Hi Dongwei and Jin,

Thank you for your report. Read all the comments. In the resubmitted version all comments marked as TO BE RESUBMITTED must be addressed.

- Each cell in this notebook contains either code or text.
- You can run a cell by pressing Ctrl-Enter, or run and advance to the next cell with Shift-Enter.
- Code cells will print their output, including images, below the cell. Running it again deletes the previous output, so be careful if you want to save some results.
- You don't have to rerun all cells to test changes, just rerun the cell you have made changes to. Some exceptions might apply, for example if you overwrite variables from previous cells, but in general this will work.
- If all else fails, use the "Kernel" menu and select "Restart Kernel and Clear All Output". You can also use this menu to run all cells.
- A useful debug tool is the console. You can right-click anywhere in the notebook and select "New console for notebook". This opens a python console which shares the environment with the notebook, which let's you easily print variables or test commands.

Setup

In [ ]:
```
%reload_ext autoreload
%autoreload 2

import numpy as np
from matplotlib import pyplot as plt
from scipy import io as sio

from utils import GenerateHaarFeatureMasks, GenerateTrainTestData, Extrac
from utils import PlotErrorGraphs, PlotClassifications, PlotSelectedHaarF
plt.rcParams['figure.facecolor']='white'

# Load data
faces = sio.loadmat('Data/faces.mat')['faces'].astype("float")
nonfaces = sio.loadmat('Data/nonfaces.mat')['nonfaces'].astype("float")
```

! IMPORTANT NOTE !

Your implementation should only use the numpy module (already imported as np). The numpy module provides all the functionality you need for this assignment and makes easier debuging your code. No other modules, e.g. scikit-learn or scipy among others, are allowed and solutions using modules other than the numpy module will be sent for re-submission.

1 Introduction

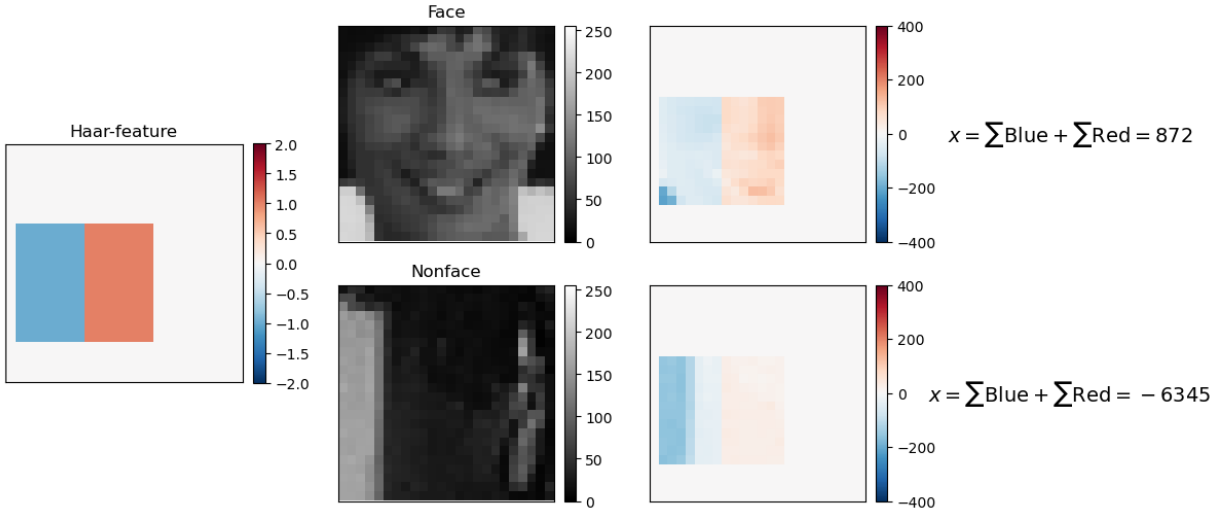
In this assignment you will explore a branch of supervised learning called *Ensemble lerning*, specifically using the algorithm known as *AdaBoost*. This algorithm trains multiple simple *weak classifiers* that are later combined into a *strong classifier* which is significantly more powerful than the individual *weak classifiers*. The *weak classifier* you will work with is a simple type of binary classifier called decision stump. These classify scalar inputs based on a single comparisson with a threshold.

In order to apply this classifier to images, we require a way to describe the features of the images using a single number that the decision stump can compare to the threshold. In a seminal article by Viola and Jones, called Robust real-time object detection, this was demonstrated using so called Haar-features, which is what we will use in this assignment.

A Haar-feature is a simple yet surprisingly effective way to quantify edges and gradients in images, by computing the weighted sum of different areas of the image. We will begin by exploring how these Haar-features work, in order to understand the fundamental building blocks of the algorithm you will implement later.

Run the following code cell to generate a random Haar-feature, select a random face and nonface image, and compute the feature values by applying the Haar-feature to the images.

In [ ]:
```
# Generate random Haar-feature and select a random face and a random non-
randomHaarFeatureMask = GenerateHaarFeatureMasks(1)[:,:,0]
randomFace = faces[:,:,np.random.randint(faces.shape[2])]
randomNonFace = nonfaces[:,:,np.random.randint(nonfaces.shape[2])]

PlotHaarFeatureDemonstration(haar=randomHaarFeatureMask, face=randomFace,
```



The Haar-features we generate in this assignment consist of either two or three areas, each with a scaling factor of -1, 1, or 2. The pixels outside the Haar-feature has a scaling of 0. The images are pixel-wise multiplied with the Haar-feature, after which the entire image is summed to a single scalar value. This weighted sum of pixels is the feature value of the image, which can be used in the weak classifier.

