Cache Flow

Team Reference Document

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Contents

1. Code Templates

1.1. .bashrc. Aliases.

```
alias nv=vim
alias o="xdg-open ."
```

1.2. .vimrc. Tabs, linenumbers, wrapping

```
set nowrap
syntax on
set tabstop=8 softtabstop=0 shiftwidth=4
set expandtab smarttab autoindent
set rnu
set number
set scrolloff=8
language en_US
```

1.3. Java Template. A Java template.

```
import java.util.*;
import java.io.*;
public class A {
    void solve(BufferedReader in) throws Exception {

    }
    int toInt(String s) {return Integer.parseInt(s);}
    int[] toInts(String s) {
        String[] a = s.split(" ");
        int[] o = new int[a.length];
        for(int i = 0; i<a.length; i++)
            o[i] = toInt(a[i]);
        return o;
    }
    void e(Object o) {
        System.err.println(o);
    }
    public static void main(String[] args)
```

```
    throws Exception {
        BufferedReader in = new BufferedReader
            (new InputStreamReader(System.in));
        (new A()).solve(in);
    }
}
```

1.4. **Python Template.** A Python template

```python
from collections import defaultdict
from collections import deque
from collections import Counter
from itertools import permutations #No repeated elements
import sys, bisect
sys.setrecursionlimit(1000000)
# q = deque([0])
# a = q.popleft()
# q.append(0)

# a = [1, 2, 3, 3, 4]
# bisect.bisect(a, 3) == 4
# bisect.bisect_left(a, 3) == 2

# reversed()
# sorted()
```

1.5. **C++ Template.** A C++ template

```cpp
#include <stdio.h>
#include <iostream>
#include <algorithm>
#include <vector>
#include <math.h>
#include <cmath>
using namespace std;
int main() {
    cout.precision(9);
    int N;
    cin >> N;
    cout << 0 << endl;
}
```

1.6. **Fast IO Java.** Kattio with easier names

```java
import java.util.StringTokenizer;
import java.io.*;
class Sc {
  public Sc(InputStream i) {
    r = new BufferedReader(new InputStreamReader(i));
  }
  public boolean hasM() {
    return peekToken() != null;
  }
  public int nI() {
    return Integer.parseInt(nextToken());
  }
  public double nD() {
    return Double.parseDouble(nextToken());
  }
  public long nL() {
    return Long.parseLong(nextToken());
  }
  public String n() {
    return nextToken();
  }
  private BufferedReader r;
  private String line;
  private StringTokenizer st;
  private String token;
  private String peekToken() {
    if (token == null)
      try {
        while (st == null || !st.hasMoreTokens()) {
          line = r.readLine();
          if (line == null) return null;
          st = new StringTokenizer(line);
        }
        token = st.nextToken();
      } catch (IOException e) { }
    return token;
  }
  private String nextToken() {
    String ans = peekToken();
```

```
    token = null;
    return ans;
  }
}
```

2. Data Structures

2.1. Binary Indexed Tree. Also called a fenwick tree. Builds in $\mathcal{O}(n \log n)$ from an array. Querry sum from 0 to i in $\mathcal{O}(\log n)$ and updates an element in $\mathcal{O}(\log n)$.

```
private static class BIT {
  long[] data;
  public BIT(int size) {
    data = new long[size+1];
  }
  public void update(int i, int delta) {
    while(i< data.length) {
      data[i] += delta;
      i += i&-i; // Integer.lowestOneBit(i);
    }
  }
  public long sum(int i) {
    long sum = 0;
    while(i>0) {
      sum += data[i];
      i -= i&-i;
    }
    return sum;
  }
}
```

2.2. Segment Tree. More general than a fenwick tree. Can adapt other operations than sum, e.g. min and max.

```
private static class ST {
  int li, ri;
  int sum; //change to max/min
  ST lN;
  ST rN;
}
static ST makeSgmTree(int[] A, int l, int r) {
  if(l == r) {
    ST node = new ST();
    node.li = l;
    node.ri = r;
    node.sum = A[l]; //max/min
    return node;
  }
  int mid = (l+r)/2;
  ST lN = makeSgmTree(A,l,mid);
  ST rN = makeSgmTree(A,mid+1,r);
  ST root = new ST();
  root.li = lN.li;
  root.ri = rN.ri;
  root.sum = lN.sum + rN.sum; //max/min
  root.lN = lN;
  root.rN = rN;
  return root;
}
static int getSum(ST root, int l, int r) {//max/min
  if(root.li>=l && root.ri<=r)
    return root.sum; //max/min
  if(root.ri<l || root.li > r)
    return 0; //minInt/maxInt
  else //max/min
    return getSum(root.lN,l,r) + getSum(root.rN,l,r);
}
static int update(ST root, int i, int val) {
  int diff = 0;
  if(root.li==root.ri && i == root.li) {
    diff = val-root.sum; //max/min
    root.sum=val; //max/min
    return diff; //root.max
  }
  int mid = (root.li + root.ri) / 2;
  if (i <= mid) diff = update(root.lN, i, val);
  else diff = update(root.rN, i, val);
  root.sum+=diff; //ask other child
  return diff; //and compute max/min
}
```

