POSSESSION OF MOBILES IN EXAM IS UFM PRACTICE

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Jaypee Institute of Information Technology, Noida
T-2 Examination, Even Sem 2023
B. Tech. IV Semester

Course Title: Probability and Random Processes **Maximum Time: 1 Hour**
Course Code: 15B11MA301 **Maximum Marks: 20**

After pursuing the course, students will be able to
CO1: explain the basic concepts of probability, conditional probability and Bayes' theorem.
CO2: identify and explain one and two dimensional random variables along with their distributions and statistical averages.
CO3: apply some probability distributions to various discrete and continuous problems.
CO4: solve the problems related to the component and system reliabilities.
CO5: identify the random processes and compute their averages.
CO6: solve the problems on Ergodic process, Poisson process and Markov chain.

All questions are compulsory.

1. The number of flaws on a magnetic cassette tape produced continuously at a factory follows Poisson distribution with parameter $\lambda = 2.5$. What is the probability that there are
   (i) at least two flaws in a single cassette tape?
   (ii) exactly six flaws in a single cassette tape?
   [CO3, 3 Marks]

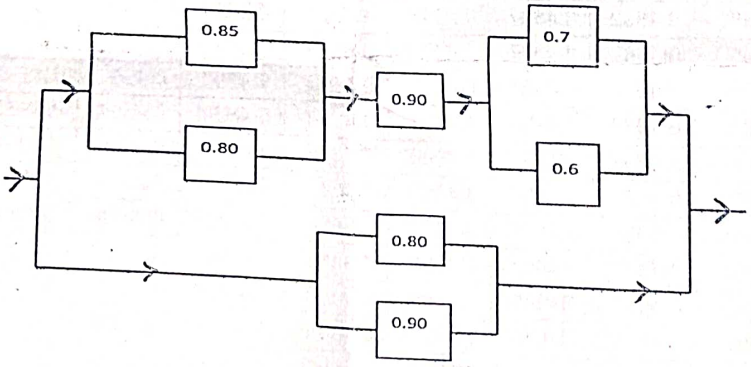
2. (i) Identify the distribution and its parameter if the M.G.F. of the random variable X is given by $M_X(t) = e^t(5 - 4e^t)^{-1}$. **[CO3, 2 Marks]**
   (ii) The digits after the decimal point of a random number between 0 and 1 are numbers selected at random, with replacement, independently, and successively from the set $\{0, 1, 2, 3, \ldots, 9\}$. In a random number from $(0, 1)$, on the average, how many digits are there before the fifth 3? **[CO3, 1 Mark]**

3. Suppose that the time (in hours) taken by a mechanic to repair a machine is a random variable X following Erlang distribution with parameters $k = 2, \lambda = 2$. Find the probability that repair takes
   (i) at most 4 hours,
   (ii) at least 3 hours,
   (iii) between 1 and 3.5 hours.
   [CO3, 3 Marks]

4. The marks scored by the students of B. Tech. II year in the course 'Probability and random processes' of a reputed institute follows normal distribution with a mean of 527 and standard deviation 112.
   (i) What is the probability of a particular student scoring above 500?
   (ii) At least how many marks must a student score to be in top 5% ?
   [CO3, 3 Marks]

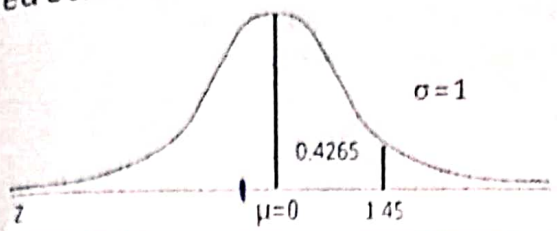
5. The density function of time to failure (in years) of an appliance is given by
   $$f(t) = 2\,a\,t\,e^{-a\,t^2}, \quad a > 0, \quad t > 0$$
   Find (i) the reliability function $R(t)$
   (ii) MTTF
   (iii) Hazard rate function $\lambda(t)$
   (iv) the design life for a reliability of e^{-7a}, given 3 years of wear-in period. **[CO4, 4 Marks]**

6. Calculate the reliability of the following system- **[CO4, 4 Marks]**



Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score. For example, when Z score = 1.45 the area = 0.4265.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

$$\frac{\frac{1}{2}e^t}{5-4e^t} = \frac{\frac{1}{2}e^t}{5\left(1-\frac{4}{5}e^t\right)} = \frac{\frac{1}{2}e^t}{1-\frac{4}{5}e^t}$$