The Noisy Euclidean Traveling Salesman Problem: A Computational Analysis

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Summary. Consider a truck that visits n households each day. The specific households (and their locations) vary slightly from one day to the next. In the noisy traveling salesman problem, we develop a rough (skeleton) route which can then be adapted and modified to accommodate the actual node locations that need to be visited from day to day. In this paper, we conduct extensive computational experiments on problems with n = 100, 200, and 300 nodes in order to compare several heuristics for solving the noisy traveling salesman problem including a new method based on quad trees. We find that the quad tree approach generates high-quality results quickly.

Key words: Traveling salesman problem; computational analysis; average trajectory.

1 Introduction

The Euclidean Traveling Salesman Problem (TSP) is a well-known combinatorial optimization problem that is easy to state – given a complete graph $G = \{N, E\}$, where N is the set of nodes, E is the set of edges, and the distances are Euclidean, find the shortest tour that visits every node in N exactly once – and difficult to solve optimally. Algorithmic developments and computational results are covered by Jünger et al. [6], Johnson and McGeoch [5], Coy et al. [2], Pepper et al. [9], and others.

Recently, Braun and Buhmann [1] introduced the following variant of the TSP which they refer to as the Noisy Traveling Salesman Problem (NTSP). "Consider a salesman who makes weekly trips. At the beginning of each week, the salesman has a new set of appointments for the week, for which he has to plan the shortest round-trip. The location of the appointments will not be completely random, because there are certain areas which have a higher probability of containing an appointment, for example cities or business districts within cities. Instead of solving the planning problem each week from scratch, a clever salesman will try to exploit the underlying density and have a rough trip pre-planned, which he will only adapt from week to week." In this paper, we consider a salesman who makes daily trips.

Braun and Buhmann viewed each node in a TSP as being sampled from a probability distribution, so that many TSP instances could be drawn from the same distribution. They used the sampled instances to build an *average trajectory* that was not forced to visit every node. For Braun and Buhmann, the average trajectory was "supposed to capture the essential structure of the underlying probability density." The average trajectory would then be used as the "seed" to generate an actual tour for each new day of appointments. Braun and Buhmann applied their average trajectory approach to a problem with 100 nodes.

To make the problem more concrete, consider the following. Each day, companies such as Federal Express and United Parcel Service send thousands of trucks to make local deliveries to households all across the United States. Let's focus on one of these trucks. Each day, it visits approximately the same number of households, in the same geographic region. The specific households may change from one day to the next, but the basic outline of the route remains the same. For example, if the truck visits the household located at 10 Main Street today, it might visit 15 Main Street instead (across the street) tomorrow. In the noisy traveling salesman problem, we develop a rough (skeleton) route which can then be adapted and modified to accommodate the actual node locations that need to be visited, from day to day.

We point out that the NTSP is similar to, but different from, the Probabilistic Traveling Salesman Problem (PTSP). In the PTSP, only a subset k ($0 \leq k \leq n$) out of n demand points needs to be visited on a daily basis. The demand point locations are known with certainty. See Jaillet [4] for details. Connections between the PTSP and the NTSP are discussed by Li [8].

In this paper, we conduct extensive computational experiments using three different data sets with different underlying structures to test Braun and Buhmann's approach, a simple convex hull, cheapest insertion heuristic, and a new heuristic (called the *quad tree approach*) that we develop for generating an average trajectory.

In Section 2, we describe Braun and Buhmann's approach. We show how they generate an average trajectory and then use it to produce an actual tour. We present their limited computational results. In Section 3, we conduct our extensive computational experiments. In Section 4, we develop the quad tree approach and test its performance. In Section 5, we present our conclusions and directions for future research.