2.3. **Lazy Segment Tree.** More general implementation of a segment tree where its possible to increase whole segments by some diff, with lazy propagation. Implemented with arrays instead of nodes, which probably has less overhead to write during a competition.

```
class LazySegmentTree {
  private int n;
  private int[] lo, hi, sum, delta;
  public LazySegmentTree(int n) {
    this.n = n;
    lo = new int[4*n + 1];
    hi = new int[4*n + 1];
    sum = new int[4*n + 1];
    delta = new int[4*n + 1];
    init();
  }
  public int sum(int a, int b) {
    return sum(1, a, b);
  }
  private int sum(int i, int a, int b) {
    if(b < lo[i] || a > hi[i]) return 0;
    if(a <= lo[i] && hi[i] <= b) return sum(i);
    prop(i);
    int l = sum(2*i, a, b);
    int r = sum(2*i+1, a, b);
    update(i);
    return l + r;

  }
  public void inc(int a, int b, int v) {
    inc(1, a, b, v);
  }
  private void inc(int i, int a, int b, int v) {
    if(b < lo[i] || a > hi[i]) return;
    if(a <= lo[i] && hi[i] <= b) {
      delta[i] += v;
      return;
    }
    prop(i);
    inc(2*i, a, b, v);
    inc(2*i+1, a, b, v);
    update(i);
  }

  private void init() {
    init(1, 0, n-1, new int[n]);
  }
  private void init(int i, int a, int b, int[] v) {
    lo[i] = a;
    hi[i] = b;
    if(a == b) {
      sum[i] = v[a];
      return;
    }
    int m = (a+b)/2;
    init(2*i, a, m, v);
    init(2*i+1, m+1, b, v);
    update(i);
  }
  private void update(int i) {
    sum[i] = sum(2*i) + sum(2*i+1);
  }
  private int range(int i) {
    return hi[i] - lo[i] + 1;
  }
  private int sum(int i) {
    return sum[i] + range(i)*delta[i];
  }
  private void prop(int i) {
    delta[2*i] += delta[i];
    delta[2*i+1] += delta[i];
    delta[i] = 0;
  }
}
```

2.4. **Union Find.** This data structure is used in varoius algorithms, for example Kruskals algorithm for finding a Minimal Spanning Tree in a weighted graph. Also it can be used for backward simulation of dividing a set.

```
private class Node {
  Node parent;
  int h;
```

```java
  public Node() {
    parent = this;
    h = 0;
  }
  public Node find() {
    if(parent != this) parent = parent.find();
    return parent;
  }
}
static void union(Node x, Node y) {
  Node xR = x.find(), yR = y.find();
  if(xR == yR) return;
  if(xR.h > yR.h)
    yR.parent = xR;
  else {
    if(yR.h == xR.h) yR.h++;
    xR.parent = yR;
  }
}
```

2.5. **Monotone Queue.** Used in sliding window algorithms where one would like to find the minimum in each interval of a given length. Amortized $\mathcal{O}(n)$ to find min in each of these intervals in an array of length n. Can easily be used to find the maximum as well.

```java
private static class MinMonQue {
    LinkedList<Integer> que = new LinkedList<>();
    public void add(int i) {
        while(!que.isEmpty() && que.getFirst() > i)
            que.removeFirst();
        que.addFirst(i);
    }
    public int last() {
        return que.getLast();
    }
    public void remove(int i) {
        if(que.getLast() == i) que.removeLast();
    }
}
```

2.6. **Treap.** Treap is a binary search tree that uses randomization to balance itself. It's easy to implement, and gives you access to the internal structures of a binary tree, which can be used to find the k'th element for example. Because of the randomness, the average height is about a factor 4 of a prefectly balanced tree.

```java
class Treap{
  int sz;
  int v;
  double y;
  Treap L, R;

  static int sz(Treap t) {
    if(t == null) return 0;
    return t.sz;
  }
  static void update(Treap t) {
    if(t == null) return;
    t.sz = sz(t.L) + sz(t.R) + 1;
  }
  static Treap merge(Treap a, Treap b) {
    if (a == null) return b;
    if(b == null) return a;
    if (a.y < b.y) {
      a.R = merge(a.R, b);
      update(a);
      return a;
    } else {
      b.L = merge(a, b.L);
      update(b);
      return b;
    }
  }
  //inserts middle in left half
  static Treap[] split(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.v <= x) {
      Treap[] p = split(t.R, x);
      t.R = p[0];
      p[0] = t;
      return p;
    } else {
      Treap[] p = split(t.L, x);
      t.L = p[1];
```

```
      p[1] = t;
      return p;
    }
  }
  //use only with split
  static Treap insert(Treap t, int x) {
    Treap m = new Treap();
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, x-1);
    return merge(merge(p[0],m), p[1]);
  }


  //inserts middle in left half
  static Treap[] splitK(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.sz < x) return new Treap[]{t, null};
    if (sz(t.L) >= x) {
      Treap[] p = splitK(t.L, x);
      t.L = p[1];
      p[1] = t;
      update(p[0]);
      update(p[1]);
      return p;
    } else if (sz(t.L) + 1 == x){
      Treap r = t.R;
      t.R = null;
      Treap[] p = new Treap[]{t, r};
      update(p[0]);
      update(p[1]);
      return p;
    } else {
      Treap[] p = splitK(t.R, x - sz(t.L)-1);
      t.R = p[0];
      p[0] = t;
      update(p[0]);
      update(p[1]);
      return p;
    }
  }
  //use only with splitK
  static Treap insertK(Treap t, int w, int x) {
    Treap m = new Treap();
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, w);
    t = merge(p[0], m);
    return merge(t, p[1]);
  }
  //use only with splitK
  static Treap deleteK(Treap t, int w, int x) {
    Treap[] p = splitK(t, w);
    Treap[] q = splitK(p[0], w-1);
    return merge(q[0], p[1]);
  }

  static Treap Left(Treap t) {
    if (t == null) return null;
    if (t.L == null) return t;
    return Left(t.L);
  }
  static Treap Right(Treap t) {
    if (t == null) return null;
    if (t.R == null) return t;
    return Right(t.R);
  }

}
```

