Производные

$u = u(x)$

u' – произв. по x

1. $(c)' = 0$
2. $(cu)' = cu'$
3. $(u + v - w)' = u' + v' - w'$
4. $(uv)' = u'v + uv'$
5. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$
6. $(u^n)' = nu^{n-1}u'$

$y = f(u)$, $u = \varphi(x)$

$y'_x = f'_u \cdot \varphi'_x$

$u^v = e^{v \ln u} = e^{v \ln u} \cdot (v \ln u)' =$

$= u^v \left(v' \ln u + \frac{v \cdot u'}{u}\right)$

846

$$y = \frac{1+x-x^2}{1-x+x^2} = \frac{-x^2+x-1+2}{x^2-x+1} = -1+\frac{2}{x^2-x+1}$$

$$y' = \frac{2'(x^2-x+1)-2(x^2-x+1)'}{(x^2-x+1)^2} = \frac{-4x+2}{(x^2-x+1)^2}$$

847

$$y = \frac{x}{(1-x)^2(1+x)^3}$$

$$y' = \frac{x'(1-x)^2(1+x)^3 - x\left((1-x)^2(1+x)^3\right)'}{(1-x)^4(1+x)^6} =$$

$$= \frac{(1-x)^2(1+x)^3 - x\left(((1-x)^2)'(1+x)^3+(1-x)^2((1+x)^3)'\right)}{(1-x)^4(1+x)^6} =$$

$$= \frac{(1-x)^2(1+x)^3 - x\left(-2(1-x)(1+x)^3+3(1+x)^2(1-x)^2\right)}{(1-x)^4(1+x)^6} =$$

$$= \frac{(1-x)(1+x)+2x(1+x)-3x(1-x)}{(1-x)^3(1+x)^4} = \frac{1-x+4x^2}{(1-x)^3(1+x)^4}$$

852

$$y = \frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}} = x^{-1} + x^{-\frac{1}{2}} + x^{-\frac{1}{3}}$$

$$y' = -\frac{1}{x^2} - \frac{1}{2x^{-\frac{3}{2}}} - \frac{1}{3x^{-\frac{4}{3}}}$$

854

$$y = x\sqrt{1+x^2} = x \cdot (1+x^2)^{\frac{1}{2}}$$

$$y' = \sqrt{1+x^2} + x\left((1+x^2)^{\frac{1}{2}}\right)' = \sqrt{1+x^2} +$$

$$+ x \cdot \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot (1+x^2)' =$$

$$= \sqrt{1+x^2} + \cancel{x} \cdot \frac{1}{\cancel{2}} \cdot (1+x^2)^{-\frac{1}{2}} \cdot 2x =$$

$$= \frac{1+x^2+x^2}{\sqrt{1+x^2}}$$

858

$$y = \sqrt[3]{\frac{1+x^3}{1-x^3}} = \left(\frac{1+x^3}{1-x^3}\right)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-\frac{2}{3}} \cdot \frac{3x^2(1-x)^3 + (1+x)^3 3x^2}{(1-x^3)^2} =$$

$$= \sqrt[3]{\left(\frac{1\ x^3}{1-x^3}\right)^2} \cdot \frac{3x^2}{(1-x^3)^2} = \sqrt[3]{}$$

$$\left(\left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(x+x^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \left(1+\frac{1}{2}x^{-\frac{1}{2}}\right)$$

860

$$y = \sqrt{x+\sqrt{x+\sqrt{x}}} =$$

$$= \left(x+\left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}\left(x+\left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \left(1+\left(x+x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)' =$$

$$= \frac{1+\dfrac{1+\dfrac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

863 $y = (2-x^2)\cos x + 2x\sin x$

$y' = -2x\cos x - \sin x(2-x^2) + 2\sin x + 2x - x =$

$= (\sin x)(x^2)$

867 $y = \dfrac{\sin^2 x}{\sin x^2}$

$y' = \dfrac{\sin x^2 \cdot 2\sin x \cdot \cos x - \sin^2 x \cdot 2x \cdot \cos x^2}{(\sin x^2)^2}$

