POSSESION OF MOBILES IN EXAM IS UFM PRACTICE

Name Vishwakant   Enrollment No. 2110[redacted].

Jaypee Institute of Information Technology, Noida
T-1 Examination, Even 2023
B.Tech IV Semester

Course Title: Probability and Random Processes   **Maximum Time: 1 Hour**
Course Code: 15B11MA301   **Maximum Marks: 20**

After pursuing the course, students will be able to
CO1: explain the basic concepts of probability, conditional probability and Bayes' theorem.
CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.
CO3: apply some probability distributions to various discrete and continuous problems.
CO4: solve the problems related to the component and system reliabilities.
CO5: identify the random processes and compute their averages.
CO6: solve the problems on Ergodic process, Poisson process and Markov chain.

All questions are compulsory.

1. (i) If A and B are any two events such that $P(A \cup B) = \frac{2}{3}$, $P(\bar{A} \cup \bar{B}) = \frac{5}{6}$ and $P(\bar{A}) = 2P(B)$. Find $P(A), P(B)$ and $P(B/A)$. Are A and B independent? Justify your answer. **[CO1, 4 Marks]**

   (ii) A pair of fair dice is thrown. Find the probability that the sum is 10 or greater if a 5 appears on at least one of the dice. **[CO1, 2 Marks]**

2. A certain blood test declares that 83% of the time it is positive for patients having a certain disease and 21% of the time it is also positive in healthy people. In a certain location, 40% of the people have the disease, and anybody with a positive blood test is given a drug that cures the disease. If 17% of the time the drug produces a scar, what is the probability that a person from this location who has the scar had the disease in the first place? **[CO1, 3 Marks]**

3. (a)The revenue generated by selling newspaper in a week is a random variable X with the moment generating function as $M_X(t) = \frac{1}{(1-2500t)^4}$. Find the standard deviation of X. **[CO2, 2 Marks]**

   (b) The probability density function of a continuous random variable X is given by
   $$f(x) = \begin{cases} k(1-x^2); & 0 < x < 1 \\ 0 & ; \ elsewhere. \end{cases}$$
   Find **(i)** the value of k, **(ii)** variance of X and **(iii)** $P(0.4 < X < 0.6)$. **[CO2, 3 Marks]**

4. The joint probability mass function of two random variables X and Y is given by
   $$f_{XY}(x,y) = \begin{cases} \frac{1}{55}(2x+y^2); & x = 1,2,3 \quad y = 0,1,2 \\ 0 \qquad ; & otherwise. \end{cases}$$
   Find **(i)** $P(X \le 2, Y > 1)$, **(ii)** $P(X = 2/Y = 1)$. **[CO2, 3 Marks]**

5. Suppose that continuous random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} c, & x^2 \le y \le 1, \ 0 < x < 1 \\ 0 & otherwise. \end{cases}$
   Find value of c, $f_X(x)$ and conditional distribution of Y given $X = \frac{1}{2}$. **[CO2, 3 Marks]**

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