3. Graph Algorithms

3.1. Djikstras algorithm. Finds the shortest distance between two Nodes in a weighted graph in $\mathcal{O}(|E|\log|V|)$ time.

```
//Requires java.util.LinkedList and java.util.TreeSet
private static class Node implements Comparable<Node>{
  LinkedList<Edge> edges = new LinkedList<>();
  int w;
```

```
  int id;
  public Node(int id) {
    w = Integer.MAX_VALUE;
    this.id = id;
  }
  public int compareTo(Node n) {
    if(w != n.w) return w - n.w;
    return id - n.id;
  }
  //Asumes all nodes have weight MAXINT.
  public int djikstra(Node x) {
    this.w = 0;
    TreeSet<Node> set = new TreeSet<>();
    set.add(this);
    while(!set.isEmpty()) {
      Node curr = set.pollFirst();
      if(x == curr) return x.w;
      for(Edge e: curr.edges) {
        Node other = e.u == curr? e.v : e.u;
        if(other.w > e.cost + curr.w) {
          set.remove(other);
          other.w = e.cost + curr.w;
          set.add(other);
        }
      }
    }
    return -1;
  }
}
private static class Edge {
  Node u,v;
  int cost;
  public Edge(Node u, Node v, int c) {
    this.u = u; this.v = v;
    cost = c;
  }
}
```

3.2. **Bipartite Graphs.** The Hopcroft-Karp algorithm finds the maximal matching in a bipartite graph. Also, this matching can together with Könings theorem be used to construct a minimal vertex-cover, which as we all know is the complement of a maximum independent set. Runs in $\mathcal{O}(|E|\sqrt{|V|})$.

```
import java.util.*;
class Node {
  int id;
  LinkedList<Node> ch = new LinkedList<>();
  public Node(int id) {
    this.id = id;
  }
}
public class BiGraph {
  private static int INF = Integer.MAX_VALUE;
  LinkedList<Node> L, R;
  int N, M;
  Node[] U;
  int[] Pair, Dist;
  int nild;
  public BiGraph(LinkedList<Node> L, LinkedList<Node> R){
    N = L.size(); M = R.size();
    this.L = L; this.R = R;
    U = new Node[N+M];
    for(Node n: L) U[n.id] = n;
    for(Node n: R) U[n.id] = n;
  }
  private boolean bfs() {
    LinkedList<Node> Q = new LinkedList<>();
    for(Node n: L)
      if(Pair[n.id] == -1) {
        Dist[n.id] = 0;
        Q.add(n);
      }else
        Dist[n.id] = INF;

    nild = INF;
    while(!Q.isEmpty()) {
      Node u = Q.removeFirst();
      if(Dist[u.id] < nild)
        for(Node v: u.ch) if(distp(v) == INF){
          if(Pair[v.id] == -1)
            nild = Dist[u.id] + 1;
```

```java
        else {
          Dist[Pair[v.id]] = Dist[u.id] + 1;
          Q.addLast(U[Pair[v.id]]);
        }
      }
    }
    return nild != INF;
  }
  private int distp(Node v) {
    if(Pair[v.id] == -1) return nild;
    return Dist[Pair[v.id]];
  }
  private boolean dfs(Node u) {
    for(Node v: u.ch) if(distp(v) == Dist[u.id] + 1) {
      if(Pair[v.id] == -1 || dfs(U[Pair[v.id]])) {
        Pair[v.id] = u.id;
        Pair[u.id] = v.id;
        return true;
      }
    }
    Dist[u.id] = INF;
    return false;
  }
  public HashMap<Integer, Integer> maxMatch() {
    Pair = new int[M+N];
    Dist = new int[M+N];
    for(int i = 0; i<M+N; i++) {
      Pair[i] = -1;
      Dist[i] = INF;
    }
    HashMap<Integer, Integer> out = new HashMap<>();
    while(bfs()) {
      for(Node n: L) if(Pair[n.id] == -1)
        dfs(n);
    }
    for(Node n: L) if(Pair[n.id] != -1)
      out.put(n.id, Pair[n.id]);
    return out;
  }
  public HashSet<Integer> minVTC() {
    HashMap<Integer, Integer> Lm = maxMatch();
    HashMap<Integer, Integer> Rm = new HashMap<>();
    for(int x: Lm.keySet()) Rm.put(Lm.get(x), x);
    boolean[] Z = new boolean[M+N];
    LinkedList<Node> bfs = new LinkedList<>();
    for(Node n: L) {
      if(!Lm.containsKey(n.id)) {
        Z[n.id] = true;
        bfs.add(n);
      }
    }
    while(!bfs.isEmpty()) {
      Node x = bfs.removeFirst();
      int nono = -1;
      if(Lm.containsKey(x.id))
        nono = Lm.get(x.id);
      for(Node y: x.ch) {
        if(y.id == nono || Z[y.id]) continue;
        Z[y.id] = true;
        if(Rm.containsKey(y.id)){
          int xx = Rm.get(y.id);
          if(!Z[xx]) {
            Z[xx] = true;
            bfs.addLast(U[xx]);
          }
        }
      }
    }
    HashSet<Integer> K = new HashSet<>();
    for(Node n: L) if(!Z[n.id]) K.add(n.id);
    for(Node n: R) if(Z[n.id]) K.add(n.id);
    return K;
  }
}
```

