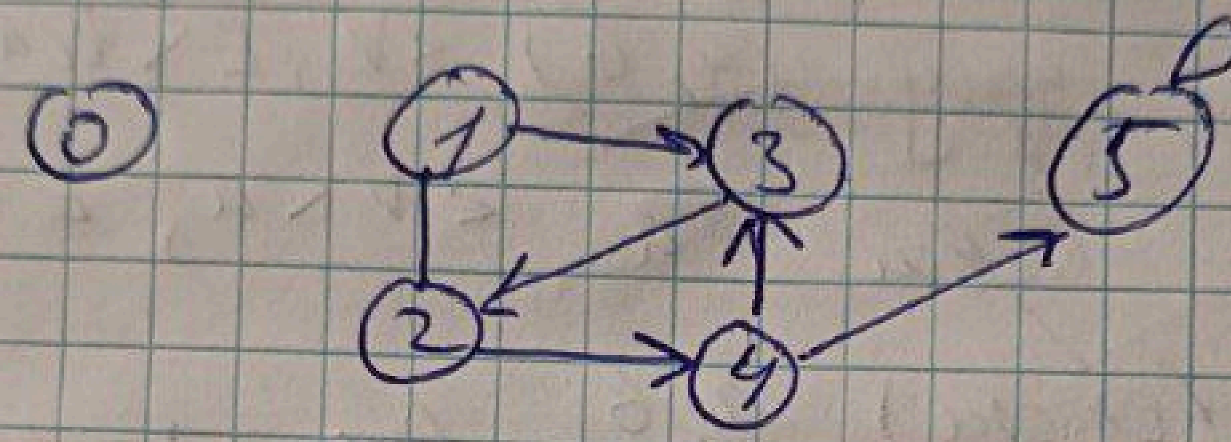
при отож: петли

б)

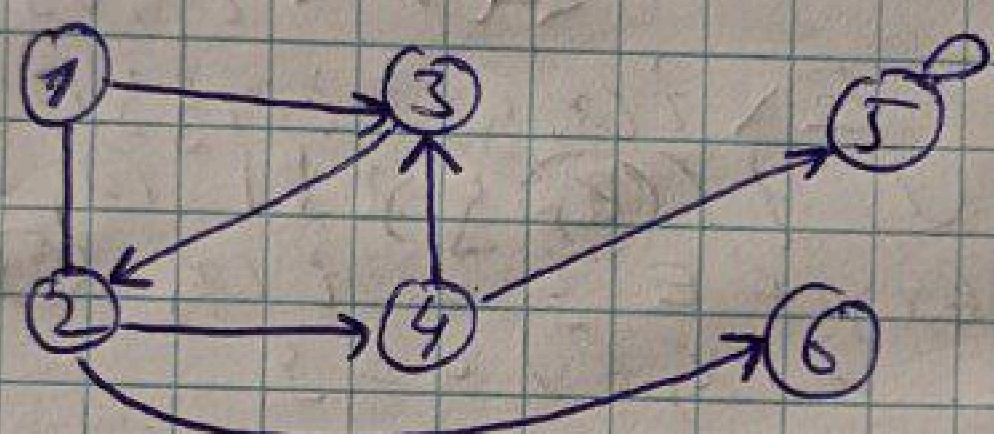


1) доб 0
2) доб (2, 6)
3) уд 1
4) отож 2, 5

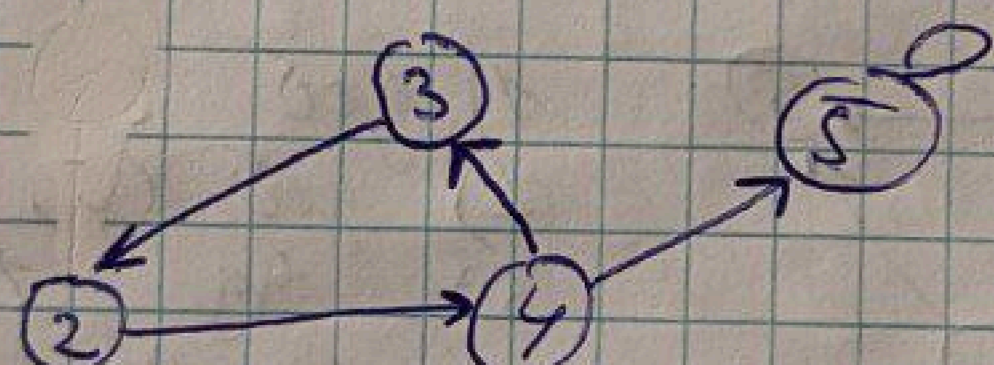
1)



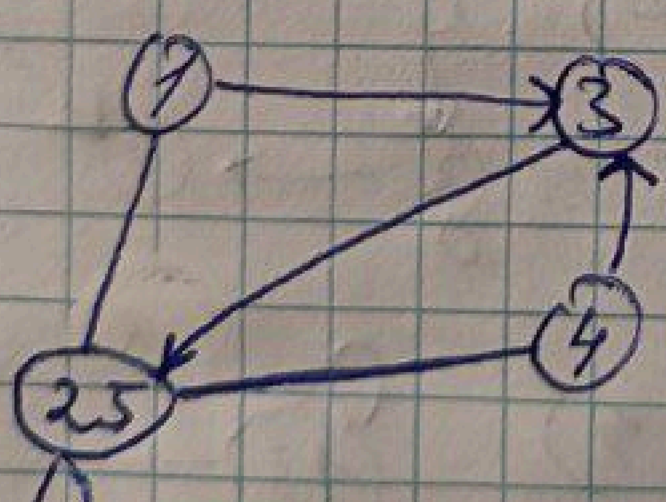
2)



3)



4)



$V_1 \cap V_2 \neq \emptyset$

Операции для группы графов

$$a_1 \cap a_2 = \langle V_1 \cap V_2, E_1 \cap E_2 \rangle$$

$$a_1 \cup a_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \rangle$$

$$a_1 \oplus a_2 = \langle V_1 \cup V_2, E_1 \oplus E_2 \rangle$$

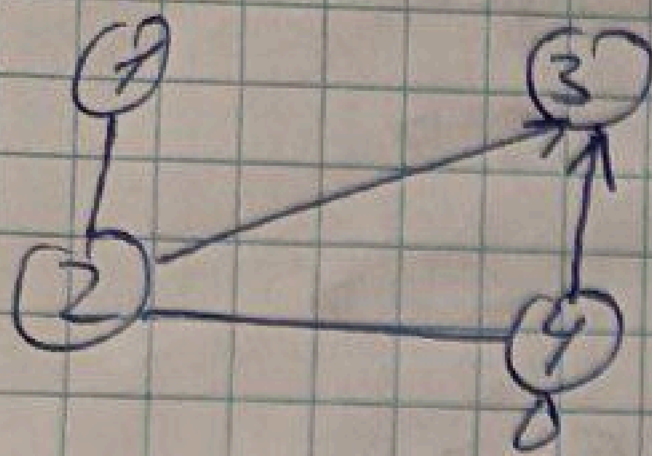
$$A \oplus B = (A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

$$a_1 + a_2 = \langle V_1 \cup V_2, E_1 \cup E_2 \cup \{[a, b] \mid a \in V_1, b \in V_2, a \neq b\} \rangle$$

