Cache Flow

Team Reference Document

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Contents

1. Acieving AC on a solved problem

1.1. WA.

- Check that minimal input passes.
- Can an int overflow?
- Reread the problem statement.
- Start creating small testcases with python.
- Does cout print with high enough precision?
- Abstract the implementation.

1.2. TLE.

- Is the solution sanity checked?
- Use pypy instead of python.
- Rewrite in C++ or Java.
- Can we apply DP anywhere?
- To minimize penalty time you should create a worst case input (if easy) to test on.
- Binary Search over the answer?

1.3. RTE.

- Recursion limit in python?
- Arrayindex out of bounds?
- Division by 0?
- Modifying iterator while iterating over it?
- Not using a well definied operator for Collections.sort?

- If nothing makes sense and the end of the contest is approaching you can binary search over where the error is with try-except.

1.4. MLE.

- Create objects outside recursive function.
- Rewrite recursive solution to itterative with an own stack.

2. Ideas

2.1. A TLE solution is obvious.

- If doing dp, drop parameter and recover from others.
- Use a sorted datastructure.
- Is there a hint in the statement saying that something more is bounded?

2.2. Try this on clueless problems.

- Try to interpret problem as a graph (D - NCPC2017).
- Can we apply maxflow, with mincost?
- How does it look for small examples, can we find a pattern?
- Binary search over solution.
- If problem is small, just brute force instead of solving it cleverly. Some times its enough to iterate over the entire domains instead of using xgcd.

3. Code Templates

3.1. .bashrc. Aliases.

```
alias p2=python2
alias p3=python3
alias nv=vim
alias o="xdg-open ."
setxkbmap -option 'nocaps:ctrl'
```

3.2. .vimrc. Tabs, linenumbers, wrapping

```
set nowrap
syntax on
set tabstop=8 softtabstop=0 shiftwidth=4
set expandtab smarttab
set autoindent smartindent
set rnu number
set scrolloff=8
filetype plugin indent on
```

3.3. run.sh. Bash script to run all tests in a folder.

```
#!/bin/bash
# make exacutable: chmod +x run.sh
# run: ./run.sh A pypy A.py
# or
# ./run.sh A ./a.out
folder=$1;shift
for f in $folder/*.in; do
    echo $f
    pre=${f%.in}
    out=$pre.out
    ans=$pre.ans
    $* < $f > $out
    diff $out $ans
done
```

3.4. Java Template. A Java template.

```
import java.util.*;
import java.io.*;
public class A {
    void solve(BufferedReader in) throws Exception {
    }
    int toInt(String s) {return Integer.parseInt(s);}
    int[] toInts(String s) {
        String[] a = s.split(" ");
        int[] o = new int[a.length];
        for(int i = 0; i<a.length; i++)
            o[i] = toInt(a[i]);
        return o;
    }
    public static void main(String[] args)
    throws Exception {
        BufferedReader in = new BufferedReader
            (new InputStreamReader(System.in));
        (new A()).solve(in);
    }
}
```

3.5. Python Template.
A Python template

```
from collections import *
from itertools import permutations #No repeated elements
import sys, bisect
sys.setrecursionlimit(10**5)
inp = raw_input

def err(s):
    sys.stderr.write('{}\n'.format(s))

def ni():
    return int(inp())

def nl():
    return [int(_) for _ in inp().split()]

# q = deque([0])
# a = q.popleft()
# q.append(0)

# a = [1, 2, 3, 3, 4]
# bisect.bisect(a, 3) == 4
# bisect.bisect_left(a, 3) == 2
```

3.6. C++ Template.
A C++ template

```
#include <stdio.h>
#include <iostream>
#include <algorithm>
#include <vector>
#include <math.h>
#include <cmath>
using namespace std;
int main() {
    cout.precision(9);
    int N;
    cin >> N;
    cout << 0 << endl;
}
```

4. Data Structures

4.1. Binary Indexed Tree.
Also called a fenwick tree. Builds in $\mathcal{O}(n \log n)$ from an array. Querry sum from 0 to i in $\mathcal{O}(\log n)$ and updates an element in $\mathcal{O}(\log n)$.

```
private static class BIT {
  long[] data;
  public BIT(int size) {
    data = new long[size+1];
  }
  public void update(int i, int delta) {
    while(i< data.length) {
      data[i] += delta;
      i += i&-i; // Integer.lowestOneBit(i);
    }
  }
  public long sum(int i) {
    long sum = 0;
    while(i>0) {
      sum += data[i];
      i -= i&-i;
    }
    return sum;
  }
}
```

