# End Term Examination, Even Sem 2021-2022
## B.Tech. IV Semester

**Course Title: Probability and Random Processes** — **Maximum Time: 2 Hrs.**
**Course Code: 15B11MA301** — **Maximum Marks: 35**

After pursuing this course student will be able to:

CO1: explain the basic concepts of probability, conditional probability and Bayes' theorem.

CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.

CO3: apply some probability distributions to various discrete and continuous problems.

CO4: solve the problems related to the component and system reliabilities.

CO5: identify the random processes and compute their averages.

CO6: solve the problems on Ergodic process, Poisson process and Markov chain.

**Note:** Attempt all the questions. The use of non programmable calculator is allowed.

**Q1.** (a) Assume that a noisy channel independently transmits symbols, say 0's, 70% of the time and 1's, 30% of the time. At the receiver, there is 1% chance of obtaining any particular symbol distorted. What is the probability of receiving a 1, irrespective of which symbol is transmitted? **[2, CO1]**

(b) The probabilities of the events $A$ and $B$ associated to a random experiment are given as follows: $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$, $P(not\ B) = \frac{1}{2}$.
Are the events $A$ and $B$ independent? Prove your answer. **[2, CO1]**

**Q2.** The cumulative distribution function of a random variable $X$, taking values -3, 0, 3 and 4, is given by

$$F(x) = \begin{cases} 0, & if\ x < -3 \\ \frac{3}{8}, & if -3 \le x < 0 \\ \frac{1}{2} & if\ 0 \le x < 3 \\ \frac{3}{4} & if\ 3 \le x < 4 \\ 1 & if\ x \ge 4. \end{cases}$$

Calculate $E(X), E(X^2 - 2|X|)$ and $E(X|X|)$. **[4, CO2]**

**Q3.** (a) Find the probability of failure free performance of the roller bearings over a period of 5000 hours, if the life expectancy of the bearings is defined by Weibull distribution with parameters $\alpha = 10^{-7}$ and $\beta = 2$. **[2, CO3]**

(b) Find the moment generating function of the Erlang distribution. **[2, CO3]**

**Q4.** Eight identical components with constant failure rates are connected in

(i) high level redundancy with 4 components in each subsystem.

(ii) low level redundancy with 2 components in each subsystem.

Determine the component MTTF in each case necessary to provide a system reliability of 0.84 after 100 hours of operation. **[4, CO4]**

PTO

**Q5.** **(i)** Let $X(t) = A + B\sin(\omega t + \varphi)$, where $A, B$ and $\phi$ are independent random variables and $\omega$ is a constant. The variable $\phi$ is uniformly distributed in $(0, 2\pi)$; $A$ and $B$ are uniformly distributed in $(0,\ 2)$. Is $\{X(t)\}$ a WSS process? Justify your answer. **[4, CO5]**

**(ii)** Find the spectral density function of a WSS process $\{X(t)\}$ whose auto correlation function is $e^{-\left(\frac{\alpha^2\tau^2}{4}\right)}$, where $\alpha$ is a constant. **[3, CO5]**

**(iii)** Find the mean and variance of a random process $\{X(t)\}$ whose auto correlation function is $R(\tau) = 36 + \frac{4}{1+3\tau^2}$. **[2, CO5]**

**Q6.** **(a)** Cell phone calls processed by a certain wireless station arrive according to a Poisson process with an average of 12 per minute. Find the probability that

(i) more than 3 calls arrive in an interval of length 2 minutes.

(ii) exactly 2 calls arrive during a 1 minute interval. **[3, CO6]**

**(b)** A WSS process $\{X(t)\}$ is given by $X(t) = 5\cos(25\,t + \beta)$ where $\beta$ is uniformly distributed over $[-\pi, \pi]$. Prove that $\{Y(t)\}$ is a correlation ergodic process. **[4, CO6]**

**(c)** A man goes to his office by three means, namely by bus, train and own car. He never goes by same mean in successive days. If he goes by bus, the next day he is twice likely to go by train, than by car. However if he goes by train or car, the next day he is equally likely to go by other means. Considering it to be a Markov process, write the transition probability matrix (TPM) of it and classify the states of the Markov chain. **[3, CO6]**

**********