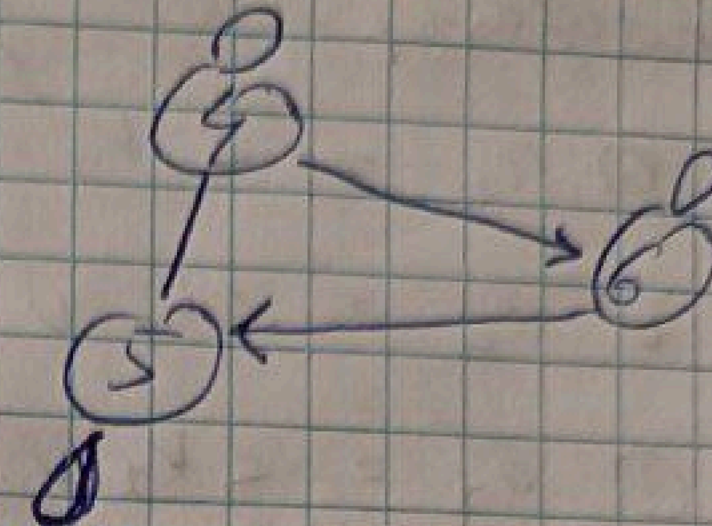
$$a_1 \times a_2 = \langle V_1 \times V_2, \{((a_1, b_1), (a_2, b_2)) \mid (a_1 = a_2, (b_1, b_2) \in E_2) \vee ((a_1, a_2) \in E_1, b_1 = b_2\} \rangle$$

$$a_1 [a_2] = \langle V_1 \times V_2, \{((a_1, b_1), (a_2, b_2)) \mid (a_1, a_2) \in E_1 \vee ((a_1 = a_2), (b_1, b_2) \in E_2\} \rangle$$

