Chapter :- 2

Q1. Define magnetic dipole moment. Derive an expression for torque on a current loop in terms of dipole moment and applied magnetic field.

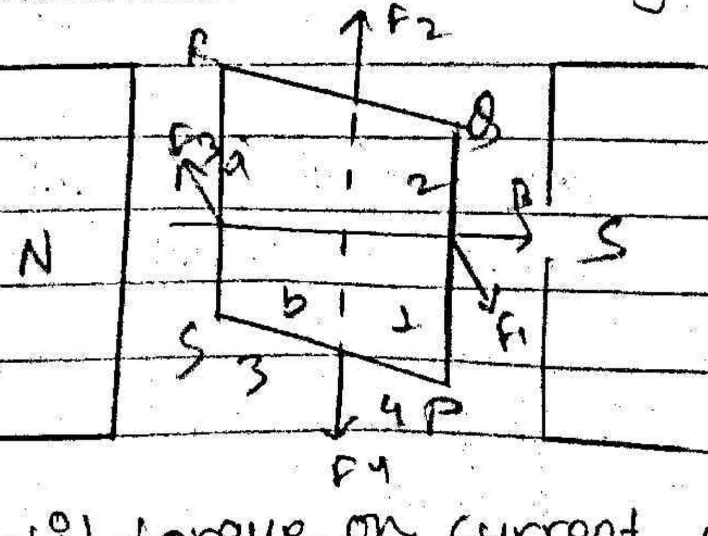
→ First part:-

The magnetic dipole moment μ is defined as product of current through the loop and the area of loop.

Mathematically,

$$\mu = IA$$

where, A = Area of loop,
I = Current through the loop.



fig(i) torque on current carrying rectangular loop of wire on pivot rod in a magnetic field

Let us consider a rectangular loop of PQRS of length 'a' and breadth 'b' connected to a pivot rod and suspended in uniform magnetic field '$\vec{B}$' as shown in fig(i).

According to flemming's left hand rule for the sides 2 and 4 magnitude of force are same but directions are opposite and their line of action is same.

$$|\vec{F_2}| = |\vec{F_4}|$$

$\therefore$ There is no net force on sides 2&4.

For sides 1 and 3 magnitudes of force are same.

$$|\vec{F_1}| = |\vec{F_3}| = I(\vec{a} \times \vec{B})$$
$$= I\, aB\sin\theta$$
$$= IaB\sin 90^\circ$$
$$= IaB.$$

where θ is angle between a & b.

$\therefore \theta = 90^\circ$ (from fig).

$$\therefore |\vec{F_1}| = |\vec{F_3}| = IaB \quad \text{--- (i)}.$$

According to flemming's left hand rule the force, F_1 & F_3 acts in opposite direction but there line of actions are different.

$\therefore$ The resultant force of $\vec{F_1}$ & $\vec{F_3}$ is not zero and they form a couple called a deflecting couple.

The torque is given by,

Torque(C) = magnitude of force × perpendicular distance betn line of two force.

$= IaB \times b\sin\theta$

$= IabB\sin\theta$

$\therefore C = IAB\sin\theta$

In place of single current loop if we take a current carrying coil of N terms then torque acting on coil.

Case-I

$\therefore$ If $\theta = 90°$ i.e A and B are perpendicular

$\therefore \tau_{max} = BIN_{max}$.

Case-II

If $\theta = 0°$, i.e A and B are parallel

$\therefore \tau_{min} = 0$

we have,

$M = IA$

$\therefore \tau = MB\sin\theta$

$$\boxed{\therefore C = \vec{\mu} + \vec{B}} \quad \text{(III)}$$

Eqn (III) is the required expression for torque on a current loop in terms of dipole moment & applied magnetic field.

