Jaypee Institute of Information Technology, Noida

T2 Examination, Even 2022

B.Tech. IV Semester

Course Title: Probability and Random Processes | **Maximum Time: 1 hr**

Course Code: 15B11MA301 | **Maximum Marks: 20**

After pursuing this course, the students will be able to:

CO1: explain the basic concepts of probability, conditional probability and Bayes theorem.

CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.

CO3: apply some probability distributions to various discrete and continuous problems.

CO4: solve the problems related to the component and system reliabilities.

CO5: identify the random processes and compute their averages.

CO6: solve the problems on Ergodic process, Poisson process and Markov chain.

Note: All questions are compulsory. The use of non-programmable calculator is allowed.

Q1. i. Prove that the Poisson distribution is the limiting case of binomial distribution.

ii. Suppose that an Indian army trainee soldier shoots a target according to geometric distribution. If the probability that a target is shoot in any shot is 0.8, find the probability that it takes an odd number of shots. *[CO3, 2+2 M]*

Q2. i. A company produces 5% defective items out of its total production. What is the probability that at least 5 items are to be examined in order to get 3 defective items?

ii. A random variable X is uniformly distributed in the interval $[-2, 2]$. Find the value of $P\left(|X-1| \geq \frac{1}{2}\right)$. [CO3, 2+1 M]

Q3. i. The time required (in hours) to repair a laptop is exponentially distributed with mean $\frac{1}{2}$. What is the probability that the repair time takes at least 10 hours given that its duration exceeds 9 hours?

ii. The marks obtained by a class of B. Tech. second year students in a mathematics course are found to be normally distributed with mean 64.5 and standard deviation 5. If the class strength is 300, then find the number of students having marks (a) less than 57, (b) between 57 to 72. Given that $P(0 < Z < 1.50) = 0.4332$ (Z – table value).

iii. It is given that X and Y are independent normal variate with $X \sim N(1, 4)$ and $Y \sim N(3, 16)$. Now if $4X - Y \sim N(\mu, \sigma^2)$, then find its mean ($\mu$) and variance ($\sigma^2$). [CO3, 2+2+1 M]

Q4. The probability density function of the time to failure (in years) of an electronic machine is given by $f(t) = \frac{2}{(t+1)^3}, t \geq 0$. Find

a. the reliability of the machine for first year of operation,

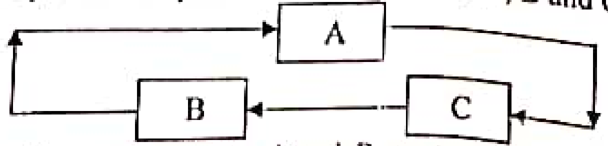
b. the mean time to failure (MTTF),

c. the design life of machine for a reliability of 0.25,

d. the reliability of the machine for 2 years of operation given that the burn-in period is of one year. [CO4, 4M]

Q5. i. A system, consisting of two components in series, is designed to operate for 100 days. The first component has a Weibull failure distribution with shape parameter 1.2 and scale parameter 840 days while the second component has a constant failure rate of 0.0001. Compute the reliability of the system.

ii. Consider the system composed of 3-components A, B and C as given in the following figure:



It is given that the components A and B are equally reliable and the reliability of C is 0.95. If the reliability of system is 0.50, what will be the reliability of component A?. [CO4, 2+2 M]