$f(u) = \sin u, \quad u = x^2$

$\sin(x^2) = \cos(x^2)(x^2)'$

$(\sin x)^2 = 2\sin x(\sin x)'$

$f(u) = u^2$

$u = \sin x$

870

$$y = \frac{\sin x - x\cos x}{\cos x + x\sin x}; \quad y' = x^2$$

$$y' = \frac{(\sin x - x\cos x)'(\cos x + x\sin x) - (\cos x + x\sin x)'(\sin x - x\cos x)}{(\cos x + x\sin x)^2}$$

$$= \frac{(\cos x - (1\cdot\cos x + x\cdot(-\sin x)))\cdot(\cos x + x\sin x)}{(\cos x + x\sin x)^2} -$$

$$- \frac{(-\sin x + (1\cdot\sin x + x\cdot\cos x))(\sin x - x\cos x)}{(\cos x + x\sin x)^2} =$$

$$= \frac{-x\cdot\sin x\cdot(\cos x + x\sin x) - x\cdot\cos x(\sin x - x\cos x)}{(\cos x + x\sin x)^2} =$$

$$= \frac{-x(\sin x(x\sin x + \cos x) + \cos x(\sin x - x\cos x))}{(x\sin x + \cos x)^2}$$

876

$(a^x)' = a^x \ln a$

$(e^x)' = e^x$

$e^{u(x)} = e^u (u)'$

$y = e^{-x^2}$

$y' = \left(e^{-x^2}\right)' = e^{-x^2} \cdot (-2x) = -2xe^{-x^2}$

890

$y = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$

$y' = \frac{1}{4} \cdot \frac{x^2+1}{x^2-1} \cdot \left(\frac{x^2-1}{x^2+1}\right)' =$

$= \frac{1}{4} \cdot \frac{x^2+1}{x^2-1} \cdot \frac{(x^2-1)'(x^2+1) - (x^2+1)'(x^2-1)}{(x^2+1)^2} =$

$= \frac{1}{4} \cdot \frac{x^2+1}{x^2-1} \cdot \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} =$

$= \frac{1}{4} \cdot \frac{x^2+1}{x^2-1} \cdot \frac{2x(x^2+1-x^2+1)}{(x^2+1)^2} =$

$= \frac{1}{4} \cdot \frac{x^2+1}{x^2-1} \cdot \frac{4x}{(x^2+1)^2} = \frac{x}{x^4-1}$

835

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$y' = \left(\ln\left(x + \sqrt{x^2+1}\right)\right)' =$$

$$= \frac{1}{x + \sqrt{x^2+1}} \cdot \left(x + \sqrt{x^2+1}\right)' =$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2}(x^2+1)^{-\frac{1}{2}}\right) =$$

$$= \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) =$$

901

$$y = \ln \operatorname{tg} \frac{x}{2}$$

$$y' = \left(\ln \operatorname{tg} \frac{x}{2}\right)' = \operatorname{ctg} \frac{x}{2} \cdot \frac{1}{2\cos^2 \frac{x}{2}} =$$

$$= \frac{\cos \frac{x}{2}}{\sin \frac{x}{2} \cdot 2 \cdot \cos^2 \frac{x}{2}} = \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \frac{1}{\sin x}$$

917

$$\boxed{(\operatorname{arctg} x)' = \frac{1}{1+x^2}}$$

$$y = \sqrt{x} - \operatorname{arctg} \sqrt{x}$$

$$y' = \frac{1}{2} x^{-\frac{1}{2}} - (\operatorname{arctg} \sqrt{x})' =$$

$$= \frac{1}{2\sqrt{x}} - \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}\left(1 - \frac{1}{1+x}\right) =$$

$$= \frac{1}{2\sqrt{x}} \cdot \frac{x}{1+x} = \frac{\sqrt{x}}{2+2x}$$

920

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos \frac{1}{x}$$

$$y' = -\frac{1}{\sqrt{1 - \frac{1}{x^2}}} \cdot (-1) \cdot \frac{1}{x^2} =$$

$$\left|\frac{1}{x\sqrt{x^2-1}}\right|$$

$$= \frac{1}{\sqrt{1 - \frac{1}{x^2}} \cdot x^2} = \frac{1}{\sqrt{\frac{x^2-1}{x^2}}\, x^2} = \pm \frac{1}{x\sqrt{x^2-1}}$$