a) a_1

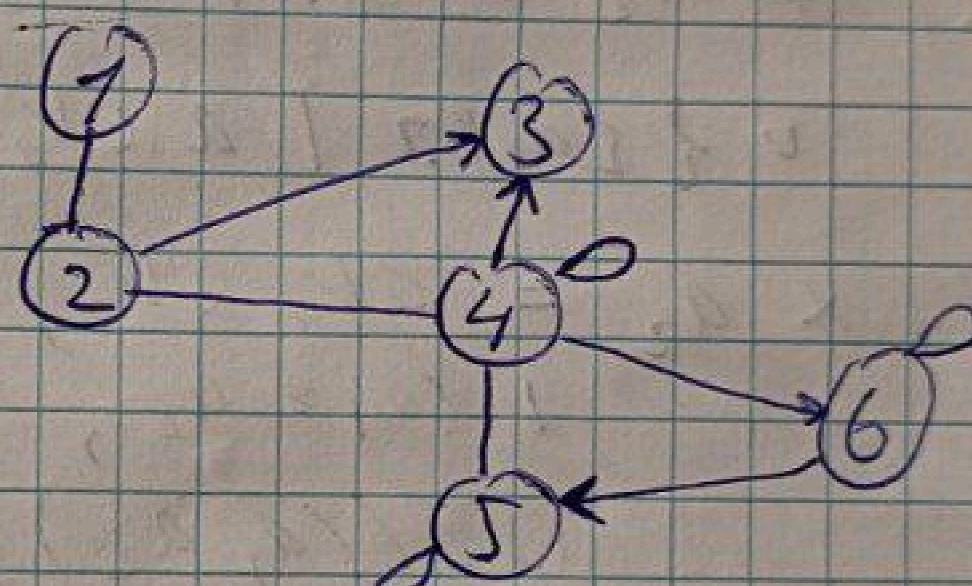


a_2

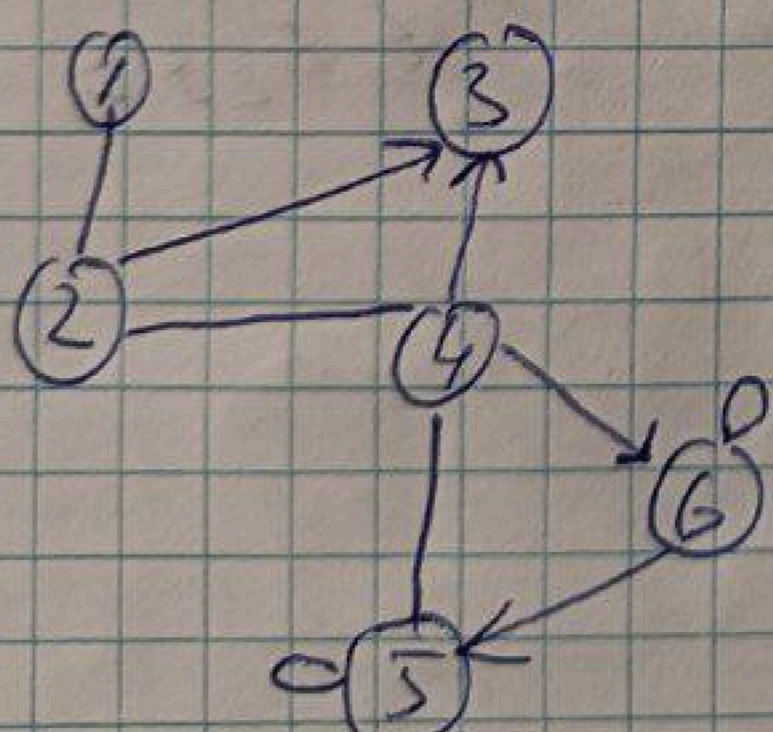


$a_1 \cap a_2$

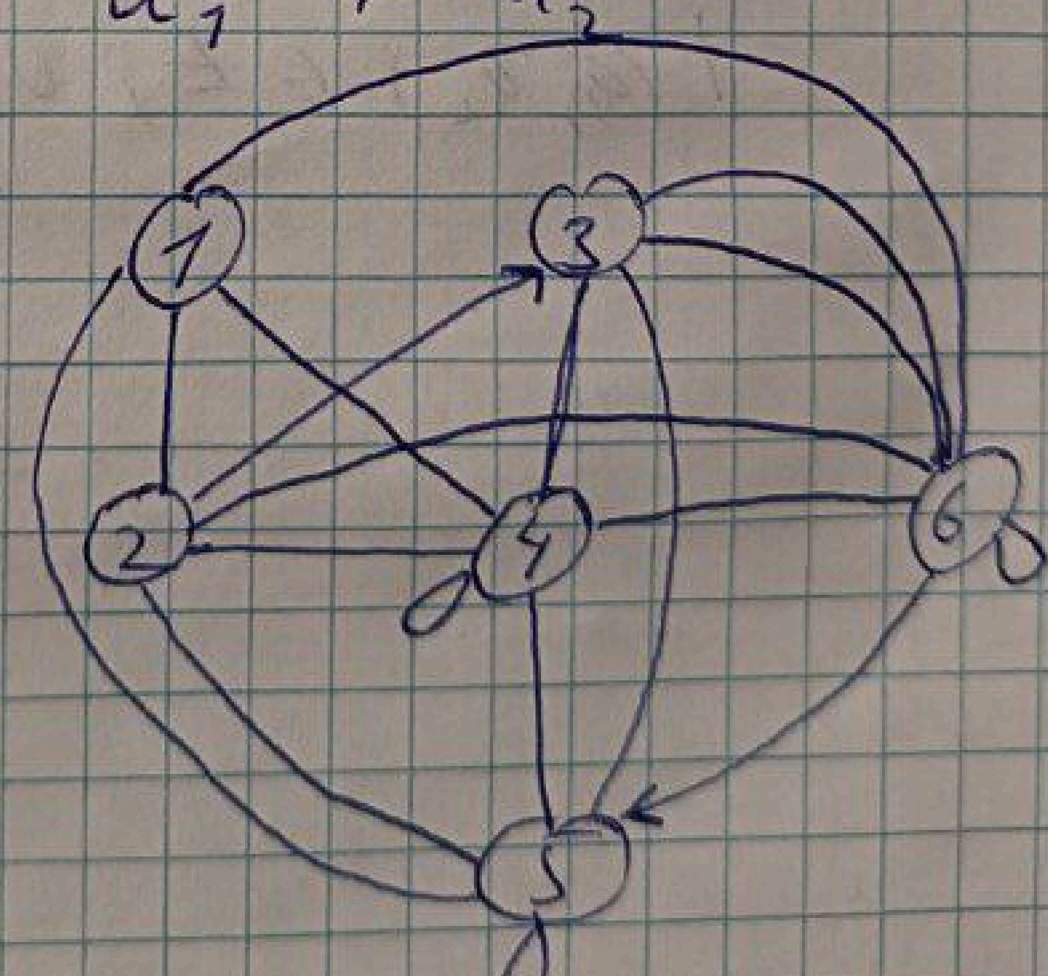
$a_1 \cup a_2$



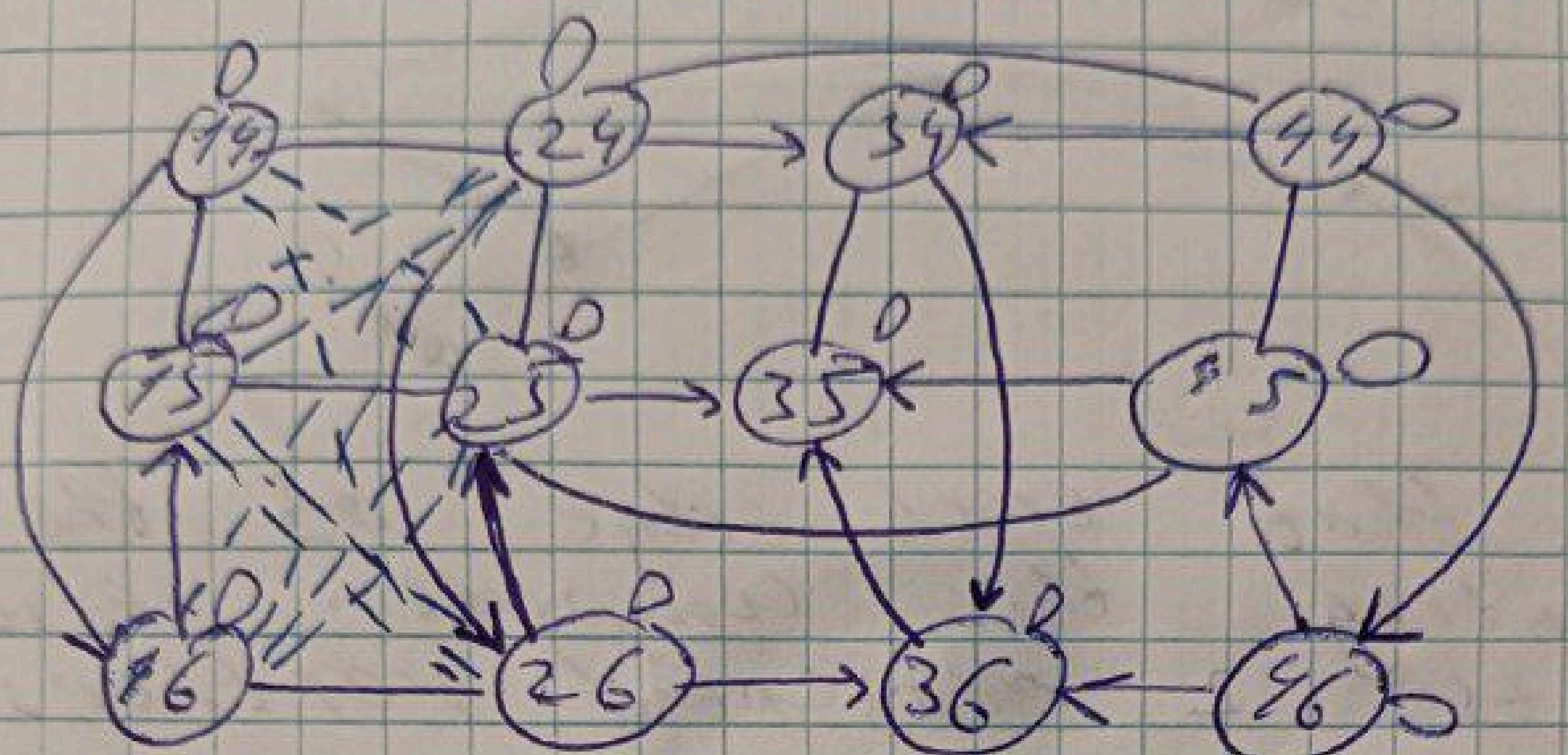
$a_1 \oplus a_2$



$a_1 + a_2$



$a_1 \times a_2$



Graphs and Graph Models

Definition 1

A graph (граф) $G = (V, E)$ consists of V, a nonempty set of vertices (вершин) (or nodes) and E, a set of edges (рёбер).

Each edge has either 1 or 2 vertices associated with it, called its endpoints (концы).

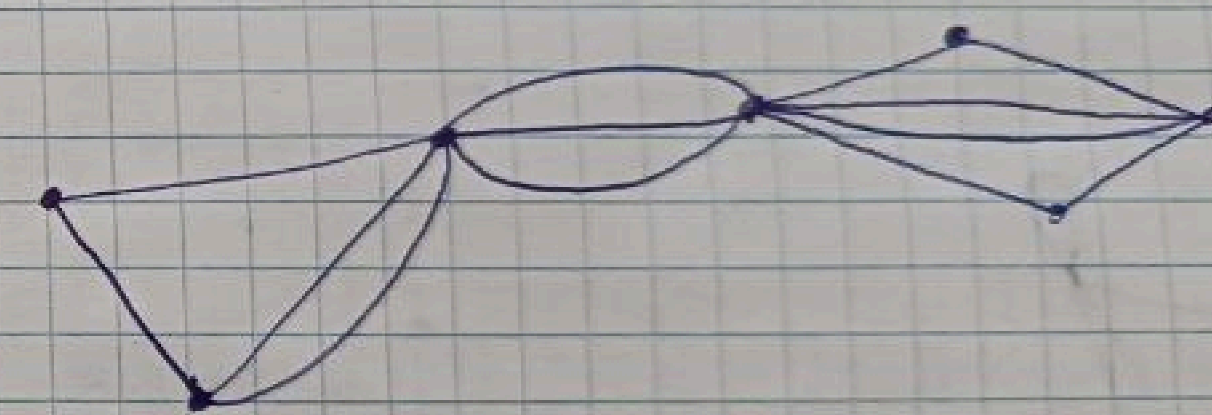
An edge is said to connect (соединяет) its endpoints.

Remark: The set of vertices V of graph G may be infinite. A graph with

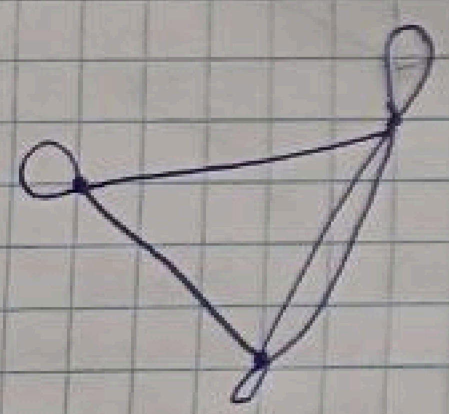
Простой граф / Simple graph

Note that each edge of the graph representing this computer network connects 2 different vertices.