3.3. **Network Flow.** The Floyd Warshall algorithm for determining the maximum flow through a graph can be used for a lot of unexpected problems. Given a problem that can be formulated as a graph, where no ideas are found trying, it might help trying to apply network flow. The running time is $\mathcal{O}(C \cdot m)$ where C is the maximum flow and m is the amount of edges in the graph. If C is very large we can change the running time to $\mathcal{O}(\log Cm^2)$ by only studying edges with a large enough capacity in the beginning.

```java
import java.util.*;
class Node {
  LinkedList<Edge> edges = new LinkedList<>();
  int id;
  boolean visited = false;
  Edge last = null;
  public Node(int id) {
    this.id = id;
  }
  public void append(Edge e) {
    edges.add(e);
  }
}
class Edge {
  Node source, sink;
  int cap;
  int id;
  Edge redge;
  public Edge(Node u, Node v, int w, int id){
    source = u; sink = v;
    cap = w;
    this.id = id;
  }
}
class FlowNetwork {
  Node[] adj;
  int edgec = 0;
  HashMap<Integer,Integer> flow = new HashMap<>();
  ArrayList<Edge> real = new ArrayList<Edge>();
  public FlowNetwork(int size) {
    adj = new Node[size];
    for(int i = 0; i<size; i++) {
      adj[i] = new Node(i);
    }
  }
  void add_edge(int u, int v, int w, int id){
    Node Nu = adj[u], Nv = adj[v];
    Edge edge = new Edge(Nu, Nv, w, edgec++);
    Edge redge = new Edge(Nv, Nu, 0, edgec++);
    edge.redge = redge;
    redge.redge = edge;
    real.add(edge);
    adj[u].append(edge);
    adj[v].append(redge);
    flow.put(edge.id, 0);
    flow.put(redge.id, 0);
  }

  void reset() {
    for(int i = 0; i<adj.length; i++) {
      adj[i].visited = false; adj[i].last = null;
    }
  }

  LinkedList<Edge> find_path(Node s, Node t,
          List<Edge> path){
    reset();
    LinkedList<Node> active = new LinkedList<>();
    active.add(s);
    while(!active.isEmpty() && !t.visited) {
      Node now = active.pollFirst();
      for(Edge e: now.edges) {
        int residual = e.cap - flow.get(e.id);
        if(residual>0 && !e.sink.visited) {
          e.sink.visited = true;
          e.sink.last = e;
          active.addLast(e.sink);
        }
      }
    }
    if(t.visited) {
      LinkedList<Edge> res = new LinkedList<>();
      Node curr = t;
      while(curr != s) {
        res.addFirst(curr.last);
        curr = curr.last.sink;
      }
      return res;
```

```
    } else return null;
  }

  int max_flow(int s, int t) {
    Node source = adj[s];
    Node sink = adj[t];
    LinkedList<Edge> path = find_path(source, sink,
            new LinkedList<Edge>());
    while (path != null) {
      int min = Integer.MAX_VALUE;
      for(Edge e : path) {
        min = Math.min(min, e.cap - flow.get(e.id));
      }
      for (Edge e : path) {
        flow.put(e.id, flow.get(e.id) + min);
        Edge r = e.redge;
        flow.put(r.id, flow.get(r.id) - min);
      }
      path = find_path(source, sink,
              new LinkedList<Edge>());
    }
    int sum = 0;
    for(Edge e: source.edges) {
      sum += flow.get(e.id);
    }
    return sum;
  }

  LinkedList<Edge> min_cut(int s, int t) {
    HashSet<Node> A = new HashSet<>();
    LinkedList<Node> bfs = new LinkedList<>();
    bfs.add(adj[s]);
    A.add(adj[s]);
    while(!bfs.isEmpty()) {
      Node i = bfs.removeFirst();
      for(Edge e: i.edges) {
        int c = e.cap - flow.get(e.id);
        if(c > 0 && !A.contains(e.sink)) {
          bfs.add(e.sink);
          A.add(e.sink);
          if(e.sink.id == t) return null;
        }
      }
    }
    LinkedList<Edge> out = new LinkedList<>();
    for(Node n: A) for(Edge e: n.edges)
      if(!A.contains(e.sink) && e.cap != 0)
        out.add(e);
    return out;
  }
}
```