961

$y = x + x^x + x^{x^x}$

$((1-x)^2)' = 2(1-x)\cdot(-1) = -2+2x$

Д/з: 963, 964, 848, 855, 859, 861, 864, 881, 889, 903, 905, 915, 928, 965

$y' = 1 + x^x(\ln x + 1) + (x^x)^x \cdot$

$\cdot\left(x\ln x + \dfrac{x\cdot x^x(\ln x+1)}{x^x}\right) =$

$= 1 + x^x(\ln x + 1) + x^{x^x}(2x\ln x$

$(u\cdot v)' = u'v + uv'$

$((2-x^2)(2-x^3))' = (2-x^2)'(2-x^3) + (2-x^2)(2-x^3)' =$

$= -2x(2-x^3) - 3x^2(2-x^2) = -4x + 2x^4 - 6x^2 + 3x^4 = 5x^4 - 6x^2 - 4x$

848 $y = \dfrac{(2-x^2)(2-x^3)}{(1-x)^2}$ $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$y' = \dfrac{((2-x^2)(2-x^3))'(1-x)^2 - (2-x^2)(2-x^3)((1-x)^2)'}{(1-x)^4} =$

$= \dfrac{(5x^4 - 6x^2 - 4x)(1-x)^2 - (2-x^2)(2-x^3)(1-x)\cdot(-2)}{(1-x)^4} =$

$= \dfrac{(5x^4-6x^2-4x)(1-x) + 2(2-x^2)(2-x^3)}{(1-x)^3}$

$(\sqrt{2+x^2}\cdot\sqrt{3+x^3})' = \dfrac{2x}{2\sqrt{2+x^2}}\cdot\sqrt{3+x^3} + \dfrac{3x^2}{2\sqrt{3+x^3}}\cdot\sqrt{2+x^2}$

855 $y = (1+x)\sqrt{2+x^2}\sqrt[3]{3+x^3}$

$y' = 1\cdot\sqrt{2+x^2}\cdot\sqrt[3]{3+x^3} + (1+x)(\sqrt{2+x^2}\sqrt[3]{3+x^3})' =$

$= \sqrt{2+x^2}\cdot\sqrt[3]{3+x^3} + (1+x)\dfrac{3x^3}{2}$

858

$$y = \frac{1}{\sqrt{1+x^2}\,(x+\sqrt{1+x^2})} \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{\sqrt{1+x^2}\,(x+\sqrt{1+x^2})}$$

$$y' = \frac{-\left(\sqrt{1+x^2}\,(x+\sqrt{1+x^2})\right)'}{(1+x^2)\,(x+\sqrt{1+x^2})^2} =$$

$$= \frac{-\left(\dfrac{x^2 + x\sqrt{1+x^2}}{\sqrt{1+x^2}} + \sqrt{1+x^2} + x\right)}{(1+x^2)\,(x+\sqrt{1+x^2})^2}$$

$$\left(\sqrt{1+x^2}\cdot(x+\sqrt{1+x^2})\right)' = \frac{2x\cdot(x+\sqrt{1+x^2})}{2\sqrt{1+x^2}} +$$

$$+ \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)\cdot\sqrt{1+x^2} =$$

$$= \frac{x^2 + x\sqrt{1+x^2}}{\sqrt{1+x^2}} + \sqrt{1+x^2} + x$$

861

$$y = \sqrt[3]{1+\sqrt[3]{1+\sqrt[3]{x}}}$$

$$y' = \frac{1}{3\sqrt[3]{(1+\sqrt[3]{1+\sqrt[3]{x}})^2}} \cdot \frac{1}{3\sqrt[3]{1+\sqrt[3]{x}}^{\,2}} \cdot$$

$$\cdot \frac{1}{3\sqrt[3]{x^2}} = \frac{1}{\sqrt[3]{\left((1+\sqrt[3]{1+\sqrt[3]{x}})(1+\sqrt[3]{x})\,x\right)^2}}$$

