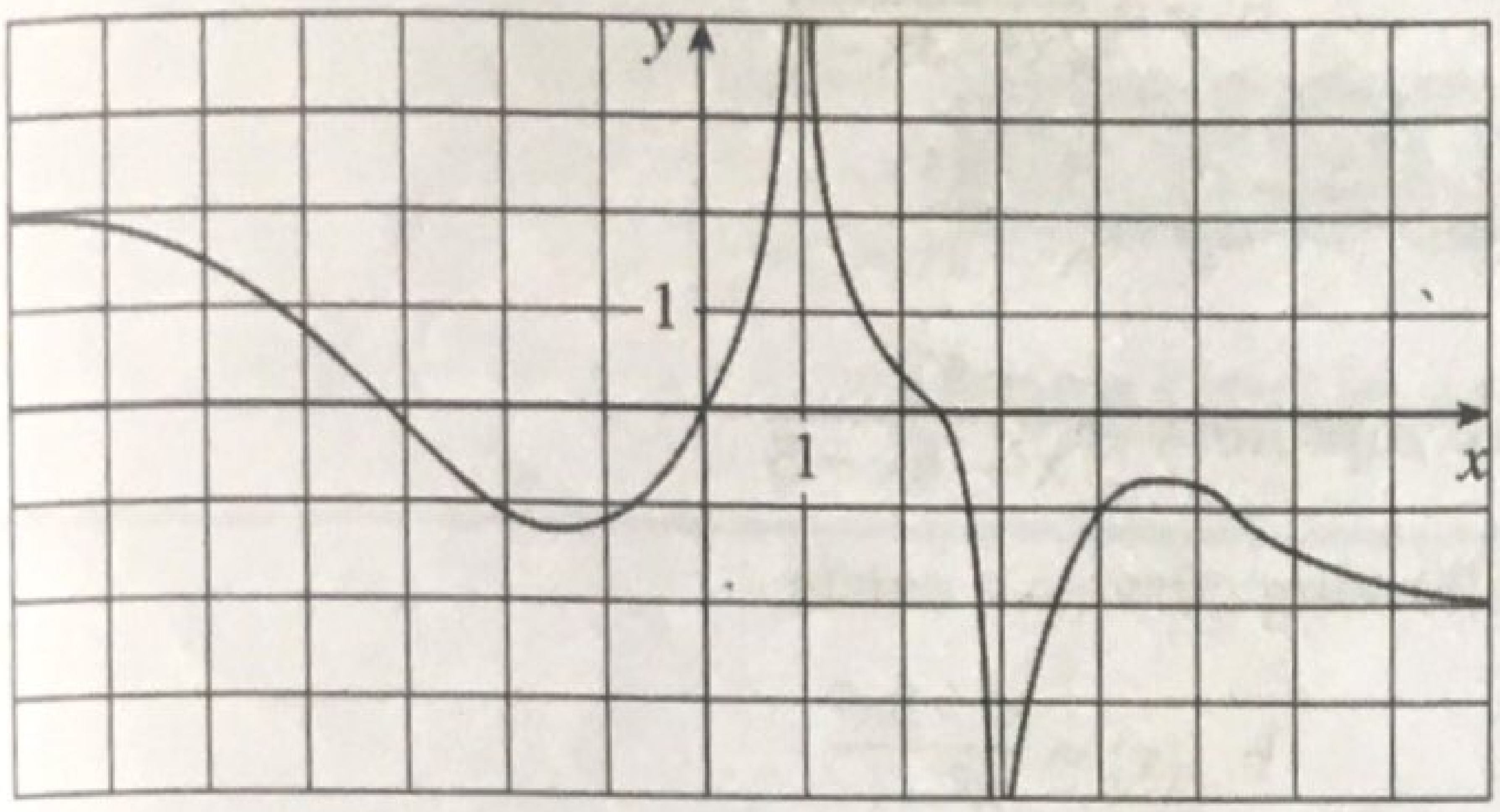
1. Explain in your own words the meaning of each of the following

(a) $\lim_{x \to \infty} f(x) = 5$ (b) $\lim_{x \to \infty} f(x) = 3$

2. For the function f whose graph is given, state the following:

(a) $\lim_{x \to \infty} f(x)$ (b) $\lim_{x \to -\infty} f(x)$ (c) $\lim_{x \to 1} f(x)$