4.2. Segment Tree.
More general than a fenwick tree. Can adapt other operations than sum, e.g. min and max.

```
private static class ST {
  int li, ri;
  int sum; //change to max/min
  ST lN;
  ST rN;
}
static ST makeSgmTree(int[] A, int l, int r) {
  if(l == r) {
    ST node = new ST();
    node.li = l;
    node.ri = r;
    node.sum = A[l]; //max/min
    return node;
```

```java
  }
  int mid = (l+r)/2;
  ST lN = makeSgmTree(A,l,mid);
  ST rN = makeSgmTree(A,mid+1,r);
  ST root = new ST();
  root.li = lN.li;
  root.ri = rN.ri;
  root.sum = lN.sum + rN.sum; //max/min
  root.lN = lN;
  root.rN = rN;
  return root;
}
static int getSum(ST root, int l, int r) {//max/min
  if(root.li>=l && root.ri<=r)
    return root.sum; //max/min
  if(root.ri<l || root.li > r)
    return 0; //minInt/maxInt
  else //max/min
    return getSum(root.lN,l,r) + getSum(root.rN,l,r);
}
static int update(ST root, int i, int val) {
  int diff = 0;
  if(root.li==root.ri && i == root.li) {
    diff = val-root.sum; //max/min
    root.sum=val; //max/min
    return diff; //root.max
  }
  int mid = (root.li + root.ri) / 2;
  if (i <= mid) diff = update(root.lN, i, val);
  else diff = update(root.rN, i, val);
  root.sum+=diff; //ask other child
  return diff; //and compute max/min
}
```

4.3. **Lazy Segment Tree.** More general implementation of a segment tree where its possible to increase whole segments by some diff, with lazy propagation. Implemented with arrays instead of nodes, which probably has less overhead to write during a competition.

```java
class LazySegmentTree {
  private int n;
  private int[] lo, hi, sum, delta;
  public LazySegmentTree(int n) {
    this.n = n;
    lo = new int[4*n + 1];
    hi = new int[4*n + 1];
    sum = new int[4*n + 1];
    delta = new int[4*n + 1];
    init();
  }
  public int sum(int a, int b) {
    return sum(1, a, b);
  }
  private int sum(int i, int a, int b) {
    if(b < lo[i] || a > hi[i]) return 0;
    if(a <= lo[i] && hi[i] <= b) return sum(i);
    prop(i);
    int l = sum(2*i, a, b);
    int r = sum(2*i+1, a, b);
    update(i);
    return l + r;

  }
  public void inc(int a, int b, int v) {
    inc(1, a, b, v);
  }
  private void inc(int i, int a, int b, int v) {
    if(b < lo[i] || a > hi[i]) return;
    if(a <= lo[i] && hi[i] <= b) {
      delta[i] += v;
      return;
    }
    prop(i);
    inc(2*i, a, b, v);
    inc(2*i+1, a, b, v);
    update(i);
  }

  private void init() {
    init(1, 0, n-1, new int[n]);
  }
```

```java
  private void init(int i, int a, int b, int[] v) {
    lo[i] = a;
    hi[i] = b;
    if(a == b) {
      sum[i] = v[a];
      return;
    }
    int m = (a+b)/2;
    init(2*i, a, m, v);
    init(2*i+1, m+1, b, v);
    update(i);
  }
  private void update(int i) {
    sum[i] = sum(2*i) + sum(2*i+1);
  }
  private int range(int i) {
    return hi[i] - lo[i] + 1;
  }
  private int sum(int i) {
    return sum[i] + range(i)*delta[i];
  }
  private void prop(int i) {
    delta[2*i] += delta[i];
    delta[2*i+1] += delta[i];
    delta[i] = 0;
  }
}
```

4.4. **Union Find.** This data structure is used in varoius algorithms, for example Kruskals algorithm for finding a Minimal Spanning Tree in a weighted graph. Also it can be used for backward simulation of dividing a set.

```java
private class Node {
  Node parent;
  int h;
  public Node() {
    parent = this;
    h = 0;
  }
  public Node find() {
    if(parent != this) parent = parent.find();
    return parent;
  }
}
static void union(Node x, Node y) {
  Node xR = x.find(), yR = y.find();
  if(xR == yR) return;
  if(xR.h > yR.h)
    yR.parent = xR;
  else {
    if(yR.h == xR.h) yR.h++;
    xR.parent = yR;
  }
}
```

4.5. **Monotone Queue.** Used in sliding window algorithms where one would like to find the minimum in each interval of a given length. Amortized $\mathcal{O}(n)$ to find min in each of these intervals in an array of length n. Can easily be used to find the maximum as well.

```java
private static class MinMonQue {
    LinkedList<Integer> que = new LinkedList<>();
    public void add(int i) {
        while(!que.isEmpty() && que.getFirst() > i)
            que.removeFirst();
        que.addFirst(i);
    }
    public int last() {
        return que.getLast();
    }
    public void remove(int i) {
        if(que.getLast() == i) que.removeLast();
    }
}
```