864

$y' =$

$+$

863

$y =$

$y' =$

$+$

<u>864</u>

$$y = \sin(\cos^2 x) \cdot \cos(\sin^2 x) \qquad (\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$

$$y' = \Big(\cos(\cos^2 x) \cdot \overbrace{(\cos^2 x)'}^{-2\cos(x)\sin(x)}\Big) \cdot \cos(\sin^2 x) +$$

$$+ \sin(\cos^2 x) \cdot \Big(-\sin(\sin^2 x) \cdot \underbrace{(\sin^2 x)'}_{2\cos(x)\sin(x)}\Big)$$

<u>863</u>

$$y = \frac{1}{x^x}$$

$$y' = \left(\frac{1}{x^x}\right)' = \left(\frac{1}{e^{x \ln x}}\right)' = e^{\frac{1}{x}\ln x} \cdot \Big(-\frac{1}{x^2}\ln x +$$

$$+ \frac{1}{x^2}\Big) = x^{\frac{1}{x}}$$

964 $u^{v} = e^{v \ln u} = e^{v \ln u} \cdot (v \ln u)' =$

$= u^{v}\left(v' \ln u + \frac{v u'}{u}\right)$

$y = (\sin x)^{\cos x} + (\cos x)^{\sin x} =$

$= e^{\cos x \ln \sin x} + e^{\sin x \ln \cos x} =$

$=$

Логарифмические производные

$y = f(x)$

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$$

$$\Rightarrow f'(x) = f(x) \cdot (\ln f(x))$$

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964 а

$$y = x\sqrt{\frac{1-x}{1+x}}$$

$$\ln y = \ln x\sqrt{\frac{1-x}{1+x}} = \ln x + \ln\sqrt{\frac{1-x}{1+x}} =$$

$$= \ln x + \frac{1}{2}(\ln(1-x)) - \ln(1+x))$$

$$(\ln y)' = \frac{1}{x} - \frac{1}{2} \cdot \frac{1}{1-x} - \frac{1}{2} \cdot \frac{1}{1+x}$$

Производная x по y , 1041

$y = f(x)$

$$x'_y = \frac{1}{y'_x}$$

$$\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases}$$

$$y'_x = [\psi(\varphi^{-1}(x))]' = \frac{y'_t}{x'_t}$$

$x = a\cos t$
$y = b\sin t$, (парам. задание $f(x)$)

$$\frac{dy}{dt} = b\cos t \quad \frac{dx}{dt} = -a\cos t$$

$$\frac{dy}{dx} = -\frac{b}{a}\operatorname{ctg} t$$

Функция в неявном виде

$$F(x, y(x)) = 0$$

Чтобы найти y'_x:

$$\frac{d}{dx} F(x, y(x)) = 0$$

1048 $x^2 + 2xy - y^2 = 2x$

$$x^2 + 2xy - y^2 - 2x = 0$$

$$F' = 2x + 2y + 2xy' - 2yy' - 2 = 0$$

$$2y'(x-y) = 2 - 2x$$

$$y' = \frac{1-x}{x-y}$$

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991 Показать, что функция

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{при } x \neq 0 \\ 0, & x = 0 \end{cases}$$

имеет разрывную производную.

$$f'(x) = \begin{cases} 2x \cdot \sin\frac{1}{x} + x^2 \cdot \cos\frac{1}{x} \cdot \left(-\frac{1}{x^2}\right) = 2x\sin\frac{1}{x} - \cos\frac{1}{x} \\ 0, & x = 0 \end{cases}$$

$$f'(0) = \lim_{\Delta x \to 0} \frac{f(0 + \Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x^2 \sin\frac{1}{\Delta x}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \Delta x \sin\frac{1}{\Delta x} = 0$$

$$\lim_{x\to 0} \left(2x \sin\frac{1}{x} - \cos\frac{1}{x}\right)$$

$$x_n = \frac{1}{\pi n} \to 0$$

Геометрический смысл производной

$y = f(x)$

Касательная, проходящая через (x_0, y_0)

$$y = f'(x_0)(x - x_0) + y_0$$

Нормаль к $f(x)$ в точке (x_0, y_0)

$$y = -\frac{1}{f'(x_0)}(x - x_0) + y_0$$

1055 (а, б) $\quad y = (1+x)\sqrt[3]{3-x}$

$$y' = \sqrt[3]{3-x} - \frac{1+x}{3\sqrt[3]{3-x}^{\,2}}$$

а) $A(-1; 0)$

Касательная к $(-1; 0)$:

$$y = \sqrt[3]{4}\,(x+1)$$

Нормаль:

$$y = -\frac{x+1}{\sqrt[3]{4}}$$

б) $B(2; 3)$

$$y = -\frac{1}{f(x_0)}(1 - x_0) + y_0$$

$$(y - y_0) f'(x_0) = -(x - x_0)$$

Кас.:

$y = 3$

Норм.:

$-(x - x_0) = 0; \quad x = 2$

1056

а) $y = 2 + x - x^2$

Касательная параллельна Ox

$y' = -2x + 1$

$f(x_0)' = 0$

$-2x_0 + 1 = 0$

$x_0 = \frac{1}{2}$; $y_0 = 2\frac{1}{4}$

б) касательная параллельна биссектрисе первого координатного угла

$f(x_0)' = 1$

$-2x_0 + 1 = 1$

$x_0 = 0$; $y_0 = 2$

Дифференциал:

$y = f(x)$

$\Delta y = f(x + \Delta x) - f(x) = \underbrace{A\Delta x}_{\text{дифференциал функции } y = f(x)} + O(\Delta x)$

функция переменной Δx

$dy = A \cdot \Delta x$ линейная часть приращения

$dy = f'(x)\Delta x = f'(x)dx$

если x - независимая переменная, то $dx = \Delta x$

$f(x + \Delta x) - f(x) \approx f'(x)\Delta x$

$dc = 0$

$d(cu) = c\,du$

$d(u + v - w) = du + dv - dw$

$d(uv) = v\,du + u\,dv$

$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$

$d(f(u)) = f'(u) \cdot du$

$y = f(x) = e^x$

$dy = f'(x)dx =$
$= e^x dx$

1050 (а, г, и)

а) $d(xe^x) = e^x dx + x\,de^x = e^x dx +$

$+ x\,\underbrace{e^x dx} = e^x(1+x)\underline{\underline{dx}}$

упрощаем вплоть до того, пока не появится дифференциал независимой переменной

г) $d\left(\frac{\ln x}{\sqrt{x}}\right) = \frac{\sqrt{x}\,d\ln x - \ln x\,d\sqrt{x}}{x} =$

$= \sqrt{x}\,\frac{dx}{x} \ldots$

$= \frac{(2 - \ln x)\,dx}{2\sqrt{x} \cdot x}$

11) $d \ln(1-x^2) = \ln(1-x^2)' \cdot d(1-x^2) =$

$= \frac{(-2x)}{1-x^2} \cdot (-2x) \cdot dx =$

$= \frac{4x^2}{1-x^2} \cdot dx$

1094 $\qquad \boxed{\arctan x = \frac{1}{1+x^2}}$

$y = \operatorname{arctg} \frac{u}{v}$

$d \operatorname{arctg} \frac{u}{v} = \frac{1}{1+\left(\frac{u}{v}\right)^2} \cdot d\left(\frac{u}{v}\right) =$

$= \frac{1}{1+\left(\frac{u}{v}\right)^2} \cdot \frac{v\,du - u\,dv}{v^2} =$

$= \frac{v\,du - u\,dv}{v^2+u^2}$

$f(x+\Delta x) - f(x) \approx f'(x)\Delta x$;

$f(x_0+\Delta x) \approx f(x_0) + f'(x_0)\Delta x$

1099 $\sqrt[3]{1{,}02}$

$f(x) = \sqrt[3]{x}$

$x_0 = 1$

$x_0 + \Delta x = 1{,}02$

$\Delta x = 0{,}02$

$\sqrt[3]{1{,}02} - \sqrt[3]{1} \approx$

$\approx f'(x_0) \cdot 0{,}02$

$f'(x) = \frac{1}{3}(x)^{-\frac{2}{3}}$

$f'(x_0) = \frac{1}{3}$

$f'(x_0) \cdot 0{,}02 = \frac{1}{3} \cdot \frac{2}{100} =$

$= \frac{2}{300} = \frac{1}{150}$

$\sqrt[3]{1{,}02} \approx 1 + \frac{1}{150}$

1100

$\sin 29°$

$f(x) = \sin x$

$x_0 = 30° = \frac{\pi}{6}$

$\Delta x = -1° = -\frac{\pi}{180}$

$f'(x) = \cos x$

$\sin 29° - \sin\frac{\pi}{6} \approx \cos\frac{\pi}{6}\left(-\frac{\pi}{180}\right) - \ln x\, d\sqrt{x} =$