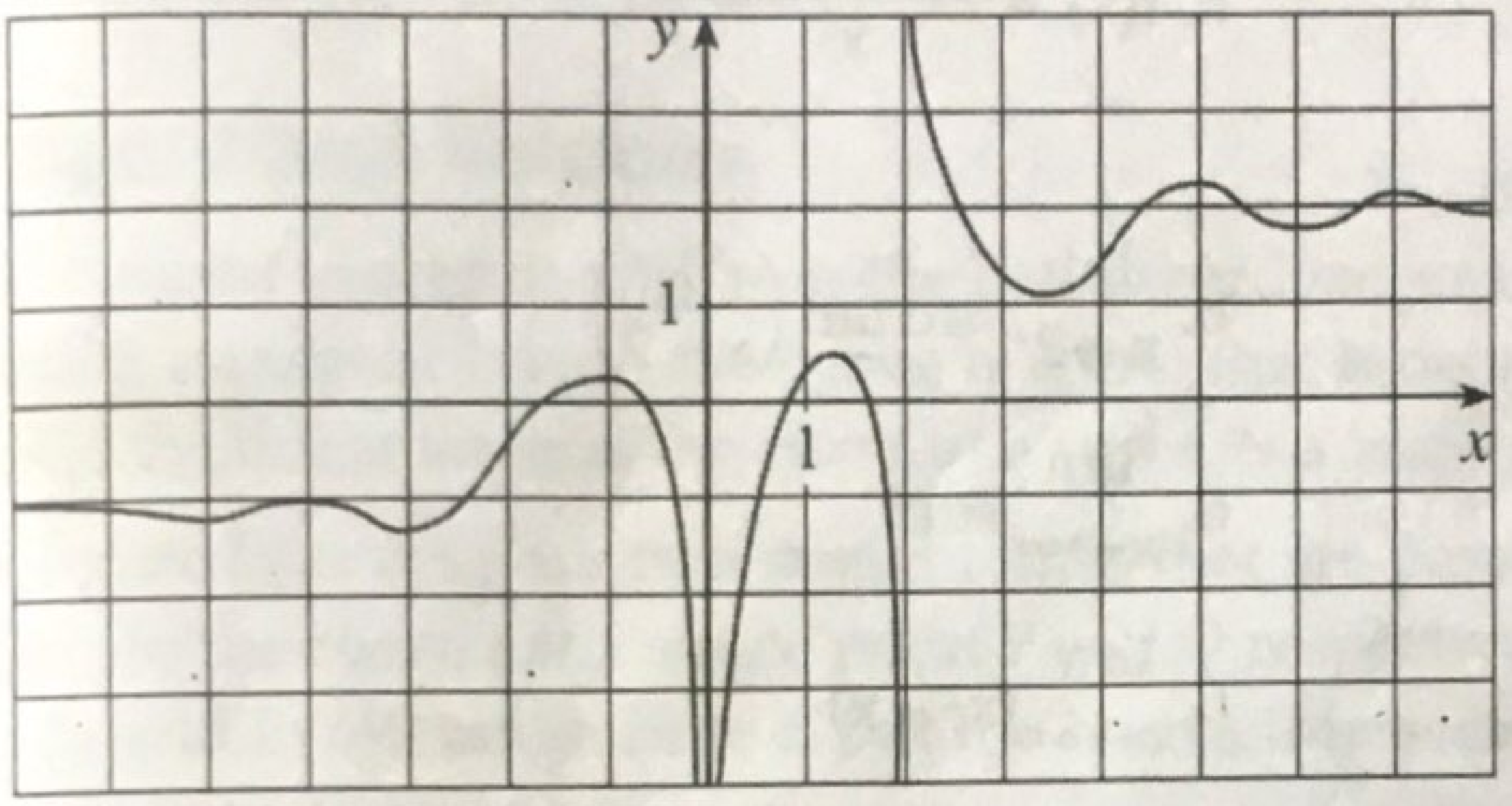
(d) $\lim\limits_{x\to 3} f(x)$ (e) the equation of the asymptotes?