4.6. **Treap.** Treap is a binary search tree that uses randomization to balance itself. It's easy to implement, and gives you access to the internal structures of a binary tree, which can be used to find the k'th element for example. Because of the randomness, the average height is about a factor 4 of a prefectly balanced tree.

```java
class Treap{
  int sz;
  int v;
```

```
  double y;
  Treap L, R;

  static int sz(Treap t) {
    if(t == null) return 0;
    return t.sz;
  }
  static void update(Treap t) {
    if(t == null) return;
    t.sz = sz(t.L) + sz(t.R) + 1;
  }
  static Treap merge(Treap a, Treap b) {
    if (a == null) return b;
    if(b == null) return a;
    if (a.y < b.y) {
      a.R = merge(a.R, b);
      update(a);
      return a;
    } else {
      b.L = merge(a, b.L);
      update(b);
      return b;
    }
  }
  //inserts middle in left half
  static Treap[] split(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.v <= x) {
      Treap[] p = split(t.R, x);
      t.R = p[0];
      p[0] = t;
      return p;
    } else {
      Treap[] p = split(t.L, x);
      t.L = p[1];
      p[1] = t;
      return p;
    }
  }
  //use only with split
  static Treap insert(Treap t, int x) {
    Treap m = new Treap();
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, x-1);
    return merge(merge(p[0],m), p[1]);
  }


  //inserts middle in left half
  static Treap[] splitK(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.sz < x) return new Treap[]{t, null};
    if (sz(t.L) >= x) {
      Treap[] p = splitK(t.L, x);
      t.L = p[1];
      p[1] = t;
      update(p[0]);
      update(p[1]);
      return p;
    } else if (sz(t.L) + 1 == x){
      Treap r = t.R;
      t.R = null;
      Treap[] p = new Treap[]{t, r};
      update(p[0]);
      update(p[1]);
      return p;
    } else {
      Treap[] p = splitK(t.R, x - sz(t.L)-1);
      t.R = p[0];
      p[0] = t;
      update(p[0]);
      update(p[1]);
      return p;
    }
  }
  //use only with splitK
  static Treap insertK(Treap t, int w, int x) {
    Treap m = new Treap();
```

```
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, w);
    t = merge(p[0], m);
    return merge(t, p[1]);
  }
  //use only with splitK
  static Treap deleteK(Treap t, int w, int x) {
    Treap[] p = splitK(t, w);
    Treap[] q = splitK(p[0], w-1);
    return merge(q[0], p[1]);
  }

  static Treap Left(Treap t) {
    if (t == null) return null;
    if (t.L == null) return t;
    return Left(t.L);
  }
  static Treap Right(Treap t) {
    if (t == null) return null;
    if (t.R == null) return t;
    return Right(t.R);
  }

}
```

4.7. **RMQ.** $\mathcal{O}(1)$ queries of interval min, max, gcd or lcm. $\mathcal{O}(n \log n)$ building time.

```
import math
class RMQ:
    def __init__(self, arr, func=min):
        self.sz = len(arr)
        self.func = func
        MAXN = self.sz
        LOGMAXN = int(math.ceil(math.log(MAXN + 1, 2)))
        self.data = [[0]*LOGMAXN for _ in range(MAXN)]
        for i in range(MAXN):
            self.data[i][0] = arr[i]
        for j in range(1, LOGMAXN):
            for i in range(MAXN - (1<<j)+1):
                self.data[i][j] = func(self.data[i][j-1],
                        self.data[i + (1<<(j-1))][j-1])

    def query(self, a, b):
        if a > b:
            # some default value when query is empty
            return 1
        d = b - a + 1
        k = int(math.log(d, 2))
        return self.func(self.data[a][k], self.data[b-(1<<k)+1][k])
```

5. Graph Algorithms

5.1. **Dijkstras algorithm.** Finds the shortest distance between two Nodes in a weighted graph in $\mathcal{O}(|E| \log |V|)$ time.

```
from heapq import heappop as pop, heappush as push
# adj: adj-list where edges are tuples (node_id, weight):
# (1) --2-- (0) --3-- (2) has the adj-list:
# adj = [[(1, 2), (2, 3)], [(0, 2)], [0, 3]]
def dijk(adj, S, T):
    N = len(adj)
    INF = 10**10
    dist = [INF]*N
    pq = []
    dist[S] = 0
    push(pq, (0, S))

    while pq:
        D, i = pop(pq)
        if D != dist[i]: continue
        for j, w in adj[i]:
            alt = D + w
            if dist[j] > alt:
                dist[j] = alt
                push(pq, (alt, j))

    return dist[T]
```