$= \frac{\sqrt{3}}{2}\cdot\left(-\frac{\pi}{180}\right) = -\frac{\sqrt{3}\pi}{360}$

$\sin 29° \approx \frac{-\sqrt{3}\pi}{360} + \frac{1}{2}$

1113 Найти y'', если $y = e^{-x^2}$

$y' = e^{-x^2}\cdot(-2x)$

$y'' = (y')' = e^{-x^2}\cdot(-2) +$

$+ (-2x)\cdot e^{-x^2}(-2x) =$

1114

$y = \operatorname{tg} x$

$y' = \frac{1}{\cos^2 x}$

$y'' = \left(\frac{1}{\cos^2 x}\right)' =$

984 Логарифмическая производная $\frac{f'(x)}{f(x)}$

г) $y = (x + \sqrt{1+x^2})^n$

$y' = n(x + \sqrt{1+x^2})^{n-1} \cdot (x + \sqrt{1+x^2})' =$

$= n(x + \sqrt{1+x^2})^{n-1} \cdot (1 + \frac{1}{2\sqrt{1+x^2}} \cdot (1+x^2)') =$

$= n(x + \sqrt{1+x^2})^{n-1} \cdot (1 + \frac{2x}{2\sqrt{1+x^2}})$

1043

$\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t \end{cases}$ $\quad y'_x = \frac{y'_t}{x'_t}$

$\cancel{y'_t = a 3 \cos^2 t}$

$\cancel{x'_t = (a \cdot 3 \sin^2 t)}$; $\cancel{y'_x = \frac{a \cdot 3 \cdot \cos^2 t}{a \cdot 3 \cdot \sin^2 t} = \operatorname{ctg}^2 t}$

$y'_t = a 3 \sin^2 t \cdot \cos t$

$x'_t = a 3 \cos^2 t \cdot (-\sin t)$

$y'_x = \frac{a 3 \sin^2 t \cos t}{-a 3 \cos^2 t \sin t} = -\operatorname{tg} t$

1051

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ (парабола)

$\sqrt{x} + \sqrt{y} - \sqrt{a} = 0$

$F' = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y' - \frac{1}{2\sqrt{a}} = 0$

$y' = -\frac{\sqrt{y}}{\sqrt{x}}$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y' = 0$

$-\frac{1}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{1} = -\frac{\sqrt{y}}{\sqrt{x}}$

$y' =$

1102

$\operatorname{arctg} 1{,}05$

$f(x) = \operatorname{arctg}(x)$

$x_0 = 1 = \frac{\pi}{4}$

$\Delta x = 0{,}05 = \frac{\pi}{80}$

$f'(x) = \frac{1}{1+x^2}$

$\operatorname{arctg} \cancel{1{,}}\ \operatorname{arctg} 0{,}05 \approx$

984 •
1043 •
1051 •
992 x
1071 x
1083 x
1090 x
~~1095 x~~
1102

$dc = 0$

$d(cu) = c\,du$

$d(u + v - w) = du + dv - dw$

$d(uv) = v\,du + u\,dv$

$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$

$d(f(u)) = f'(u) \cdot du$

$y = f(x) = e^x$

$dy = f'(x)dx = e^x dx$

1090 (а, г, и)

а) $d(xe^x) = e^x dx + x\,de^x = e^x dx + x\underbrace{e^x dx} = e^x(1+x)\underline{dx}$

упрощаем вплоть до того, пока не появится дифференциал независимой переменной

г) $d\left(\frac{\ln x}{\sqrt{x}}\right) = \frac{\sqrt{x}\,d\ln x - \ln x\,d\sqrt{x}}{x} =$

$= \sqrt{x}\,\frac{dx}{x}$ . . .