3. For the function of whose graph is given, state the following:

(a) $\lim\limits_{x\to\infty} g(x)$ (b) $\lim\limits_{x\to -\infty} g(x)$ (c) $\lim\limits_{x\to 0} g(x)$

(d) $\lim\limits_{x\to 2-} g(x)$ (e) $\lim\limits_{x\to 2+}$ (f) the equations of asymptotes



4. Sketch the graph of an example of a function f that satisfies all of the given conditions

(a) $\lim\limits_{x\to 0} f(x) = -\infty$, $\lim\limits_{x\to -\infty} f(x) = 5$, $\lim\limits_{x\to\infty} f(x) = -5$

5. Guess the value of limit $\lim\limits_{x\to\infty} \frac{x^2}{2^x}$ by evaluating the function $f(x) = \frac{x^2}{2^x}$ for x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 50 and 100. Then use the graph to support your guess.

6. Evaluate the limit and justify each step by indicating the properties of limits

a. $\lim\limits_{x\to\infty} \frac{3x^2 - x + 14}{2x^2 + 5x - 8}$ b. $\lim\limits_{x\to\infty} \sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}}$

7. Find the limit or show that it does not exists:

a. $\lim\limits_{x\to\infty} \frac{3x - 2}{2x + 1}$ b. $\lim\limits_{t\to\infty} \frac{\sqrt{t + t^2}}{2t - t^2}$

c. $\lim\limits_{x\to\infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$ d. $\lim\limits_{x\to\infty} (\sqrt{9x^2 + x} - 3x)$

e. $\lim\limits_{x\to\infty} \arctan(e^x)$

8. Find the horizontal and vertical asymptotes of each curve.

a. $y = \dfrac{2x+1}{x-2}$ b. $y = \dfrac{x^2+1}{2x^2-3x-2}$

c. $y = \dfrac{2x^2+x-1}{x^2+x-2}$ d. $y = \dfrac{1+x^4}{x^2-x^4}$

9. Find the vertical and horizontal asymptote: $y = \dfrac{x^3-x}{x^2-6x+5}$

10. Find the slant asymptote of the following curves: if exists.

a. $f(x) = \dfrac{2x^2}{1-x}$ b. $f(x) = \dfrac{x^3-3x^2}{x^2-1}$

c. $f(x) = \dfrac{(2+x)(2-3x)}{(2x+3)^2}$ d. $f(x) = \dfrac{x^3-1}{x^2-x-2}$

e. $f(x) = \dfrac{x^2-2x}{x^3+1}$ f. $f(x) = \dfrac{1-x^3}{x}$

g. $f(x) = \dfrac{x^3-1}{2(x^2-1)}$ h. $f(x) = \dfrac{x^4-2x^3+1}{x^2}$

11. Evaluate the following limits:

a. $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x$ b. $\lim\limits_{x\to 2^+} \arctan\left(\dfrac{1}{x-2}\right)$

c. $\lim\limits_{x\to 0^-} e^{1/x}$ d. $\lim\limits_{x\to\infty} \sin x$

e. $\lim\limits_{x\to\infty} x^3$ and $\lim\limits_{x\to-\infty} x^3$ f. $\lim\limits_{x\to\infty} (x^2 - x)$

g. $\lim\limits_{x\to\infty} \dfrac{x^2+x}{3-x}$

Answers:

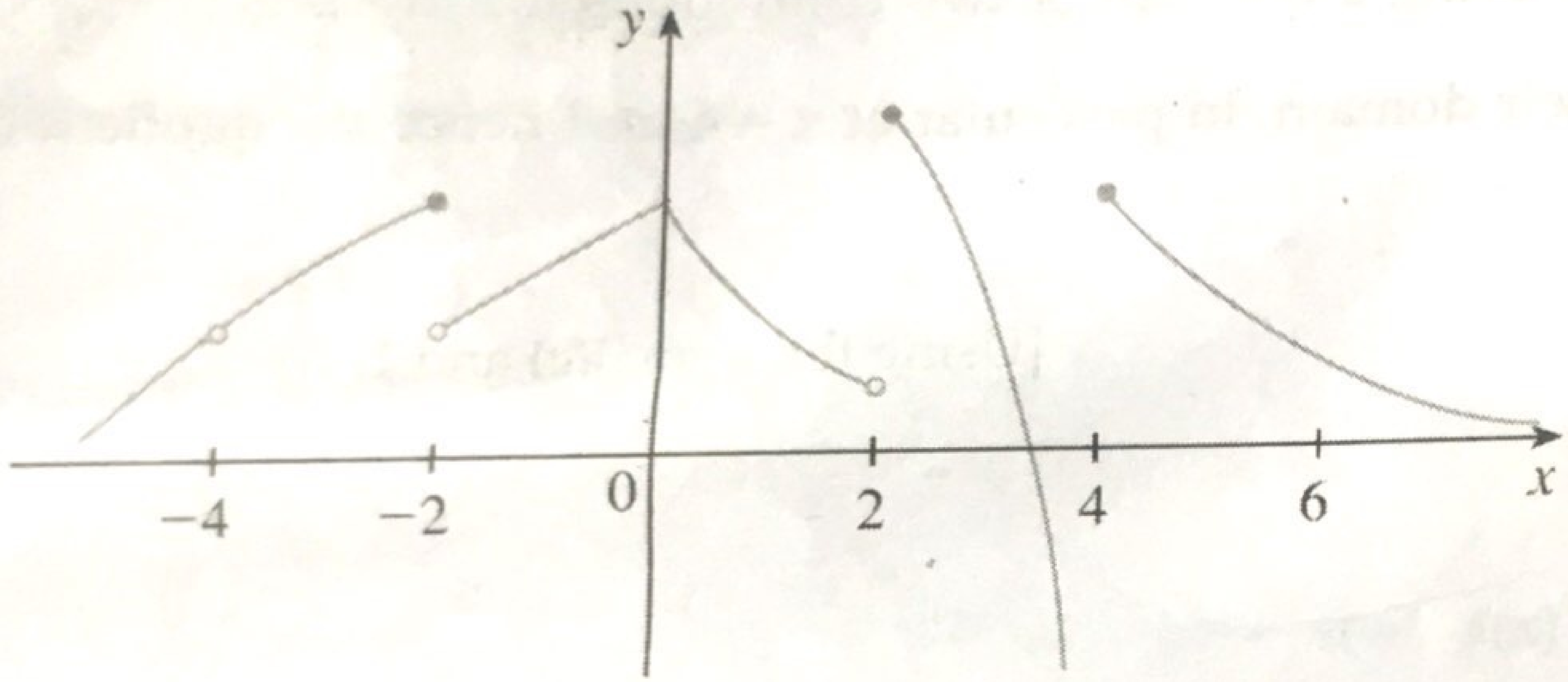
1. (a) As x increases toward positive infinity, the value of f(x) becomes very close to 5 yet never reaches it.
2. (a) $\lim\limits_{x\to\infty} f(x) = -2$ (b) $\lim\limits_{x\to-\infty} f(x) = 2$ (c) $\lim\limits_{x\to 1} f(x) = \infty$ (d) $\lim\limits_{x\to 3} f(x) = -\infty$ (e) $x = 1, x = 3, y = 2, y = -2$
3. (a) 2 (b) -1 (c) $-\infty$ (d) $-\infty$ (e) ∞ (f) $x = 0, x = 2, y = -1, y = 2$
6. (a) 3/2 (b) 2
7. (a) 3/2 (b) -1 (c) 3 (d) 1/6 (e) $\pi/2$
8. (a) Vertical asymptote; $x = 2$, Horizontal asymptote $y = 2$
   (b) Vertical asymptote; $x = -1/2, x = 2$, Horizontal asymptote $y = 1/2$
   (c) Vertical asymptote; $x = 1, x = -2$, Horizontal asymptote $y = 2$
   (d) Vertical asymptote; $x = 0, x = 1, x = -1$, Horizontal asymptote $y = -1$
9. Vertical asymptote; $x = 5$, No horizontal asymptote
10. (a) $y + 2x + 2 = 0$ (b) $y = x - 3$ (c) Does not exist (d) $y = x + 1$ (e) Does not exist (f) $y = -x^2$ (g) $y = \dfrac{x}{2}$ (h) $y^2 = x^2 - 2x$
11. (a) 0 (b) $\pi/2$ (c) 0 (d) Does not exist (e) $\infty, -\infty$ (f) ∞ (g) $-\infty$