Мультиграф / Multigraph



Псевдограф / Pseudo grap

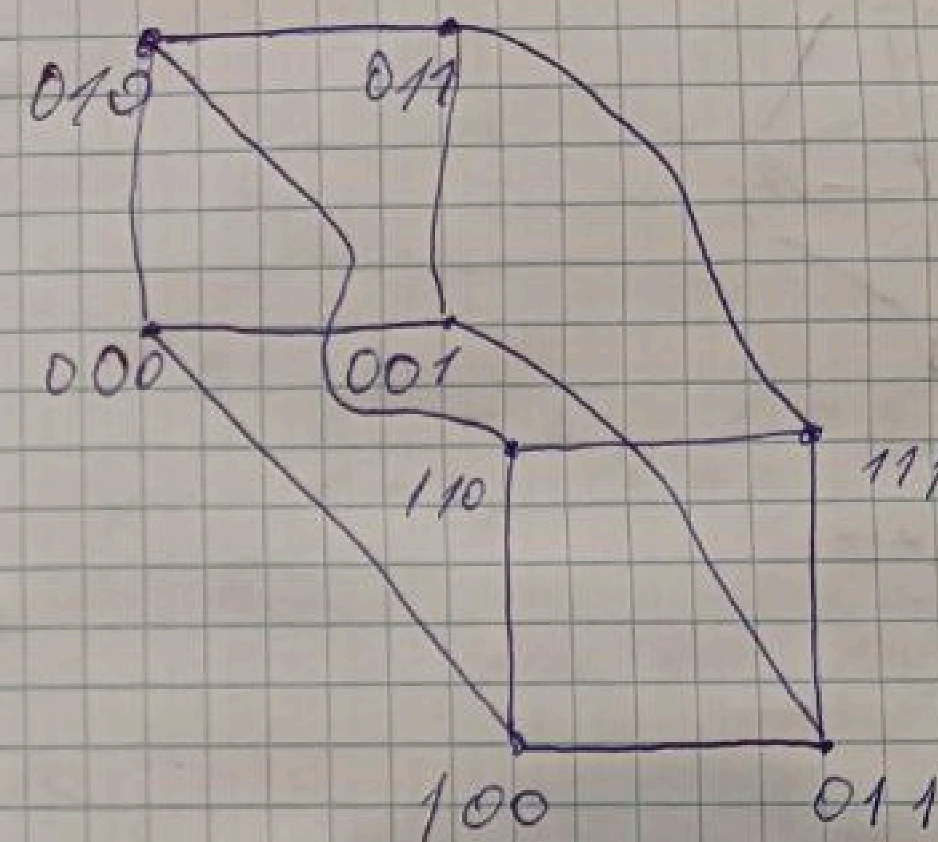


Ориентированный граф / Directed graph

Definition 2

A directed graph (ориентированный / направленный граф) (or digraph, орграф) (V, E) const

n - Cubes (n-мерные кубы)



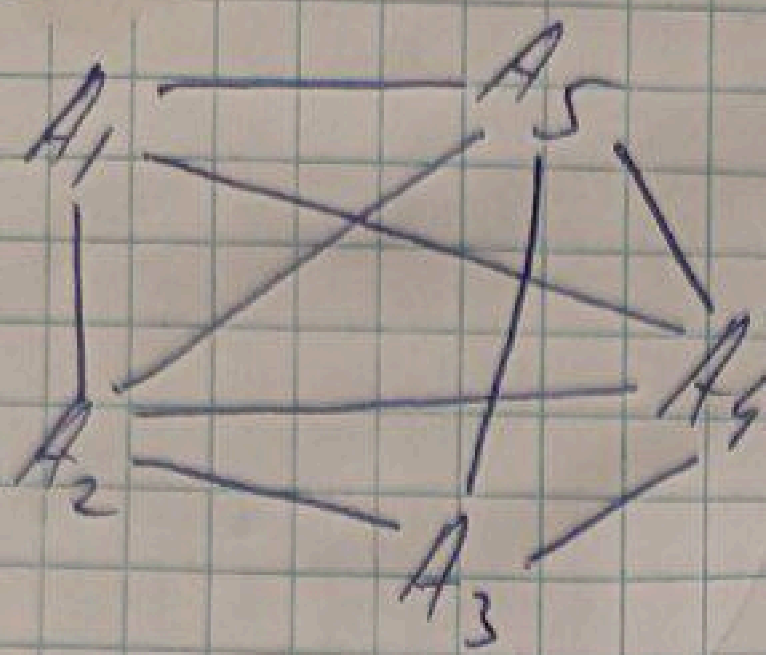
Spanning subgraph – остовный подграф

Induced subgraph – индуцированный подграф

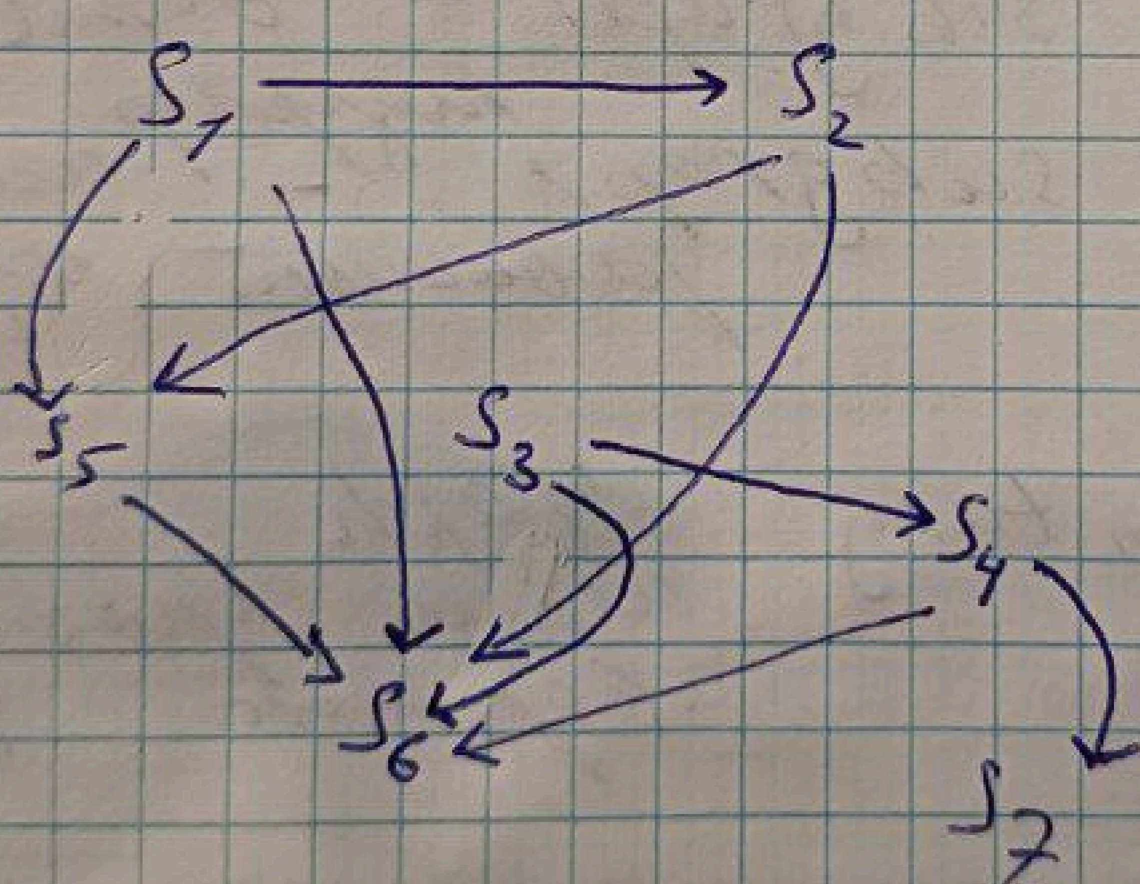
Complementary graph – дополнение графа с n вершин до K_n.

3.

a)



8. $S_1 : x := 0$
$S_2 : x := x + 1$
$S_3 : y := 2$
$S_4 : z := y$
$S_5 : x := x + 2$
$S_6 : y := x + z$
$S_7 : z := 4$

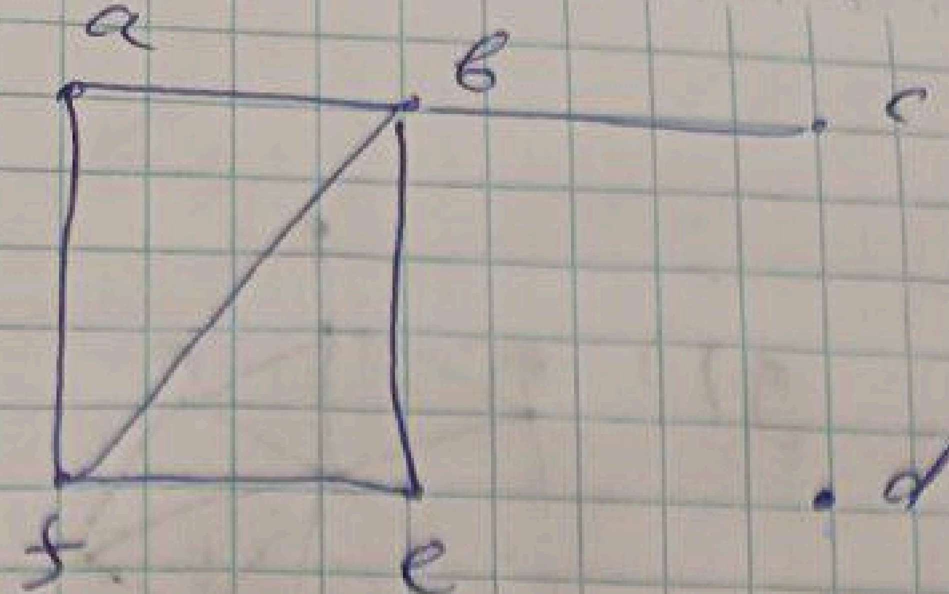


9.