5.2. **Bipartite Graphs.** The Hopcroft-Karp algorithm finds the maximal matching in a bipartite graph. Also, this matching can together with Könings theorem be used to construct a minimal vertex-cover, which as we all know is the complement of a maximum independent set. Runs in $\mathcal{O}(|E|\sqrt{|V|})$.

```java
import java.util.*;
class Node {
  int id;
  LinkedList<Node> ch = new LinkedList<>();
  public Node(int id) {
    this.id = id;
  }
}
public class BiGraph {
  private static int INF = Integer.MAX_VALUE;
  LinkedList<Node> L, R;
  int N, M;
  Node[] U;
  int[] Pair, Dist;
  int nild;
  public BiGraph(LinkedList<Node> L, LinkedList<Node> R){
    N = L.size(); M = R.size();
    this.L = L; this.R = R;
    U = new Node[N+M];
    for(Node n: L) U[n.id] = n;
    for(Node n: R) U[n.id] = n;
  }
  private boolean bfs() {
    LinkedList<Node> Q = new LinkedList<>();
    for(Node n: L)
      if(Pair[n.id] == -1) {
        Dist[n.id] = 0;
        Q.add(n);
      }else
        Dist[n.id] = INF;

    nild = INF;
    while(!Q.isEmpty()) {
      Node u = Q.removeFirst();
      if(Dist[u.id] < nild)
        for(Node v: u.ch) if(distp(v) == INF){
          if(Pair[v.id] == -1)
            nild = Dist[u.id] + 1;
          else {
            Dist[Pair[v.id]] = Dist[u.id] + 1;
            Q.addLast(U[Pair[v.id]]);
          }
        }
    }
    return nild != INF;
  }
  private int distp(Node v) {
    if(Pair[v.id] == -1) return nild;
    return Dist[Pair[v.id]];
  }
  private boolean dfs(Node u) {
    for(Node v: u.ch) if(distp(v) == Dist[u.id] + 1) {
      if(Pair[v.id] == -1 || dfs(U[Pair[v.id]])) {
        Pair[v.id] = u.id;
        Pair[u.id] = v.id;
        return true;
      }
    }
    Dist[u.id] = INF;
    return false;
  }
  public HashMap<Integer, Integer> maxMatch() {
    Pair = new int[M+N];
    Dist = new int[M+N];
    for(int i = 0; i<M+N; i++) {
      Pair[i] = -1;
      Dist[i] = INF;
    }
    HashMap<Integer, Integer> out = new HashMap<>();
    while(bfs()) {
      for(Node n: L) if(Pair[n.id] == -1)
        dfs(n);
    }
    for(Node n: L) if(Pair[n.id] != -1)
      out.put(n.id, Pair[n.id]);
    return out;
  }
  public HashSet<Integer> minVTC() {
```

```java
    HashMap<Integer, Integer> Lm = maxMatch();
    HashMap<Integer, Integer> Rm = new HashMap<>();
    for(int x: Lm.keySet()) Rm.put(Lm.get(x), x);
    boolean[] Z = new boolean[M+N];
    LinkedList<Node> bfs = new LinkedList<>();
    for(Node n: L) {
      if(!Lm.containsKey(n.id)) {
        Z[n.id] = true;
        bfs.add(n);
      }
    }
    while(!bfs.isEmpty()) {
      Node x = bfs.removeFirst();
      int nono = -1;
      if(Lm.containsKey(x.id))
        nono = Lm.get(x.id);
      for(Node y: x.ch) {
        if(y.id == nono || Z[y.id]) continue;
        Z[y.id] = true;
        if(Rm.containsKey(y.id)){
          int xx = Rm.get(y.id);
          if(!Z[xx]) {
            Z[xx] = true;
            bfs.addLast(U[xx]);
          }
        }
      }
    }
    HashSet<Integer> K = new HashSet<>();
    for(Node n: L) if(!Z[n.id]) K.add(n.id);
    for(Node n: R) if(Z[n.id]) K.add(n.id);
    return K;
  }
}
```

5.3. **Network Flow.** The Floyd Warshall algorithm for determining the maximum flow through a graph can be used for a lot of unexpected problems. Given a problem that can be formulated as a graph, where no ideas are found trying, it might help trying to apply network flow. The running time is $\mathcal{O}(C \cdot m)$ where C is the maximum flow and m is the amount of edges in the graph. If C is very large we can change the running time to $\mathcal{O}(\log Cm^2)$ by only studying edges with a large enough capacity in the beginning.

```python
from collections import defaultdict
class Flow:
    def __init__(self, sz):
        self.G = [defaultdict(int) for _ in range(sz)]

    def add_edge(self, i, j, w):
        self.G[i][j] += w

    def dfs(self, s, t, FLOW):
        if s in self.V: return 0
        if s == t: return FLOW
        self.V.add(s)
        for u, w in self.G[s].items():
            if w and u not in self.dead:
                F = self.dfs(u, t, min(FLOW, w))
                if F:
                    self.G[s][u] -= F
                    self.G[u][s] += F
                    return F
        self.dead.add(s)
        return 0

    def max_flow(self, s, t):
        flow = 0
        self.dead = set()
        while True:
            pushed = self.bfs(s, t)
            if not pushed: break
            flow += pushed
        return flow
```

5.4. **Dinitz Algorithm.** Faster flow algorithm.

```python
from collections import defaultdict
class Dinitz:
    def __init__(self, sz, INF=10**10):
        self.G = [defaultdict(int) for _ in range(sz)]
        self.sz = sz
        self.INF = INF
```

```python
    def add_edge(self, i, j, w):
        self.G[i][j] += w

    def bfs(self, s, t):
        level = [0]*self.sz
        q = [s]
        level[s] = 1
        while q:
            q2 = []
            for u in q:
                for v, w in self.G[u].items():
                    if w and level[v] == 0:
                        level[v] = level[u] + 1
                        q2.append(v)
            q = q2
        self.level = level
        return level[t] != 0

    def dfs(self, s, t, FLOW):
        if s in self.V: return 0
        if s == t: return FLOW
        self.V.add(s)
        L = self.level[s]
        for u, w in self.G[s].items():
            if u in self.dead: continue
            if w and L+1==self.level[u]:
                F = self.dfs(u, t, min(FLOW, w))
                if F:
                    self.G[s][u] -= F
                    self.G[u][s] += F
                    return F
        self.dead.add(s)
        return 0


    def max_flow(self, s, t):
        flow = 0
        while self.bfs(s, t):
            self.dead = set()
            while True:
                self.V = set()
                pushed = self.dfs(s, t, self.INF)
                if not pushed: break
                flow += pushed
        return flow
```