Q. Derive an expression for magnetic energy of a dipole placed in a uniform magnetic field.

→ The magnetic dipole moment is the product of current through the loop and the area of loop. Mathematically,

$$M = IA \quad \text{--- (1)}$$

where,

A = area of loop

I = current flow through the loop.

we have,

torque experienced by rectangular current carrying wire placed in magnetic field as.

$$\tau = IBA\sin\theta \quad \text{--- (11)}$$

Using eqn (1) & (11), we get

$$\tau = MB\sin\theta \quad \text{--- (111)}$$

In vector form, eqn (111) can be written as

$$|\vec{\tau}| = |\vec{M} \times \vec{B}|$$

the magnetic dipole experienced a torque when placed in an external field, work must be done to change its orentation and this workdone is reffered as energy of dipole.

The term energy is given by.

$$U = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$

$$= \int_{\theta_i}^{\theta_f} \mu B \sin\theta \, d\theta.$$

Here, we said

$U = 0$, $\theta = 90°$ i.e when dipole vector is perpendicular to the magnetic field.

$$= \int_{90°}^{\theta} \mu B \sin\theta \, d\theta$$

$$= \mu B \int_{90}^{\theta} \sin\theta \, d\theta$$

$$\therefore U = -\mu B \cos\theta$$

eqn (iv) can be expressed as dot product

$$\boxed{U = -\vec{\mu} \cdot \vec{B}} \quad \text{—— (V)}$$

eqn (V) is the required expression for a magnetic energy of a dipole placed in a uniform magnetic field.

Case I:-

when $\theta = \pi$

i.e μ & B are anti aligned

$U_{max} = -\mu B \cos\pi$

Case II:-

when $\theta = 0°$

i.e μ & B are aligned

$U_{min} = -\mu B.$

Case III :-

When $\theta = 90°$

i.e μ and B are perpendicular to each other.

$$U = -\mu B \cdot \cos 90°$$
$$= 0.$$

Q3. What is Hall effect? Derive an expression for hall coefficient and establish the relation with mobility of charge carryers and conductivity of material of wire. Also explain how resistance varies with magnetic field.

→ First part:-

When a magnetic field is applied perpendicular to a current carrying conductor a voltage developed across the conductor in a direction perpendicular to both current and magnetic field. This phenomenon is called hall effect. The voltage so, developed is called Hall voltage.

Second part:-

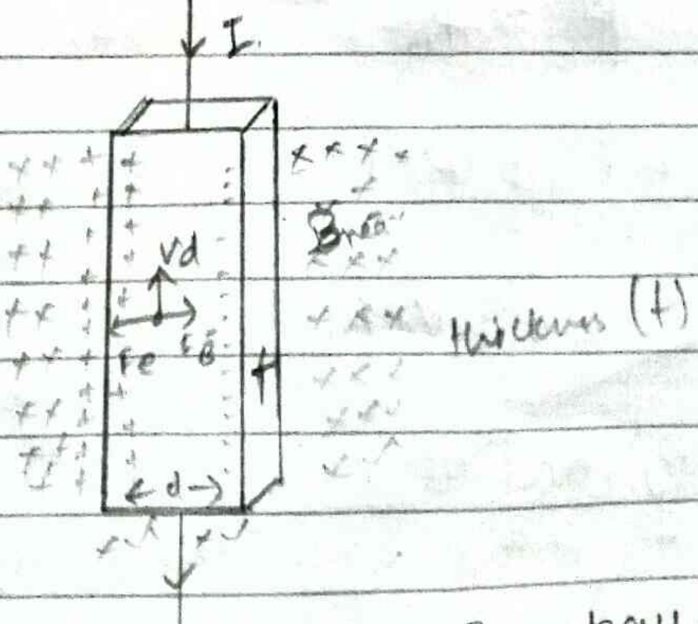


fig:- specemen showing hall effect.

Let us consider a roctangular strip of width d, thickness 't', cross sectional area 'A' carring current I as shown in fig. Since curron is flowing in downward direction. So movement of electron is in upward direction with drift velocity vd.

