Jaypee Institute of Information Technology, Noida
T1 Examination, Even 2022
B. Tech. IV Semester

Course Title: Probability and Random Processes — **Maximum Time: 1 Hr**
Course Code: 15B11MA301 — **Maximum Marks: 20**

After pursuing the course, the students will be able to:

CO1: explain the basic concepts of probability, conditional probability and Bayes theorem.
CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.
CO3: apply some probability distributions to various discrete and continuous problems.
CO4: solve the problems related to the component and system reliabilities.
CO5: identify the random processes and compute their averages.
CO6: solve the problems on ergodic process, Poisson process and Markov chain.

Note: All questions are compulsory. The use of non programmable calculator is allowed.

Q1 (i) Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are black or both are queen? [CO1, 2M]

(ii) There are 7 tickets marked with numbers from 11 to 17. If three tickets are selected at random, find the probability that the numbers are in arithmetic progression. [CO1, 2M]

Q2 The physical development of UG students of an institute has been investigated through various physical aptitude tests. Let E be the event that the student passes the test, B be the event that a boy student takes the test, G be the event that a girl student takes the test. Due to hard drive failure of computer storing the data, only partial data was recovered, which is shown in the table below.

P(E)	P(B)	P(G)	P(E|B)	P(E|G)	P(B|$\bar{E}$)	P(G|P)
0.3[illegible]	a	b	0.35	0.40	c	d

Here P(E) represents the probability that the event E occurs. Using the knowledge of probability theory, find the missing values a, b, c and d. [CO1, 3M]

Q3 (i) A lucky draw casket contains 2^n coupons. Out of these, ${}^nC_i$ coupons carry the number i; $i = 0,1,2,\dots,n$. Find the expected value of the sum of numbers, if a group of 'm' coupons is drawn from the casket. [CO1, 2M]

(ii) The cumulative distribution function (CDF) of a random variable X is defined by

$$F(X)=\begin{cases}0, & x<1\\ A(x-1), & 1\le x\le 4,\\ 1, & x>4\end{cases}$$

Find (a) the value of A and (b) $P(1\le X\le 3)$. [CO1, 2M]

Q4 Let the joint density function of X and Y is given by $f(x,y)=\begin{cases}kxy, & for\ 0<x<y<1, x+y\le 1\\ 0, & otherwise.\end{cases}$

Find (i) the value of k, (ii) $f_X(x), f_Y(y)$, (iii) $E(Y\mid X=x)$. [CO2, 4M]

Q5 (i) The joint probability mass function of two dimensional random variable (X, Y) is given as follows:

X \ Y	0	1
-1	1/8	2/8
1	3/8	2/8

Find the correlation coefficient between X and Y. [CO2, 3M]

(ii) The characteristic function of a random variable X is $e^{-2|\omega|}$, find $P(1\le X\le 3)$. [CO2, 2M]

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