5.5. **Min Cost Max Flow.** Finds the minimal cost of a maximum flow through a graph. Can be used for some optimization problems where the optimal assignment needs to be a maximum flow.

```java
class MinCostMaxFlow {
boolean found[];
int N, dad[];
long cap[][], flow[][], cost[][], dist[], pi[];

static final long INF = Long.MAX_VALUE / 2 - 1;

boolean search(int s, int t) {
Arrays.fill(found, false);
Arrays.fill(dist, INF);
dist[s] = 0;

while (s != N) {
  int best = N;
  found[s] = true;
  for (int k = 0; k < N; k++) {
    if (found[k]) continue;
    if (flow[k][s] != 0) {
      long val = dist[s] + pi[s] - pi[k] - cost[k][s];
      if (dist[k] > val) {
        dist[k] = val;
        dad[k] = s;
      }
    }
    if (flow[s][k] < cap[s][k]) {
      long val = dist[s] + pi[s] - pi[k] + cost[s][k];
      if (dist[k] > val) {
        dist[k] = val;
        dad[k] = s;
      }
```

```
    }

    if (dist[k] < dist[best]) best = k;
  }
  s = best;
}
for (int k = 0; k < N; k++)
  pi[k] = Math.min(pi[k] + dist[k], INF);
return found[t];
}

long[] mcmf(long c[][], long d[][], int s, int t) {
cap = c;
cost = d;

N = cap.length;
found = new boolean[N];
flow = new long[N][N];
dist = new long[N+1];
dad = new int[N];
pi = new long[N];

long totflow = 0, totcost = 0;
while (search(s, t)) {
  long amt = INF;
  for (int x = t; x != s; x = dad[x])
    amt = Math.min(amt, flow[x][dad[x]] != 0 ?
    flow[x][dad[x]] : cap[dad[x]][x] - flow[dad[x]][x]);
  for (int x = t; x != s; x = dad[x]) {
    if (flow[x][dad[x]] != 0) {
      flow[x][dad[x]] -= amt;
      totcost -= amt * cost[x][dad[x]];
    } else {
      flow[dad[x]][x] += amt;
      totcost += amt * cost[dad[x]][x];
    }
  }
  totflow += amt;
}
return new long[]{ totflow, totcost };
}
}
```

5.6. **2-Sat.** Solves 2sat by splitting up vertices in strongly connected components.

```
import sys
sys.setrecursionlimit(10**5)
class Sat:
    def __init__(self, no_vars):
        self.size = no_vars*2
        self.no_vars = no_vars
        self.adj = [[] for _ in range(self.size())]
        self.back = [[] for _ in range(self.size())]
    def add_imply(self, i, j):
        self.adj[i].append(j)
        self.back[j].append(i)
    def add_or(self, i, j):
        self.add_imply(i^1, j)
        self.add_imply(j^1, i)
    def add_xor(self, i, j):
        self.add_or(i, j)
        self.add_or(i^1, j^1)
    def add_eq(self, i, j):
        self.add_xor(i, j^1)

    def dfs1(self, i):
        if i in self.marked: return
        self.marked.add(i)
        for j in self.adj[i]:
            self.dfs1(j)
        self.stack.append(i)

    def dfs2(self, i):
        if i in self.marked: return
        self.marked.add(i)
        for j in self.back[i]:
            self.dfs2(j)
        self.comp[i] = self.no_c

    def is_sat(self):
```

```
        self.marked = set()
        self.stack = []
        for i in range(self.size):
            self.dfs1(i)
        self.marked = set()
        self.no_c = 0
        self.comp = [0]*self.size
        while self.stack:
            i = self.stack.pop()
            if i not in self.marked:
                self.no_c += 1
                self.dfs2(i)
        for i in range(self.no_vars):
            if self.comp[i*2] == self.comp[i*2+1]:
                return False
        return True
```

6. Dynamic Programming

6.1. Longest Increasing Subsequence. Finds the longest increasing subsequence in an array in $\mathcal{O}(n \log n)$ time. Can easility be transformed to longest decreasing/nondecreasing/nonincreasing subsequence.

```
def lis(X):
    N = len(X)
    P = [0]*N
    M = [0]*(N+1)
    L = 0
    for i in range(N):
        lo, hi = 1, L + 1
        while lo < hi:
            mid = (lo + hi) >> 1
            if X[M[mid]] < X[i]:
                lo = mid + 1
            else:
                hi = mid
        newL = lo
        P[i] = M[newL - 1]
        M[newL] = i
        L = max(L, newL)
    S = [0]*L
    k = M[L]
    for i in range(L-1, -1, -1):
        S[i] = X[k]
        k = P[k]
    return S
```