3.4. **Min Cost Max Flow.** Finds the minimal cost of a maximum flow through a graph. Can be used for some optimization problems where the optimal assignment needs to be a maximum flow.

```
class MinCostMaxFlow {
boolean found[];
int N, dad[];
long cap[][], flow[][], cost[][], dist[], pi[];

static final long INF = Long.MAX_VALUE / 2 - 1;

boolean search(int s, int t) {
Arrays.fill(found, false);
Arrays.fill(dist, INF);
dist[s] = 0;

while (s != N) {
  int best = N;
  found[s] = true;
  for (int k = 0; k < N; k++) {
    if (found[k]) continue;
    if (flow[k][s] != 0) {
      long val = dist[s] + pi[s] - pi[k] - cost[k][s];
      if (dist[k] > val) {
        dist[k] = val;
        dad[k] = s;
      }
    }
    if (flow[s][k] < cap[s][k]) {
```

```
      long val = dist[s] + pi[s] - pi[k] + cost[s][k];
      if (dist[k] > val) {
        dist[k] = val;
        dad[k] = s;
      }
    }

    if (dist[k] < dist[best]) best = k;
  }
  s = best;
}
for (int k = 0; k < N; k++)
  pi[k] = Math.min(pi[k] + dist[k], INF);
return found[t];
}

long[] mcmf(long c[][], long d[][], int s, int t) {
cap = c;
cost = d;

N = cap.length;
found = new boolean[N];
flow = new long[N][N];
dist = new long[N+1];
dad = new int[N];
pi = new long[N];

long totflow = 0, totcost = 0;
while (search(s, t)) {
  long amt = INF;
  for (int x = t; x != s; x = dad[x])
    amt = Math.min(amt, flow[x][dad[x]] != 0 ?
    flow[x][dad[x]] : cap[dad[x]][x] - flow[dad[x]][x]);
  for (int x = t; x != s; x = dad[x]) {
    if (flow[x][dad[x]] != 0) {
      flow[x][dad[x]] -= amt;
      totcost -= amt * cost[x][dad[x]];
    } else {
      flow[dad[x]][x] += amt;
      totcost += amt * cost[dad[x]][x];
    }
  }
  totflow += amt;
}

return new long[]{ totflow, totcost };
}
}
```

4. Dynamic Programming

4.1. **Longest Increasing Subsequence.** Finds the longest increasing subsequence in an array in $\mathcal{O}(n \log n)$ time. Can easility be transformed to longest decreasing/nondecreasing/nonincreasing subsequence.

```
public static int lis(int[] X) {
  int n = X.length;
  int P[] = new int[n];
  int M[] = new int[n+1];
  int L = 0;
  for(int i = 0; i<n; i++) {
    int lo = 1;
    int hi = L;
    while(lo<=hi) {
      int mid = lo + (hi - lo + 1)/2;
      if(X[M[mid]]<X[i])
        lo = mid+1;
      else
        hi = mid-1;
    }
    int newL = lo;
    P[i] = M[newL-1];
    M[newL] = i;
    if (newL > L)
      L = newL;

  }
  int[] S = new int[L];
  int k = M[L];
  for (int i = L-1; i>=0; i--) {
    S[i] = k; //or X[k]
```

```
    k = P[k];
  }
  return L; // or S
}
```

4.2. String functions. The z-function computes the longest common prefix of t and $t[i:]$ for each i in $\mathcal{O}(|t|)$. The border function computes the longest common proper (smaller than whole string) prefix and suffix of string $t[:i]$.

```python
def zfun(t):
    z = [0]*len(t)
    n = len(t)
    l, r = (0,0)
    for i in range(1,n):
        if i < r:
            z[i] = min(z[i-l], r-i+1)
        while z[i] + i < n and t[i+z[i]] == t[z[i]]:
            z[i]+=1
        if i + z[i] - 1 > r:
            l = i
            r = i + z[i] - 1
    return z

def matches(t, p):
    s = p + '#' + t
    return filter(lambda x: x[1] == len(p),
            enumerate(zfun(s)))

def boarders(s):
    b = [0]*len(s)
    for i in range(1, len(s)):
        k = b[i-1]
        while k>0 and s[k] != s[i]:
            k = b[k-1]
        if s[k] == s[i]:
            b[i] = k+1
    return b
```

5. Etc

5.1. System of Equations. Solves the system of equations $\boldsymbol{Ax} = \boldsymbol{b}$ by Gaussian elimination. This can for example be used to determine the expected value of each node in a markov chain. Runns in $\mathcal{O}(N^3)$.

```java
//Computes A^-1 * b
static double[] solve(double[][] A, double[] b) {
  int N = b.length;
  // Gaussian elimination with partial pivoting
  for (int i = 0; i < N; i++) {
    // find pivot row and swap
    int max = i;
    for (int j = i + 1; j < N; j++)
      if (Math.abs(A[j][i]) > Math.abs(A[max][i]))
        max = j;
    double[] tmp = A[i];
    A[i] = A[max];
    A[max] = tmp;
    double tmp2 = b[i];
    b[i] = b[max];
    b[max] = tmp2;
    // A doesn't have full rank
    if (Math.abs(A[i][i])<0.00001) return null;
    // pivot within b
    for (int j = i + 1; j < N; j++)
      b[j] -= b[i] * A[j][i] / A[i][i];
    // pivot within A
    for (int j = i + 1; j < N; j++) {
      double m = A[j][i] / A[i][i];
      for (int k = i+1; k < N; k++)
        A[j][k] -= A[i][k] * m;
      A[j][i] = 0.0;
    }
  }
  // back substitution
  double[] x = new double[N];
  for (int j = N - 1; j >= 0; j--) {
    double t = 0.0;
    for (int k = j + 1; k < N; k++)
      t += A[j][k] * x[k];
```

```
    x[j] = (b[j] - t) / A[j][j];
  }
  return x;
}
```