1. Use the continuity to evaluate $\lim_{x \to \pi} \frac{\sin x}{2 + \cos x}$.

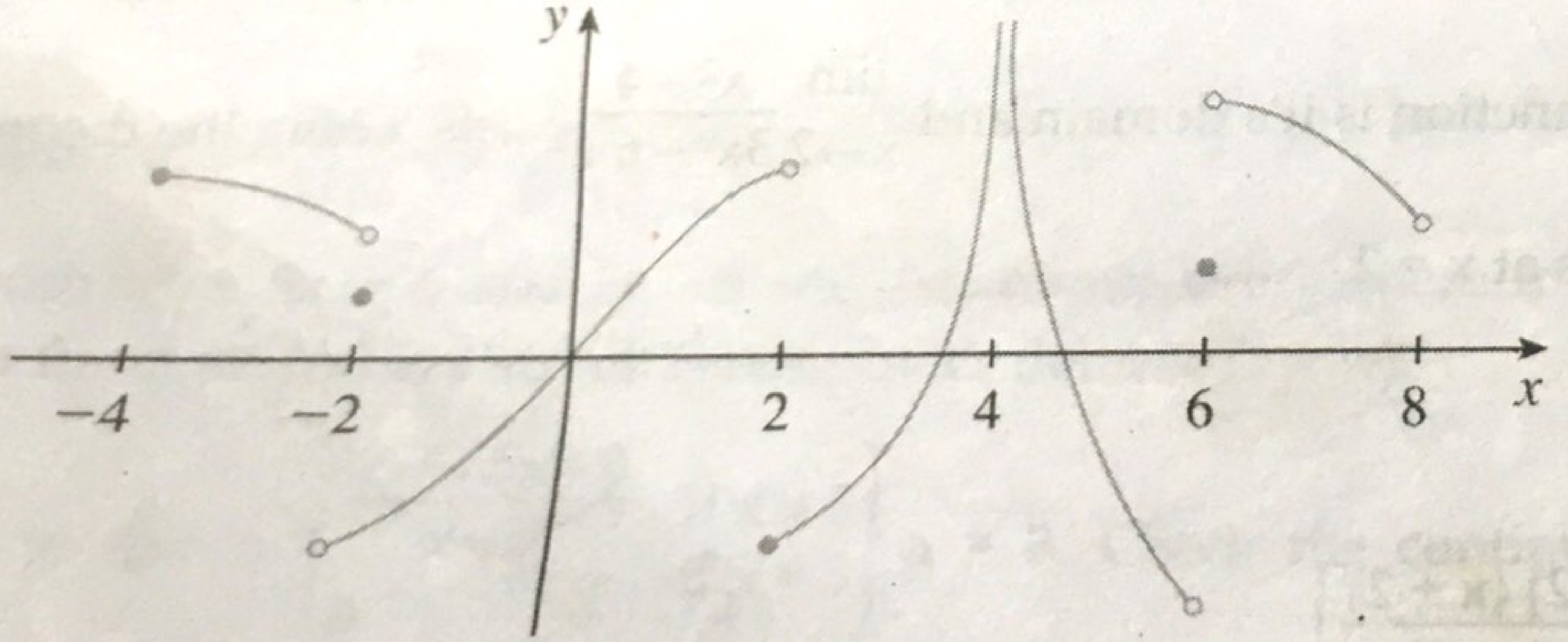
2. If f is continuous on (-∞, ∞), what can you say about it's graph?

3. (a) From the graph of f. State the numbers at which f is discontinuous and explain why.

(b) For each of the numbers stated in part (a), determine whether f is continuous from the right, or from the left.



4. From the graph of g. State the intervals on which g is continuous.



Sketch graph of a function f that is continuous expect for the stated discontinuity.

a. Discontinuous, but continuous from the right at 2.

b. Removable discontinuity at 3, Jump discontinuing at 5.

5. Use the definition of continuity and the properties of limits to show that the function is continuous at the given number *a*.

(i) $f(x) = 3x^4 - 5x + 3\sqrt{x^2+4}$, $a = 2$

(ii) $f(x) = (x + 2x^3)^4$, $a = -1$

(iii) $h(t) = \dfrac{2t - 3t^2}{1 + t^3}$, $a = 1$

6. Explain, using the theorem of continuous functions why the functions are continuous of every number in it's domain. State the domain.

a. $G(x) = \dfrac{x^2 + 1}{2x^2 - x - 1}$

b. $f(x) = \dfrac{\sqrt[3]{x - 2}}{x^3 - 2}$

Answers:

1. 0

3. (a) $f(-4)$ is not defined and $\lim\limits_{x \to a} f(x)$ for $a = -2.2$ and 4 does not exists (b) -4, neither; -2 left; 2, right; 4, right

6. (a) $D = (-\infty, \infty) - \{1, -1/2\}$ (b) $D = (-\infty, \sqrt[3]{2}) \cup (\sqrt[3]{2}, \infty)$