1) $V = 6$, $E = 6$;
$\deg(a) = 2$, $\deg(b) = 4$, $\deg(c) = 1$,
$\deg(d) = 0$, $\deg(e) = 2$, $\deg(f) = 3$

Изолированная вершина – d,
висячая – c.

10.

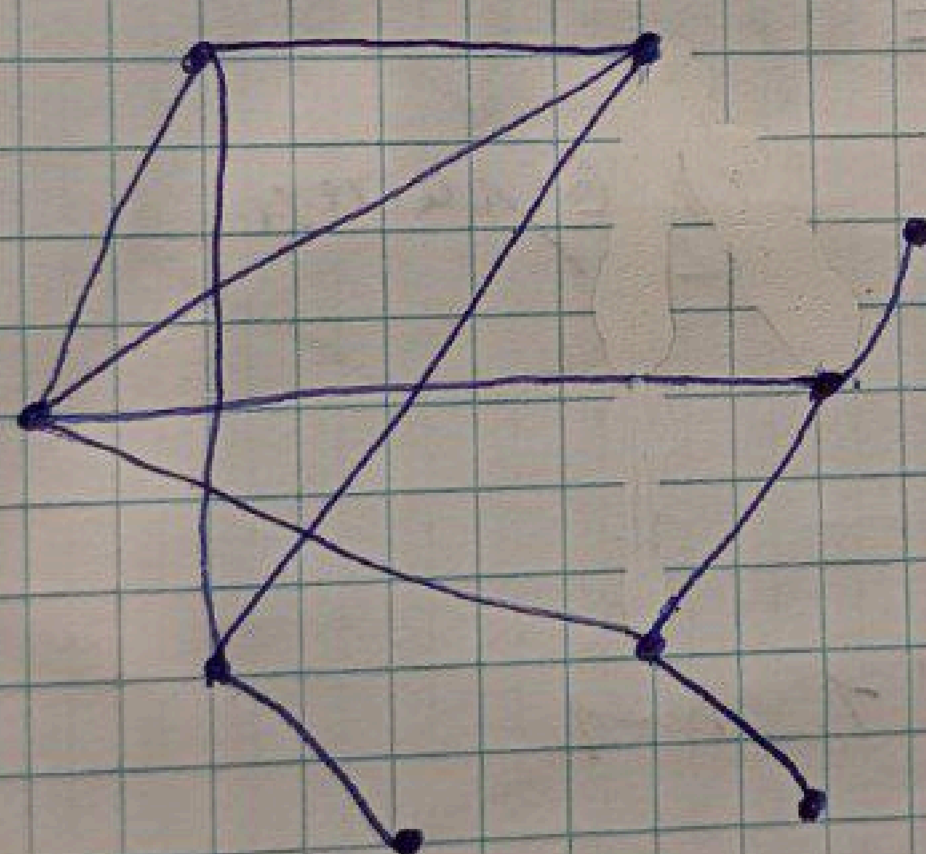


deg(x):

a = 2, b = 4, c = 1, d = 0,
e = 2, f = 3

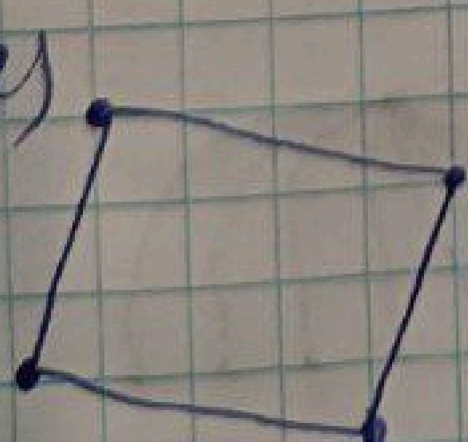
Сумма степеней: 12
кол-во ребер: 6.

12.

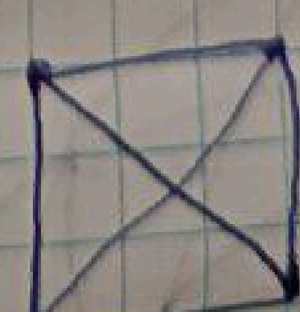


13.

а)

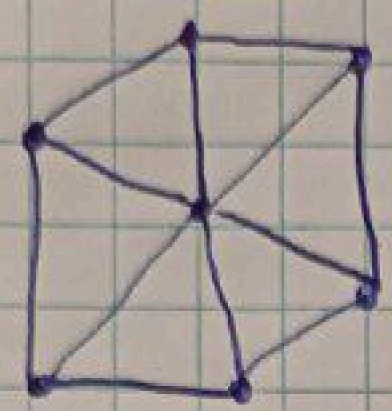


б)

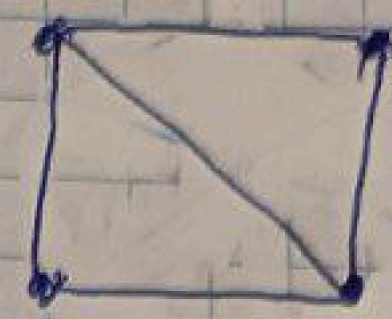


в) —

г)



д)



14.

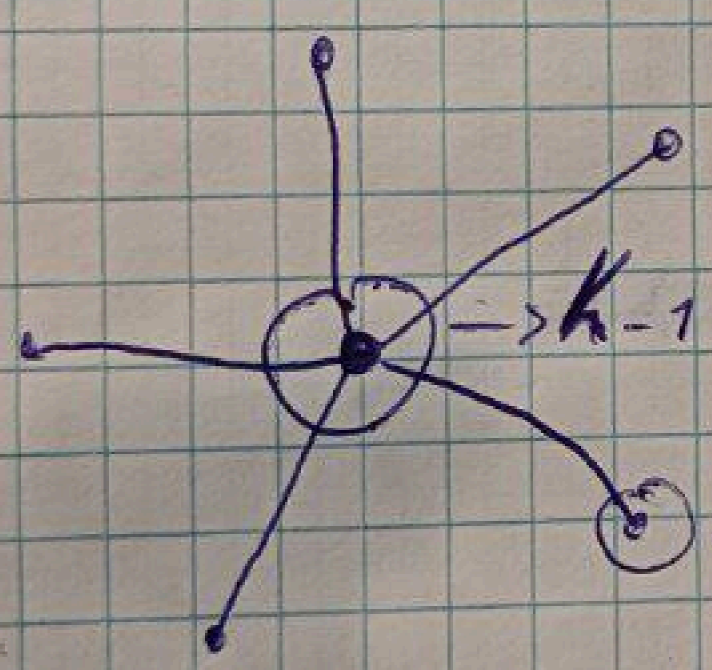
Для $n = 1$:

→ либо • • (либо 0 рёбер)