When the magnetic field B is turn on then magnetic force $\vec{F_B} = q(\vec{v_d} \times \vec{B})$ will act on each electron puching it towards the right edge or strip.

As time goes on, an excess of negative charge accumulate at right edge of strip, leaving an excess of positive charge at its left edge.

This seperation of charge create an electric field to $\vec{E}$ with the strip from left to right direction as shown in fig

This accumulation continuous until the force due to $\vec{E}$ as shown in fig is equal to force exerted by magnetic field.

$$\vec{F_E} = \vec{F_B}$$

$$e\vec{E_H} = e(\vec{v_d} \times \vec{B})$$

$$\therefore E_H = V_d B. \quad \text{—— (1)}$$

Since $\vec{v_d}$ and $\vec{B}$ are perpendicular to each other at this equilibrium radiation the force due to $\vec{B}$ & force due to $\vec{E}$ are balanced. The electron can again move freely along the conductor. The electric field and potential difference at this condition are called Hall effect field E_H and hall voltage V_H.

Since,

$$\text{current density } (J) = -V_d e n$$

$$\text{i.e } V_d = -\frac{J}{ne} \quad \text{— (ii)}$$

Substituting (ii) in (i)

$$E_H = -\frac{J}{ne} B$$

$$\text{Or, } \frac{E_H}{JB} = -\frac{1}{ne} \quad \text{— (iii)}$$

From eqn (iii), the term $\frac{E_H}{JB}$ is called hall coefficient. (R_H)

$$\therefore (R_H) := \frac{E_H}{JB} = -\frac{1}{ne}.$$

Its unit is m^3/columb.

Again from (i)

$$E_H = B v_d$$

and we also have,

$$E_H = \frac{V_H}{d} \quad \text{— (iv)}$$

where d is width of strip

$$\text{current } I = V_d e n A$$

$$\text{i.e } V_d = \frac{I}{enA} \quad \text{— (v)}$$

* Application of Hall effect:

1) It can be used to determine the nature of charge carrier.

$$R_H = \frac{1}{nq}$$

if R_H is positive, then the charge carrier is hole.

i.e $q = e$

if R_H is negative, then charge carrier is electron.

i.e $q = -e$

$$R_H = \pm \frac{1}{e}$$

(2) It can be used to determine the concentration of charge carrier. 'e' (n).

(3) It can be used to determine mobility of charge carrier.

(4) It can be used to determine weather the specement is metal, semiconductor and ins

along positive direction, with electric field oscillating parallel to the y-axis and the magnetic field oscillating parallel to z-axis (Using right hand rule). Then we can write electric field and magnetic field as sinosuidal function of position of 'x' and time 't'.

i.e $$E = E_0 \sin(kx - \omega t) \quad \text{—— (1)}$$
$$B = B_0 \sin(kx - \omega t) \quad \text{—— (2)}$$

where E_0 and B_0 are amplitudes of electric and magnetic field 'ω' and 'k' are angular frequency and a wave number of the wave.

Substituting (iv) and (v) in eqn (i).

$$\frac{V_H}{d} = \frac{BI}{enA}$$

$$\text{or, } V_H = \frac{Bd}{enA} \cdot I$$

$$\text{or, } V_H = \frac{IB}{ne} \frac{d}{A}$$

$$\text{or, } V_H = \frac{IB}{ne} \frac{d}{d \times t}$$

$$\boxed{\therefore V_H = \frac{IB}{net}} \text{ --- (vi)}$$

eqn (vi) is expression for hall voltage - If the steady state is maintained by applying an external field $\vec{E}$, Let carriers of current attains a drift velocity v_d at this state. Then mobility (μ) of carrier is defined as the drift velocity acquired per unit applied electric field.

$$\mu = \frac{v_d}{E}$$

Since $J = v_d en$, $v_d = J/en$

$$\text{or, } \mu = \frac{J}{enE}$$

$$J = \mu enE$$

we also have,

$$J = \sigma E$$

$$\text{i.e } \boxed{\sigma = \mu en}$$