$= \frac{(2 - \ln x)\,dx}{2\sqrt{x}\cdot x}$

1159 $y = \frac{x^2}{1-x}$ Найти $y^{(8)}$

$$y' = \frac{2x(1-x) + x^2}{(1-x)^2} = \frac{2x - 2x^2 + x^2}{(1-x)^2} =$$

$$= \frac{2x - x^2}{(1-x)^2} = \frac{(1-x)^2 + 1}{(1-x)^2} = -1 + \frac{1}{(1-x)^2}$$

$$y'' = \frac{2}{(1-x)^3} \qquad y''' = \frac{3!}{(1-x)^4}$$

$$y^{(8)} = \frac{8!}{(1-x)^9}$$

$$(uv)^{(n)} = \sum_{i=0}^{n} C_n^i u^{(i)} v^{(n-i)} \qquad C_{20}^2 = \frac{20!}{2!\cdot 18!} = 190$$

$$u^{(0)} = u \qquad C_{20}^1 = \frac{20!}{1!\cdot 19!} = 20$$

1161 $y = x^2 e^{2x}$; Найти $y^{(20)}$

$u = x^2$ $\qquad v = e^{2x}$ $\qquad (x^2 e^{2x}) = C_{20}^0 u^{(0)} v^{(20)} + C_{20}^1 u^{(1)} v^{(19)} + C_{20}^2 u^{(2)} v^{(18)} =$

$u' = 2x$ $\qquad v' = e^{2x} \cdot 2$

$u'' = 2$ $\qquad v'' = 2(e^{2x})' = 4e^{2x}$

$u''' = 0$ $\qquad v^{(k)} = 2^k e^{2x}$

$\ldots = 0$

$$= 1 \cdot x^2 \cdot 2^{20} e^{2x} + 20 \cdot 2x \cdot 2^{19} e^{2x} + 190 \cdot 2 \cdot 2^{18} e^{2x} =$$

$$= 2^{20} e^{2x} (x^2 + 20x + 95)$$

1173

$y = x\cos 2x$, найти $d^{10}y$

$(x\cos 2x)^{(10)} = C_{10}^{0} u^{(0)} v^{(10)} + C_{10}^{1} u^{(1)} v^{(9)} =$

$u = x$ $\quad v = \cos 2x$

$u' = 1$ $\quad v' = -2\sin 2x$

$u'' = 0$ $\quad v'' = -4\cos 2x$

$v''' = 8\sin 2x$

$v'''' = 16\cos 2x$

$v''''' = -32\sin 2x$

...

$v^{2k} = (-1)^{k}(2)^{2k}\cos 2x$

$v^{2k+1} = (-1)^{k}(-2^{2k+1})\sin 2x$

$= -x\,2^{10}\cos 2x + 10\,2^{9}\sin 2x$

$2^{10}(-x\cos 2x + 5\sin 2x)(dx)^{10}$

~~№6~~

~~Найти $y^{(n)}$~~

~~$y = \frac{ax+b}{cx+d}$~~

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$$

13/9 $\lim\limits_{x\to 0} \frac{\operatorname{ch} x - \cos x}{x^2}$

Правило Лопиталя

Работает для неопределённостей $\boxed{\frac{0}{0}}$, $\frac{\infty}{\infty}$

$f(x), g(x)$ – опр. и непр. в $U_\varepsilon(a)$, $a \in \bar{R}$

Если 1) $f, g \xrightarrow[x\to a]{} 0$

2) $\exists f', g'$ в $\overset{\circ}{U}_\varepsilon(a)$, $(f')^2 + (g')^2 \neq 0$

3) $\exists \lim\limits_{x\to a} \frac{f'}{g'}$ – конечный или бесконечный

Тогда: $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$

$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$

$\operatorname{ch}' x = \operatorname{sh} x$

$\operatorname{sh}' x = \operatorname{ch} x$

13/9

$$\lim_{x\to 0} \frac{\operatorname{ch} x - \cos x}{x^2} = \lim_{x\to 0} \frac{\operatorname{sh} x + \sin x}{2x} = \lim_{x\to 0} \frac{\operatorname{ch} x + \cos x}{2} = 1$$