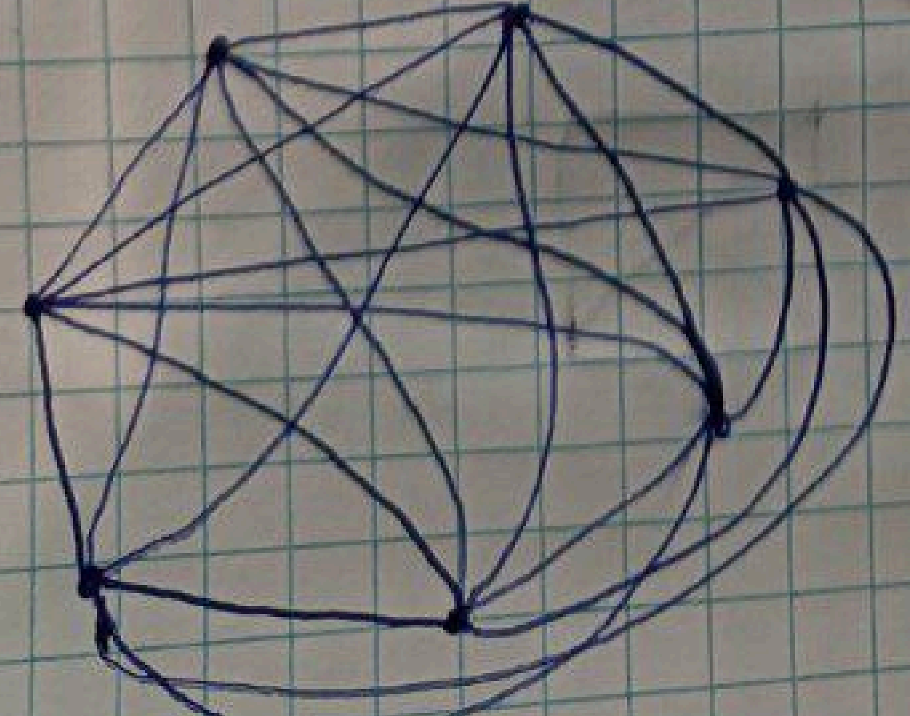
→ либо •—• (либо 1 ребро)