5.2. **Convex Hull.** From a collection of points in the plane the convex hull is often used to compute the largest distance or the area covered, or the length of a rope that encloses the points. It can be found in $\mathcal{O}(N \log N)$ time by sorting the points on angle and the sweeping over all of them.

```
import java.util.*;
public class ConvexHull {
  static class Point implements Comparable<Point> {
    static Point xmin;
    int x, y;
    public Point(int x, int y) {
      this.x = x;this.y = y;
    }
    public int compareTo(Point p) {
      int c = cross(this, xmin, p);
      if(c!=0) return c;
      double d = dist(this,xmin) - dist(p,xmin);
      return (int) Math.signum(d);
    }
  }
  static double dist(Point p1, Point p2) {
    return Math.hypot(p1.x - p2.x, p1.y - p2.y);
  }
  static int cross(Point a, Point b, Point c) {
    int dx1 = b.x - a.x;
    int dy1 = b.y - a.y;
    int dx2 = c.x - b.x;
    int dy2 = c.y - b.y;
    return dx1*dy2 - dx2*dy1;
  }
  Point[] convexHull(Point[] S) {
    int N = S.length;
    // find a point on the convex hull.
    Point xmin = S[0];
    int id = 0;
    for(int i = 0; i<N; i++) {
      Point p = S[i];
      if(xmin.x > p.x ||
         xmin.x == p.x && xmin.y > p.y) {
        xmin = p;
        id = i;
      }
    }
    S[id] = S[N-1];
    S[N-1] = xmin;
    Point.xmin = xmin;
    // Sort on angle to xmin.
    Arrays.sort(S, 0, N-1);
    Point[] H = new Point[N+1];
    H[0] = S[N-2];
    H[1] = xmin;
    for(int i = 0; i<N-1; i++)
      H[i+2] = S[i];
    int M = 1;
    // swipe over the points
    for(int i = 2; i<=N; i++) {
      while(cross(H[M-1],H[M],H[i]) <= 0) {
        if(M>1)
          M--;
        else if (i == N)
          break;
        else
          i += 1;
      }
      M+=1;
      Point tmp = H[M];
      H[M] = H[i];
      H[i] = tmp;
    }
    Point[] Hull = new Point[M];
    for(int i = 0; i<M; i++)
      Hull[i] = H[i];
    return Hull;
  }
}
```

5.3. Number Theory.

```python
def gcd(a, b):
  return b if a%b == 0 else gcd(b, a%b)

# returns g = gcd(a, b), x0, y0,
# where g = x0*a + y0*b
def xgcd(a, b):
  x0, x1, y0, y1 = 1, 0, 0, 1
  while b != 0:
    q, a, b = (a // b, b, a % b)
    x0, x1 = (x1, x0 - q * x1)
    y0, y1 = (y1, y0 - q * y1)
  return (a, x0, y0)

def crt(la, ln):
    assert len(la) == len(ln)
    for i in range(len(la)):
        assert 0 <= la[i] < ln[i]
    prod = 1
    for n in ln:
        assert gcd(prod, n) == 1
        prod *= n
    lN = []
    for n in ln:
        lN.append(prod//n)
    x = 0
    for i, a in enumerate(la):
        print(lN[i], ln[i])
        _, Mi, mi = xgcd(lN[i], ln[i])
        x += a*Mi*lN[i]
    return x % prod

# finds x^e mod m
def modpow(x, m, e):
    res = 1
    while e:
        if e%2 == 1:
            res = (res*x) % m
        x = (x*x) % m
        e = e//2
    return res

# Divides a list of digits with an int.
# A lot faster than using bigint-division.
def div(L, d):
  r = [0]*(len(L) + 1)
  q = [0]*len(L)
  for i in range(len(L)):
    x = int(L[i]) + r[i]*10
    q[i] = x//d
    r[i+1] = x-q[i]*d
  s = []
  for i in range(len(L) - 1, 0, -1):
    s.append(q[i]%10)
    q[i-1] += q[i]//10

  while q[0]:
    s.append(q[0]%10)
    q[0] = q[0]//10
  s = s[::-1]
  i = 0
  while s[i] == 0:
    i += 1
  return s[i:]


# Multiplies a list of digits with an int.
# A lot faster than using bigint-multiplication.
def mul(L, d):
  r = [d*x for x in L]
  s = []
  for i in range(len(r) - 1, 0, -1):
    s.append(r[i]%10)
    r[i-1] += r[i]//10
  while r[0]:
    s.append(r[0]%10)
    r[0] = r[0]//10
  return s[::-1]

large_primes = [
```

```
5915587277,
1500450271,
3267000013,
5754853343,
4093082899,
9576890767,
3628273133,
2860486313,
5463458053,
3367900313
        ]
```