6.2. String functions. The z-function computes the longest common prefix of t and $t[i:]$ for each i in $\mathcal{O}(|t|)$. The border function computes the longest common proper (smaller than whole string) prefix and suffix of string $t[: i]$.

```
def zfun(t):
    z = [0]*len(t)
    n = len(t)
    l, r = (0,0)
    for i in range(1,n):
        if i < r:
            z[i] = min(z[i-l], r-i+1)
        while z[i] + i < n and t[i+z[i]] == t[z[i]]:
            z[i]+=1
        if i + z[i] - 1 > r:
            l = i
            r = i + z[i] - 1
    return z

def matches(t, p):
    s = p + '#' + t
    return filter(lambda x: x[1] == len(p),
            enumerate(zfun(s)))

def boarders(s):
    b = [0]*len(s)
    for i in range(1, len(s)):
        k = b[i-1]
        while k>0 and s[k] != s[i]:
            k = b[k-1]
        if s[k] == s[i]:
            b[i] = k+1
    return b
```

6.3. Josephus problem. Who is the last one to get removed from a circle if the k'th element is continously removed?

```
# Rewritten from J(n, k) = (J(n-1, k) + k)%n
def J(n, k):
    r = 0
    for i in range(2, n+1):
        r = (r + k)%i
    return r
```

6.4. **Floyd Warshall.** Constucts a matrix with the distance between all pairs of nodes in $\mathcal{O}(n^3)$ time. Works for negative edge weights, but not if there exists negative cycles. The nxt matrix is used to reconstruct a path. Can be skipped if we don't care about the path.

```
# Computes distance matrix and next matrix given an edgelist
def FloydWarshall(n, edges):
  INF = 1000000000
  dist = [[INF]*n for _ in range(n)]
  nxt = [[None]*n for _ in range(n)]
  for e in edgs:
    dist[e[0]][e[1]] = e[2]
    nxt[e[0]][e[1]] = e[1]
  for k in range(n):
    for i in range(n):
      for j in range(n):
        if dist[i][j] > dist[i][k] + dist[k][j]:
          dist[i][j] = dist[i][k] + dist[k][j]
          nxt[i][j] = nxt[i][k]
  return dist, nxt

# Computes the path from i to j given a nextmatrix
def path(i, j, nxt):
  if nxt[i][j] == None: return []
  path = [i]
  while i != j:
    i = nxt[i][j]
    path.append(i)
  return path
```

7. Etc

7.1. **System of Equations.** Solves the system of equations $\boldsymbol{Ax} = \boldsymbol{b}$ by Gaussian elimination. This can for example be used to determine the expected value of each node in a markov chain. Runns in $\mathcal{O}(N^3)$.

```
# monoid needs to implement
# __add__, __mul__, __sub__, __div__ and isZ
def gauss(A, b, monoid=None):
  def Z(v): return abs(v) < 1e-6 if not monoid else v.isZ()

  N = len(A[0])
  for i in range(N):
    m = next(j for j in range(i, N) if Z(A[j][i]) == False)
    if i != m:
        A[i], A[m] = A[m], A[i]
        b[i], b[m] = b[m], b[i]
    for j in range(i+1, N):
      sub = A[j][i]/A[i][i]
      b[j] -= sub*b[i]
      for k in range(N):
        A[j][k] -= sub*A[i][k]

  for i in range(N-1, -1, -1):
    for j in range(N-1, i, -1):
      sub = A[i][j]/A[j][j]
      b[i] -= sub*b[j]
      A[i][k] -= sub*A[j][k]
    b[i], A[i][i] = b[i]/A[i][i], A[i][i]/A[i][i]
  return b
```

7.2. **Convex Hull.** From a collection of points in the plane the convex hull is often used to compute the largest distance or the area covered, or the length of a rope that encloses the points. It can be found in $\mathcal{O}(N \log N)$ time by sorting the points on angle and the sweeping over all of them.

```
def convex_hull(pts):
    pts = sorted(set(pts))

    if len(pts) <= 2:
        return pts

    def cross(o, a, b):
        return (a[0] - o[0]) * (b[1] - o[1]) - (a[1] - o[1]) * (b[0] - o[0])

    lo = []
    for p in pts:
```

```
        while len(lo) >= 2 and cross(lo[-2], lo[-1], p) <= 0:
            lo.pop()
        lo.append(p)

    hi = []
    for p in reversed(pts):
        while len(hi) >= 2 and cross(hi[-2], hi[-1], p) <= 0:
            hi.pop()
        hi.append(p)

    return lo[:-1] + hi[:-1]
```