ПРИНЦИП ДИРИХЛЕ

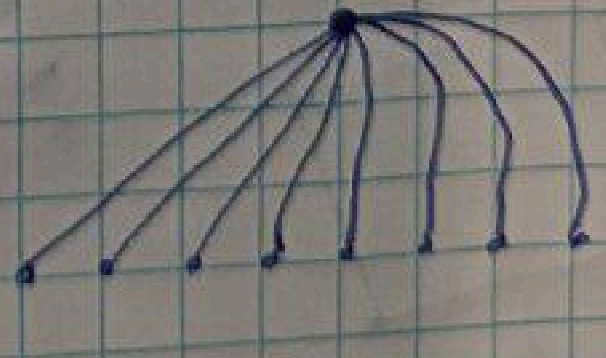
n-вершин



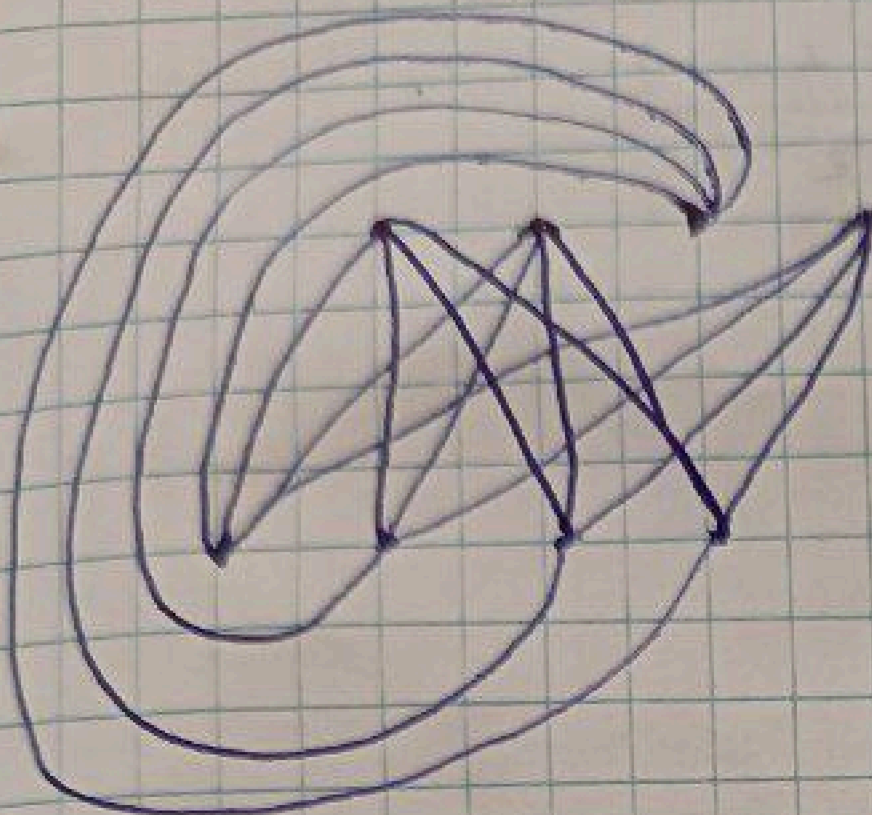
16. а) K_7



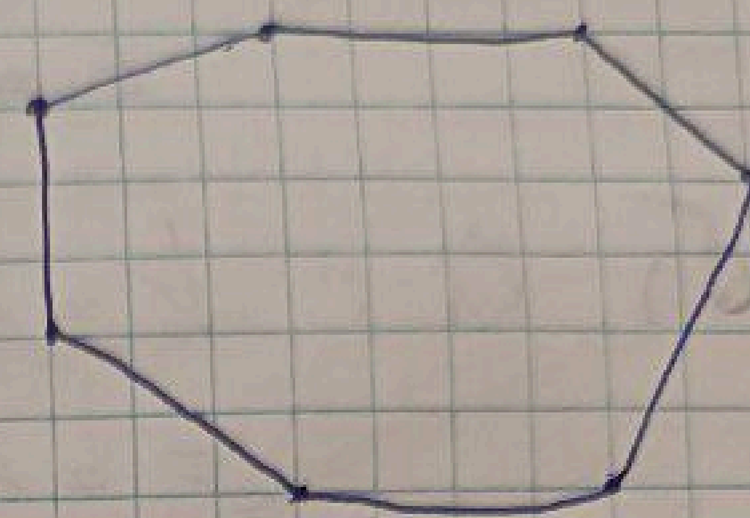
б) $K_{1,8}$



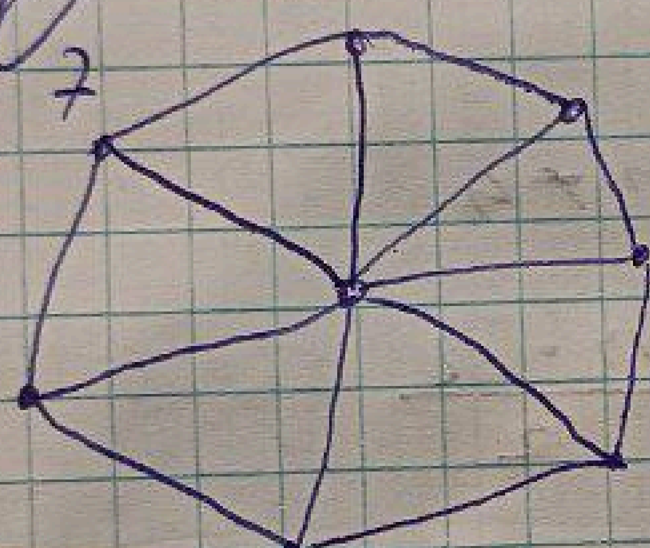
c) $K_{4,4}$



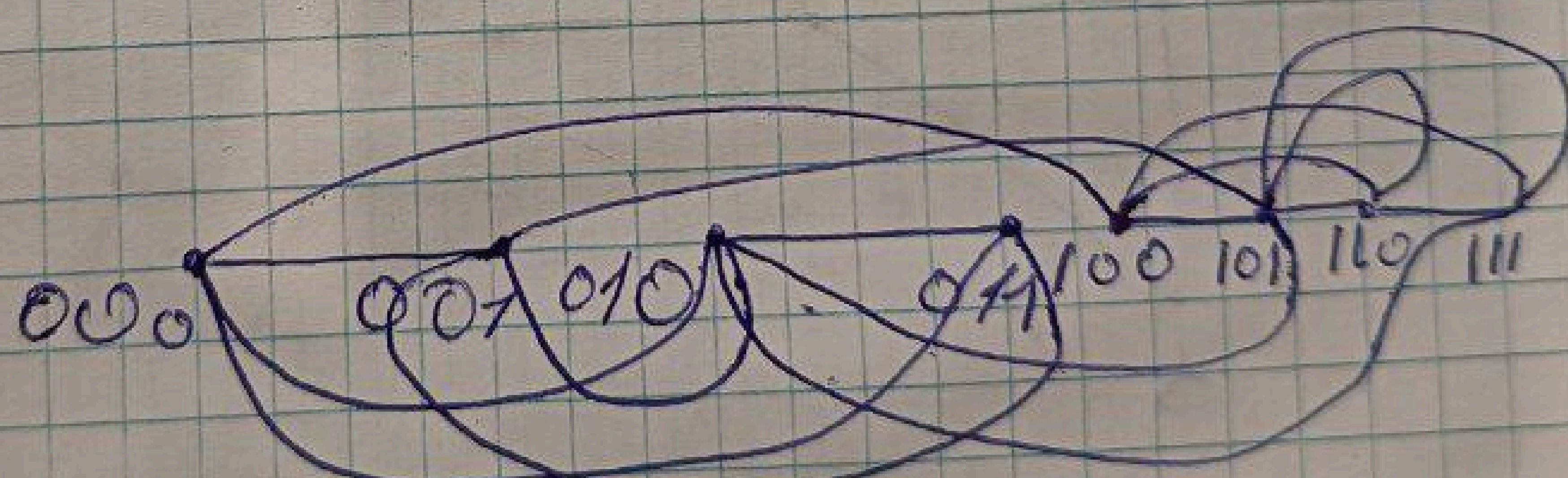
d) C_7



e) W_7



f) Q_3



17. Кол-во вершин и рёбер:

а) K_n : $V = n$

$E = \frac{n \cdot (n-1)}{2}$

б) C_n : $V = n$

$E = n$

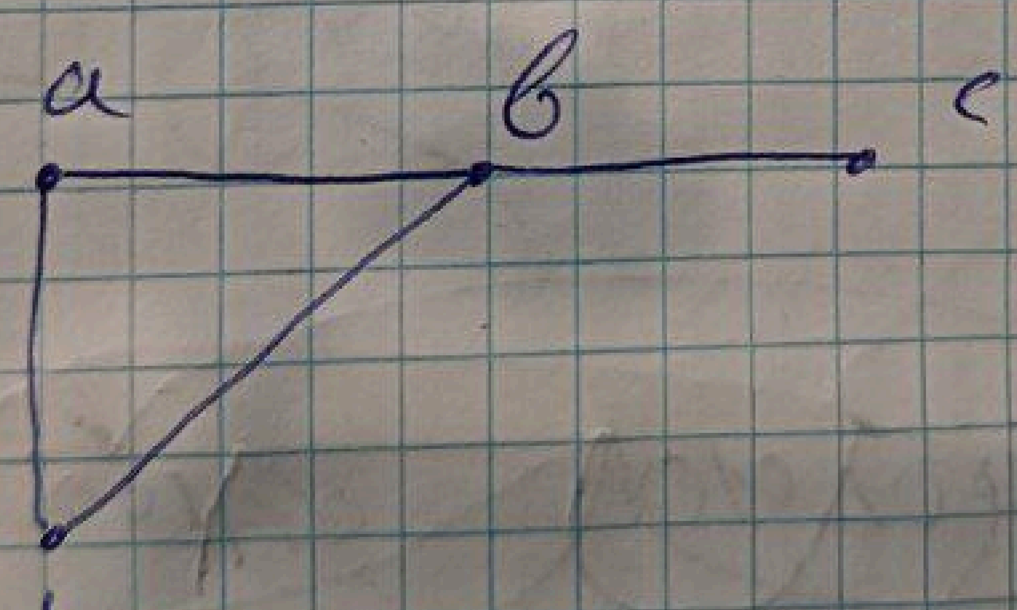
c) W_n : $V = n+1$

$E = 2n$

d $K_{m,n}$: $V = m+n$

$E = m \cdot n$

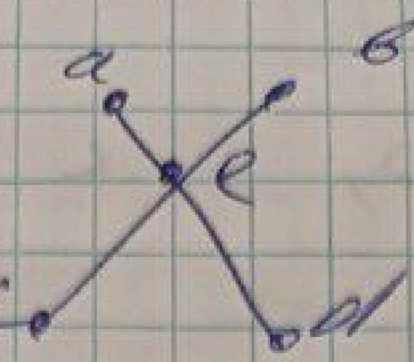
18.



19. На дом.

20.

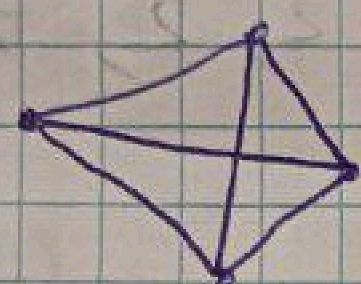
а)



4 1 1 1 1

21.

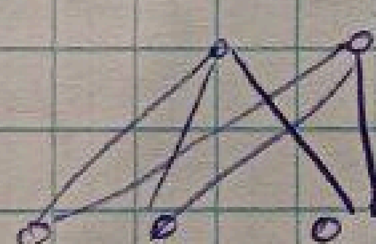
а) K_4



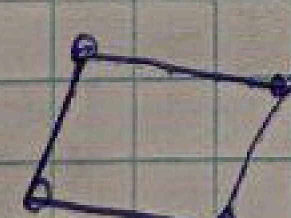
3 3 3 3

б) $K_{2,3}$

3 3 2 2 2



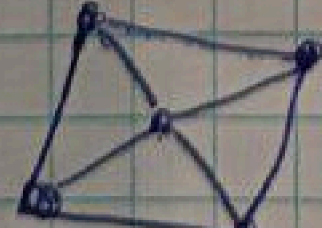
в) C_4



2 2 2 2

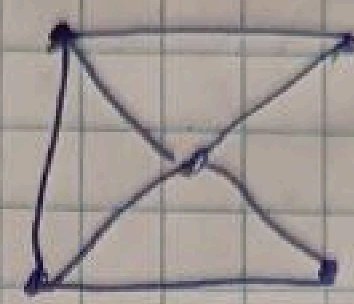
г) W_4

4 3 3 3 3

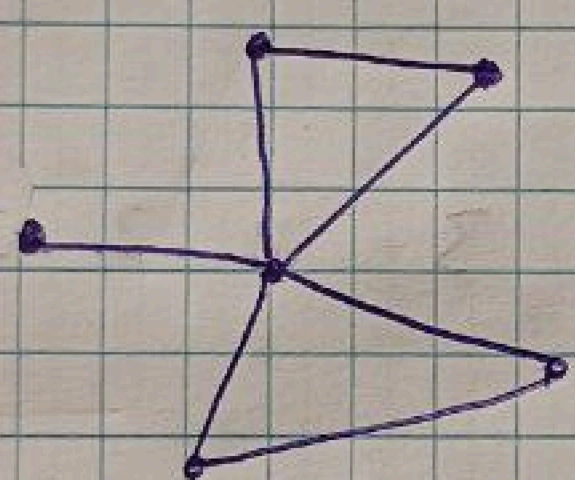


д) Q_3 3 3 3 3 3 3 3 3

24.
Deg: 4, 3, 3, 2, 2



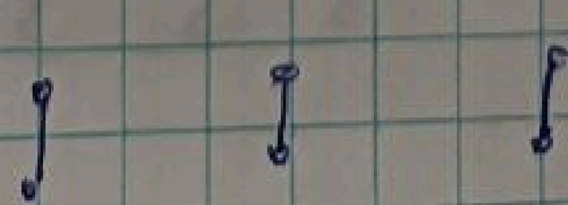
25.
Deg: 5, 2, 2, 2, 2, 1



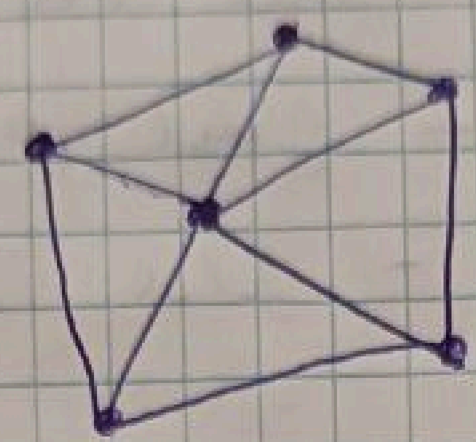
26.
b) 2, 2, 2, 2, 2, 2

g)

e)

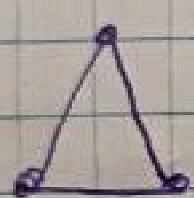


41)



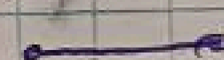
27.