6. NP tricks

6.1. **MaxClique.** The max clique problem is one of Karp's 21 NP-complete problems. The problem is to find the lagest subset of an undirected graph that forms a clique - a complete graph. There is an obvious algorithm that just inspects every subset of the graph and determines if this subset is a clique. This algorithm runns in $\mathcal{O}(n^2 2^n)$. However one can use the meet in the middle trick (one step divide and conquer) and reduce the complexity to $\mathcal{O}(n^2 2^{\frac{n}{2}})$.

```
static int max_clique(int n, int[][] adj) {
  int fst = n/2;
  int snd = n - fst;
  int[] maxc = new int[1<<fst];
  int max = 1;
  for(int i = 0; i<(1<<fst); i++) {
    for(int a = 0; a<fst; a++) {
      if((i&1<<a) != 0)
        maxc[i] = Math.max(maxc[i], maxc[i^(1<<a)]);
    }
    boolean ok = true;
    for(int a = 0; a<fst; a++) if((i&1<<a) != 0) {
      for(int b = a+1; b<fst; b++) {
          if((i&1<<b) != 0 && adj[a][b] == 0)
              ok = false;
      }
    }
    if(ok) {
      maxc[i] = Integer.bitCount(i);
      max = Math.max(max, maxc[i]);
    }
  }
  for(int i = 0; i<(1<<snd); i++) {
    boolean ok = true;
    for(int a = 0; a<snd; a++) if((i&1<<a) != 0) {
      for(int b = a+1; b<snd; b++) {
        if((i&1<<b) != 0)
          if(adj[a+fst][b+fst] == 0)
            ok = false;
      }
    }
    if(!ok) continue;
    int mask = 0;
    for(int a = 0; a<fst; a++) {
      ok = true;
      for(int b = 0; b<snd; b++) {
        if((i&1<<b) != 0) {
          if(adj[a][b+fst] == 0) ok = false;
        }
      }
      if(ok) mask |= (1<<a);
    }
    max = Math.max(Integer.bitCount(i) + maxc[mask],
            max);
  }
  return max;
}
```

7. Coordinate Geometry

7.1. **Area of a nonintersecting polygon.** The signed area of a polygon with n verticies is given by

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$$

7.2. **Intersection of two lines.** Two lines defined by

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Intersects in the point

$$P = (\frac{b_1c_2 - b_2c_1}{w}, \frac{a_2c_1 - a_1c_2}{w}),$$

where $w = a_1b_2 - a_2b_1$. If $w = 0$ the lines are parallell.

7.3. **Distance between line segment and point.** Given a linesegment between point P, Q, the distance D to point R is given by:

$$\begin{aligned}
a &= Q_y - P_y \\
b &= Q_x - P_x \\
c &= P_xQ_y - P_yQ_x \\
R_P &= (\frac{b(bR_x - aR_y) - ac}{a^2 + b^2}, \frac{a(aR_y - bR_x) - bc}{a^2 + b^2}) \\
D &= \begin{cases} \frac{|aR_x + bR_y + c|}{\sqrt{a^2+b^2}} & \text{if } (R_{P_x} - P_x)(R_{P_x} - Q_x) < 0, \\ \min |P - R|, |Q - R| & \text{otherwise} \end{cases}
\end{aligned}$$

7.4. **Picks theorem.** Find the amount of internal integer coordinates i inside a polygon with picks theorem $A = \frac{b}{2} + i - 1$, where A is the area of the polygon and b is the amount of coordinates on the boundary.