7.3. Number Theory.

```
def gcd(a, b):
  return b if a%b == 0 else gcd(b, a%b)

# returns g = gcd(a, b), x0, y0,
# where g = x0*a + y0*b
def xgcd(a, b):
  x0, x1, y0, y1 = 1, 0, 0, 1
  while b != 0:
    q, a, b = (a // b, b, a % b)
    x0, x1 = (x1, x0 - q * x1)
    y0, y1 = (y1, y0 - q * y1)
  return (a, x0, y0)

def crt(la, ln):
    assert len(la) == len(ln)
    for i in range(len(la)):
        assert 0 <= la[i] < ln[i]
    prod = 1
    for n in ln:
        assert gcd(prod, n) == 1
        prod *= n
    lN = []
    for n in ln:
        lN.append(prod//n)
    x = 0
    for i, a in enumerate(la):
        print(lN[i], ln[i])
        _, Mi, mi = xgcd(lN[i], ln[i])
        x += a*Mi*lN[i]
    return x % prod

# finds x^e mod m
def modpow(x, m, e):
    res = 1
    while e:
        if e%2 == 1:
            res = (res*x) % m
        x = (x*x) % m
        e = e//2
    return res

# Divides a list of digits with an int.
# A lot faster than using bigint-division.
def div(L, d):
  r = [0]*(len(L) + 1)
  q = [0]*len(L)
  for i in range(len(L)):
    x = int(L[i]) + r[i]*10
    q[i] = x//d
    r[i+1] = x-q[i]*d
  s = []
  for i in range(len(L) - 1, 0, -1):
    s.append(q[i]%10)
    q[i-1] += q[i]//10

  while q[0]:
    s.append(q[0]%10)
    q[0] = q[0]//10
  s = s[::-1]
  i = 0
  while s[i] == 0:
    i += 1
  return s[i:]


# Multiplies a list of digits with an int.
# A lot faster than using bigint-multiplication.
```

```
def mul(L, d):
  r = [d*x for x in L]
  s = []
  for i in range(len(r) - 1, 0, -1):
    s.append(r[i]%10)
    r[i-1] += r[i]//10
  while r[0]:
    s.append(r[0]%10)
    r[0] = r[0]//10
  return s[::-1]

large_primes = [
5915587277,
1500450271,
3267000013,
5754853343,
4093082899,
9576890767,
3628273133,
2860486313,
5463458053,
3367900313
        ]
```

7.4. **FFT.** FFT can be used to calculate the product of two polynomials of length N in $\mathcal{O}(N \log N)$ time. The FFT function requires a power of 2 sized array of size at least 2N to store the results as complex numbers.

```
import cmath
# A has to be of length a power of 2.
def FFT(A, inverse=False):
    N = len(A)
    if N <= 1:
        return A
    if inverse:
        D = FFT(A) # d_0/N, d_{N-1}/N, d_{N-2}/N, ...
        return map(lambda x: x/N, [D[0]] + D[:0:-1])
    evn = FFT(A[0::2])
    odd = FFT(A[1::2])
    Nh = N//2
    return [evn[k%Nh]+cmath.exp(2j*cmath.pi*k/N)*odd[k%Nh]
            for k in range(N)]
```

8. NP tricks

8.1. **MaxClique.** The max clique problem is one of Karp's 21 NP-complete problems. The problem is to find the lagest subset of an undirected graph that forms a clique - a complete graph. There is an obvious algorithm that just inspects every subset of the graph and determines if this subset is a clique. This algorithm runns in $\mathcal{O}(n^2 2^n)$. However one can use the meet in the middle trick (one step divide and conquer) and reduce the complexity to $\mathcal{O}(n^2 2^{\frac{n}{2}})$.

```
static int max_clique(int n, int[][] adj) {
  int fst = n/2;
  int snd = n - fst;
  int[] maxc = new int[1<<fst];
  int max = 1;
  for(int i = 0; i<(1<<fst); i++) {
    for(int a = 0; a<fst; a++) {
      if((i&1<<a) != 0)
        maxc[i] = Math.max(maxc[i], maxc[i^(1<<a)]);
    }
    boolean ok = true;
    for(int a = 0; a<fst; a++) if((i&1<<a) != 0) {
      for(int b = a+1; b<fst; b++) {
          if((i&1<<b) != 0 && adj[a][b] == 0)
              ok = false;
      }
    }
    if(ok) {
      maxc[i] = Integer.bitCount(i);
      max = Math.max(max, maxc[i]);
    }
  }
  for(int i = 0; i<(1<<snd); i++) {
    boolean ok = true;
    for(int a = 0; a<snd; a++) if((i&1<<a) != 0) {
      for(int b = a+1; b<snd; b++) {
        if((i&1<<b) != 0)
          if(adj[a+fst][b+fst] == 0)
            ok = false;
```

```
      }
    }
    if(!ok) continue;
    int mask = 0;
    for(int a = 0; a<fst; a++) {
      ok = true;
      for(int b = 0; b<snd; b++) {
        if((i&1<<b) != 0) {
          if(adj[a][b+fst] == 0) ok = false;
        }
      }
      if(ok) mask |= (1<<a);
    }
    max = Math.max(Integer.bitCount(i) + maxc[mask],
            max);
  }
  return max;
}
```

9. Coordinate Geometry

9.1. Area of a nonintersecting polygon. The signed area of a polygon with n vertices is given by

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$$

9.2. Intersection of two lines. Two lines defined by

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0$$

Intersects in the point

$$P = (\frac{b_1 c_2 - b_2 c_1}{w}, \frac{a_2 c_1 - a_1 c_2}{w}),$$

where $w = a_1 b_2 - a_2 b_1$. If $w = 0$ the lines are parallell.