K_3

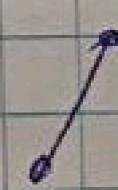


Subgraphs:

1) 

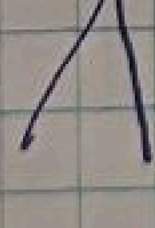
4) 

2) 

5) 

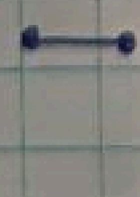
3) 

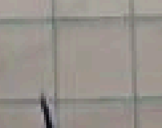
6) 

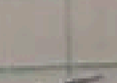
7) 

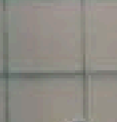
8) 

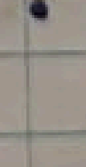
9) 

10) 

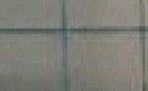
11) 

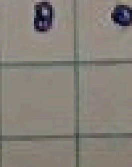
12) 

13) 

14) 

15) 

16) 

17) 

30.

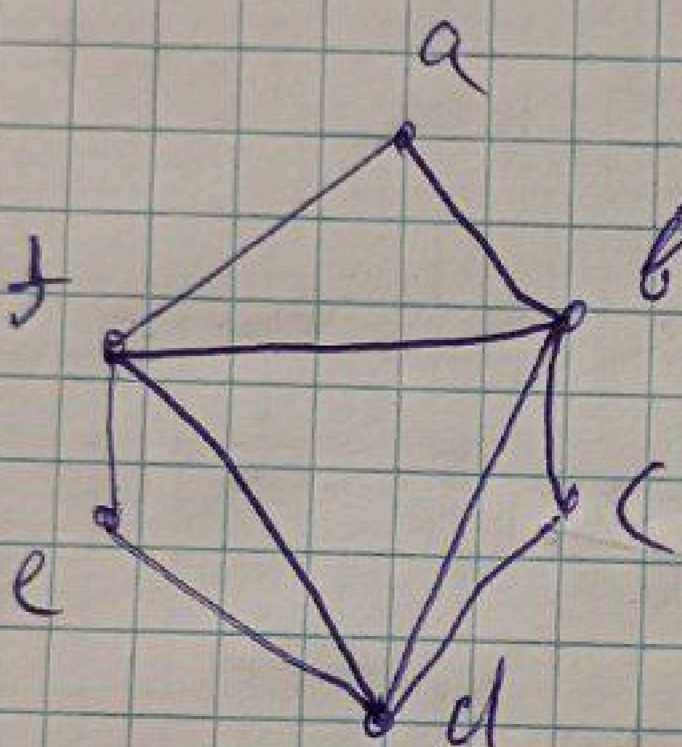
а) K_n — рег. для люб. n

б) C_n — для люб.

в) W_n — для $n = 3$

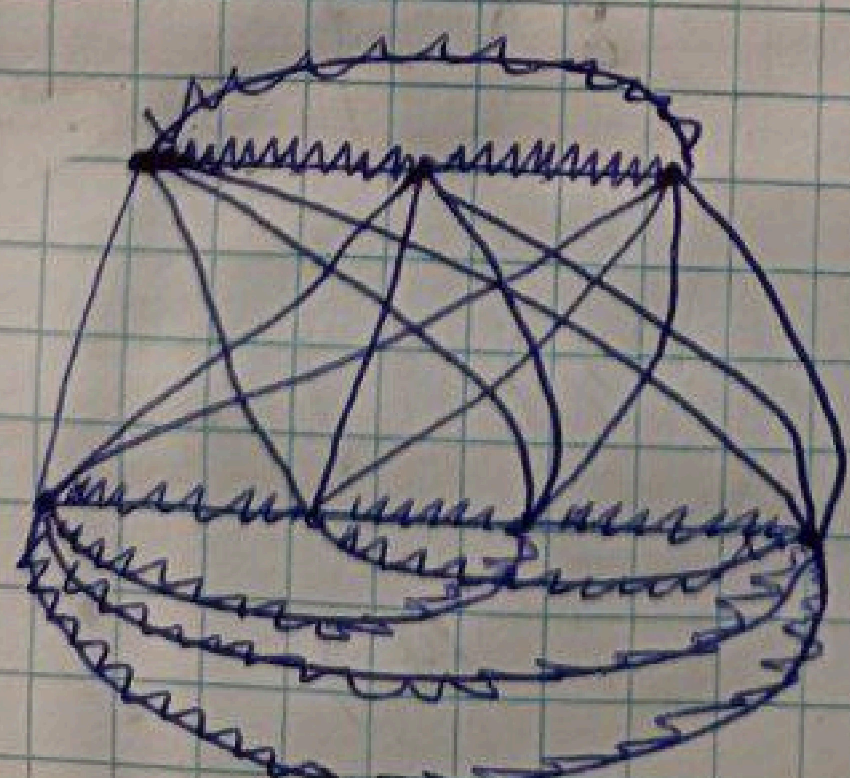
г) Q_n — для люб.

31.

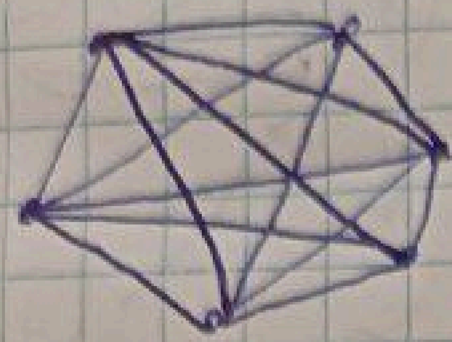


32. а) $\overline{K_n}$ — уже полный

в) $\overline{K_{m,n}} = K_m \cup K_n$



c) $\overline{C_n}$

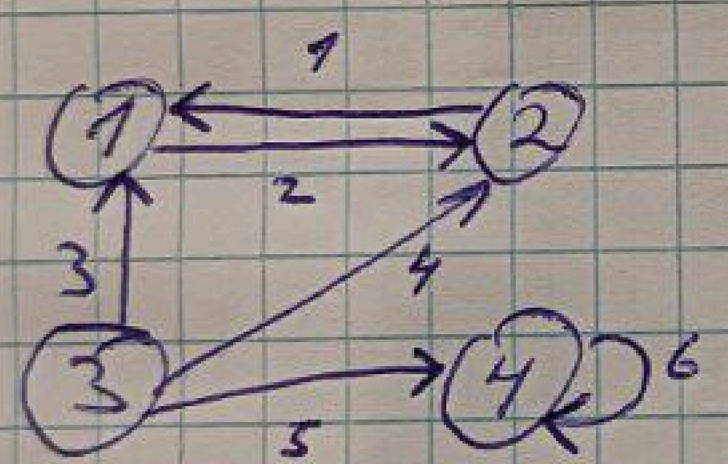


$$\frac{n(n-3)}{2}$$

Д/з №1
19, 33, 34, 35

Способы представления графов 2.0

Матрица инцидентности



	1	2	3	4	5	6
1	-1	1	-1	0	0	0
2	1	-1	0	-1	0	0
3	0	0	1	1	1	0
4	0	0	0	0	-1	1