7.5. **Implementations.**

```python
import math

# Distance between two points
def dist(p, q):
  return math.hypot(p[0]-q[0], p[1] - q[1])

# Converts two points to a line (a, b, c),
# ax + by + c = 0
# if p == q, a = b = c = 0
def pts2line(p, q):
  return (-q[1] + p[1],
      q[0] - p[0],
      p[0]*q[1] - p[1]*q[0])

# Distance from a point to a line,
# given that a != 0 or b != 0
def distl(l, p):
  return (abs(l[0]*p[0] + l[1]*p[1] + l[2])
      /math.hypot(l[0], l[1]))

# intersects two lines.
# if parallell, returnes False.
def inters(l1, l2):
  a1,b1,c1 = l1
  a2,b2,c2 = l2
  cp = a1*b2 - a2*b1
  if cp != 0:
    return ((b1*c2 - b2*c1)/cp, (a2*c1 - a1*c2)/cp)
  else:
    return False

# projects a point on a line
def project(l, p):
  a, b, c = l
  return ((b*(b*p[0] - a*p[1]) - a*c)/(a*a + b*b),
      (a*(a*p[1] - b*p[0]) - b*c)/(a*a + b*b))

# Finds the distance between a point, and
# the Segment AB, the Ray AB and the line AB.
# (distSeg, distHalfinf, distLine)
def distSegP(P, Q, R):
  a, b, c = pts2line(P, Q)
  Rpx, Rpy = project((a,b,c), R)
  dp = min(dist(P, R), dist(Q, R))
  dl = distl((a,b,c), R)
  if (inside(Rpx, P[0], Q[0]) and
          inside(Rpy, P[1], Q[1])):
    return (dl, dl, dl)
  if insideH((Rpx, Rpy), P, Q):
    return (dp, dl, dl)
  return (dp, dp, dl)

# Finds if A <= i <= B.
def inside(i, A, B):
  return (i-A)*(i-B) <= 0

# Finds if a point i on the line AB
# is on the segment AB.
def insideS(i, A, B):
```

```
  return (inside(i[0], A[0], B[0])
    and inside(i[1], A[1], B[1]))

# Finds if a point i on the line AB
# is on the ray AB.
def insideH(i, A, B):
  return ((i[0] - A[0])*(A[0] - B[0]) <= 0
    and (i[1] - A[1])*(A[1] - B[1]) <= 0)

# Finds if segment AB and CD overlabs.
def overlap(A, B, C, D):
    if A[0] == B[0]:
        return __overlap(A[1], B[1], C[1], D[1])
    else:
        return __overlap(A[0], B[0], C[0], D[0])
# Helper functions
def __overlap(x1, x2, x3, x4):
    x1, x2 = (min(x1,x2), max(x1, x2))
    x3, x4 = (min(x3,x4), max(x3, x4))
    return x2 >= x3 and x1 <= x4

# prints a point
def p(P):
    print(str(P[0]) + ' ' + str(P[1]))

# prints common points between
# two segments AB and CD.
def SegSeg(A, B, C, D):
  eqa = A == B
  eqc = C == D
  if eqa and eqc:
    if A == C:
      p(A)
      return True
    return False
  if eqc:
    eqa, A, B, C, D = (eqc, C, D, A, B)
  if eqa:
    l = pts2line(C, D)
    if (l[0]*A[0] + l[1]*A[1] + l[2] == 0 and
        inside(A[0], C[0], D[0]) and
        inside(A[1], C[1], D[1])):
        p(A)
        return True
      return False

    A, B = tuple(sorted([A,B]))
    C, D = tuple(sorted([C,D]))
    l1 = pts2line(A, B)
    l2 = pts2line(C, D)
    if l1[0]*l2[1] == l1[1]*l2[0]:
      if l1[0]*l2[2] == l1[2]*l2[0]:
        if overlap(A, B, C, D):
          if B == C:
            p(B)
            return True
          if D == A:
            p(A)
            return True
          s = sorted([A,B,C,D])
          p(s[1])
          p(s[2])
          return True
        else:
          return False
      else:
        return False
    ix, iy = inters(l1, l2)
    if (inside(ix, A[0], B[0]) and
      inside(iy, A[1], B[1]) and
      inside(ix, C[0], D[0]) and
      inside(iy, C[1], D[1])):
      p((ix, iy))
      return True
    return False

# Intersections between circles
def intersections(c1, c2):
  x1, y1, r1 = c1
  x2, y2, r2 = c2
```

```python
  if x1 == x2 and y1 == y2 and r1 == r2:
    return False
  if r1 > r2:
    x1, y1, r1, x2, y2, r2 = (x2, y2, r2, x1, y1, r1)
  dist2 = (x1 - x2)*(x1-x2) + (y1 - y2)*(y1 - y2)
  rsq = (r1 + r2)*(r1 + r2)
  if dist2 > rsq or dist2 < (r1-r2)*(r1-r2):
    return []
  elif dist2 == rsq:
    cx = x1 + (x2-x1)*r1/(r1+r2)
    cy = y1 + (y2-y1)*r1/(r1+r2)
    return [(cx, cy)]
  elif dist2 == (r1-r2)*(r1-r2):
    cx = x1 - (x2-x1)*r1/(r2-r1)
    cy = y1 - (y2-y1)*r1/(r2-r1)
    return [(cx, cy)]

  d = math.sqrt(dist2)
  f = (r1*r1 - r2*r2 + dist2)/(2*dist2)
  xf = x1 + f*(x2-x1)
  yf = y1 + f*(y2-y1)
  dx = xf-x1
  dy = yf-y1
  h = math.sqrt(r1*r1 - dx*dx - dy*dy)
  norm = abs(math.hypot(dx, dy))
  p1 = (xf + h*(-dy)/norm, yf + h*(dx)/norm)
  p2 = (xf + h*(dy)/norm, yf + h*(-dx)/norm)
  return sorted([p1, p2])

# Finds the bisector through origo
# between two points by normalizing.
def bisector(p1, p2):
  d1 = math.hypot(p1[0], p2[1])
  d2 = math.hypot(p2[0], p2[1])
  return ((p1[0]/d1 + p2[0]/d2),
          (p1[1]/d1 + p2[1]/d2))

# Distance from P to origo
def norm(P):
  return (P[0]**2 + P[1]**2 + P[2]**2)**(0.5)
```

```python
# Finds ditance between point p
# and line A + t*u in 3D
def dist3D(A, u, p):
  AP = tuple(A[i] - p[i] for i in range(3))
  cross = tuple(AP[i]*u[(i+1)%3] - AP[(i+1)%3]*u[i]
    for i in range(3))
  return norm(cross)/norm(u)
```

8. Acieving AC on a solved problem

8.1. WA.

- Check that minimal input passes.
- Can an int overflow?
- Reread the problem statement.
- Start creating small testcases with python.
- Does cout print with high enough precision?
- Abstract the implementation.

8.2. TLE.

- Is the solution sanity checked?
- Use pypy instead of python.
- Rewrite in C++ or Java.
- Can we apply DP anywhere?
- To minimize penalty time you should create a worst case input (if easy) to test on.
- Binary Search over the answer?

8.3. RTE.

- Recursion limit in python?
- Arrayindex out of bounds?
- Division by 0?
- Modifying iterator while iterating over it?
- Not using a well definied operator for Collections.sort?
- If nothing makes sense and the end of the contest is approaching you can binary search over where the error is with try-except.

8.4. MLE.

- Create objects outside recursive function.
- Rewrite recursive solution to itterative with an own stack.

9. Practice Contest Checklist

- Operations per second in py2
- Operations per second in py3
- Operations per second in java
- Operations per second in c++
- Operations per second on local machine
- Is MLE called MLE or RTE?
- What happens if extra output is added? What about one extra new line or space?
- Look at documentation on judge.
- Submit a clar.
- Print a file.
- Directory with test cases.