9.3. Distance between line segment and point. Given a linesegment between point P, Q, the distance D to point R is given by:

$$a = Q_y - P_y$$
$$b = Q_x - P_x$$
$$c = P_x Q_y - P_y Q_x$$
$$R_P = (\frac{b(bR_x - aR_y) - ac}{a^2 + b^2}, \frac{a(aR_y - bR_x) - bc}{a^2 + b^2})$$
$$D = \begin{cases} \frac{|aR_x + bR_y + c|}{\sqrt{a^2+b^2}} & \text{if } (R_{P_x} - P_x)(R_{P_x} - Q_x) < 0, \\ \min |P - R|, |Q - R| & \text{otherwise} \end{cases}$$

9.4. Picks theorem. Find the amount of internal integer coordinates i inside a polygon with picks theorem $A = \frac{b}{2} + i - 1$, where A is the area of the polygon and b is the amount of coordinates on the boundary.

9.5. Trigonometry. Sine-rule

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Cosine-rule

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$

Area-rule

$$A = \frac{a \cdot b \cdot \sin(\gamma)}{2}$$

9.6. Implementations.

```python
import math

# Distance between two points
def dist(p, q):
  return math.hypot(p[0]-q[0], p[1] - q[1])

# Square distance between two points
def d2(p, q):
  return (p[0] - q[0])**2 + (p[1] - q[1])**2

# Converts two points to a line (a, b, c),
# ax + by + c = 0
# if p == q, a = b = c = 0
def pts2line(p, q):
  return (-q[1] + p[1],
```

```
        q[0] - p[0],
        p[0]*q[1] - p[1]*q[0])

# Distance from a point to a line,
# given that a != 0 or b != 0
def distl(l, p):
  return (abs(l[0]*p[0] + l[1]*p[1] + l[2])
      /math.hypot(l[0], l[1]))

# intersects two lines.
# if parallell, returnes False.
def inters(l1, l2):
  a1,b1,c1 = l1
  a2,b2,c2 = l2
  cp = a1*b2 - a2*b1
  if cp != 0:
    return float(b1*c2 - b2*c1)/cp, float(a2*c1 - a1*c2)/cp
  else:
    return False

# projects a point on a line
def project(l, p):
  a, b, c = l
  return ((b*(b*p[0] - a*p[1]) - a*c)/(a*a + b*b),
    (a*(a*p[1] - b*p[0]) - b*c)/(a*a + b*b))

# Intersections between circles
def intersections(c1, c2):
  if c1[2] > c2[2]:
      c1, c2 = c2, c1
  x1, y1, r1 = c1
  x2, y2, r2 = c2
  if x1 == x2 and y1 == y2 and r1 == r2:
    return False

  dist2 = (x1 - x2)*(x1-x2) + (y1 - y2)*(y1 - y2)
  rsq = (r1 + r2)*(r1 + r2)
  if dist2 > rsq or dist2 < (r1-r2)*(r1-r2):
    return []
  elif dist2 == rsq:
    cx = x1 + (x2-x1)*r1/(r1+r2)
    cy = y1 + (y2-y1)*r1/(r1+r2)
    return [(cx, cy)]
  elif dist2 == (r1-r2)*(r1-r2):
    cx = x1 - (x2-x1)*r1/(r2-r1)
    cy = y1 - (y2-y1)*r1/(r2-r1)
    return [(cx, cy)]

  d = math.sqrt(dist2)
  f = (r1*r1 - r2*r2 + dist2)/(2*dist2)
  xf = x1 + f*(x2-x1)
  yf = y1 + f*(y2-y1)
  dx = xf-x1
  dy = yf-y1
  h = math.sqrt(r1*r1 - dx*dx - dy*dy)
  norm = abs(math.hypot(dx, dy))
  p1 = (xf + h*(-dy)/norm, yf + h*(dx)/norm)
  p2 = (xf + h*(dy)/norm, yf + h*(-dx)/norm)
  return sorted([p1, p2])

# Finds the bisector through origo
# between two points by normalizing.
def bisector(p1, p2):
  d1 = math.hypot(p1[0], p2[1])
  d2 = math.hypot(p2[0], p2[1])
  return ((p1[0]/d1 + p2[0]/d2),
          (p1[1]/d1 + p2[1]/d2))

# Distance from P to origo
def norm(P):
  return (P[0]**2 + P[1]**2 + P[2]**2)**(0.5)

# Finds ditance between point p
# and line A + t*u in 3D
def dist3D(A, u, p):
  AP = tuple(A[i] - p[i] for i in range(3))
  cross = tuple(AP[i]*u[(i+1)%3] - AP[(i+1)%3]*u[i]
    for i in range(3))
  return norm(cross)/norm(u)
```

10. Practice Contest Checklist

- Operations per second in py2
- Operations per second in py3
- Operations per second in java
- Operations per second in c++
- Operations per second on local machine
- Is MLE called MLE or RTE?
- What happens if extra output is added? What about one extra new line or space?
- Look at documentation on judge.
- Submit a clar.
- Print a file.
- Directory with test cases.