Of course, since the Haar-feature in this picture is chosen entierly at random, we should not expect that it makes sense as a good feature extractor. Finding a good set of Haar-features is your task, but we will get back to that soon. Before that, take a look at the decision stump classifier again. Now that we know how to extract a single feature value from an image, we can see how a desision stump can be used to classify the images as faces or nonfaces. We can write the decision stump as a function:

$$f(x \mid \tau) = \begin{cases} +1 & \text{if } x \geq \tau \\ -1 & \text{if } x < \tau \end{cases}$$

As you have already seen in the lecture on boosting and ensemble learning, this can also be extended to a more general definition by including a polarity parameter:

$$f(x \mid \tau, p) = \begin{cases} +1 & \text{if } px \geq p\tau \\ -1 & \text{if } px < p\tau \end{cases}$$

By selecting p equal to +1 or -1, we can flip the sign of the predicted output. Since this is a binary classification, this also changes the error of the classifier from e to $1 - e$. This is necessary if we want to guarantee that we can find the optimal classifier.

1.1 Generating the training data

Since a single random Haar-feature is unlikely to be a good feature extractor, we need to generate a large number of them and find the ones that are useful. But extracting the feature values from the images during training is very inefficient, so instead we precompute the feature value for each Haar-feature applied on each image. This will therefore result in a feature value matrix $X^{[R,C]}$ (i.e. R rows and C columns), where R is the number of Haar-feature and C is the number of images in the dataset, as illustrated in this picture:

Note that the training loop will only work with this matrix, never with the original images or Haar-features.

1.2 The AdaBoost algorithm

AdaBoost is an ensemble learning algorithm that successively trains many weak classifiers that are informed about the weaknesses of the previous classifiers. This is done by keeping track of a weight value d for each training image, representing how "difficult" that image has been to classify during training so far. After a weak classifier has been trained, the predictions of that classifier is used to update the weights of all images. If an image was classified correctly the weight is exponentially decreased, and the next weak classifier will therefore put less emphasis on that sample since it is already taken care of by the previous weak classifiers. The weights of misclassifed images are instead exponentially increased to make them more important for the next weak classifier.

In more detail, if a weak classifier results in an error ε, we compute the update factor

$$\alpha = \frac{1}{2}\ln\left(\frac{1-\varepsilon}{\varepsilon}\right)$$

and then update the weights according to

$$d_{t+1} = \begin{cases} d_t e^{-\alpha} & \text{if correctly classified} \\ d_t e^{\alpha} & \text{if misclassified} \end{cases}$$

We also normalize d_{t+1} such that the total weight of all samples is 1. The next weak classifier is then trained using the new weights d_{t+1}, which will put the focus on missclassified images and therefore result in a new combination of optimal parameters. This is how AdaBoost gradually covers more and more of the difficult cases in the data, even thought the same training images and Haar-feature are used for each weak classifier.

When the training of all weak classifiers is finished we use them in a weighted voting scheme to build the *strong classifier*.

$$H(x) = \text{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

Note that we use the update factor α for each weak classifier as a weight in the voting. This is because weak classifiers with lower total error should have more voting power to increase the overall performance of the strong classifier.

Question 1:

The error of a single weak classifier is always in the range 0 to 0.5 (since we otherwise flip the polarity). Consider the two edge cases:

1. If you get an optimal $\varepsilon = 0$ this single weak classifier correctly predicted the entire training set. What does this indicate about your hyperparameters (number of Haar-features, number of training images, number of weak classifiers)?
2. If you get an optimal $\varepsilon = 0.5$ the training will get stuck. Explain why this is the case.

Answer:

1. If you get an optimal $\varepsilon = 0$ this single weak classifier correctly predicted the entire training set. What does this indicate about your hyperparameters (number of Haar-features, number of training images, number of weak classifiers)?
   - It implies that we may have a small amount of Haar-features and training images(which can make it eaiser to make $\epsilon = 0$), but we might have a lot of weak classifiers to make ϵ eventually reach 0
2. If you get an optimal $\varepsilon = 0.5$ the training will get stuck. Explain why this is the case.
   - A binary classifier which $\varepsilon = 0.5$ have the same effect as flip a coin and make a decision. It will update the weight by increse those of the wrong side and decrese those of the correct side, and result in another classifier with $\varepsilon = 0.5$ but only reverse the label it gives to the data points.

1.3 Pseudo-code

With all this in mind, we can summarize the entire algorithm in the following pseudo-code:

```
TrainWeakClassifier:
    for every combination of (feature, threshold, polarity):
        Run WeakClassifier
        Measure error
        if new smallest error:
            Save parameters
    return optimal parameters


TrainStrongClassifer:
    Initialize weights for each training sample
    for number of weak classifiers:
        TrainWeakClassifier
        Measure error and get predictions
        Compute update parameter alpha
        Update training sample weights
    return optimal parameters
```

---

2. Implementing the weak classifier

You will now start by implementing the decision stump weak classifier, the weighted error function, and the training loop for a single weak classifier.

2.1 Weak classifier forward function

The forward function classifies the data according to the threshold and polarity.

In [ ]:
```python
# Implement decision stump classifier
# X - Feature values (vector)(1*Nsamples)
# t - Threshold (scalar)
# p - Polarity (1 or -1)
# Return a vector of predicted classes (1 or -1), same shape as X.
# classes-(1*Nsamples)

def WeakClassifier(X,t,p):
    # classes = []
    # for xVal in X:
    #     if xVal*p >= t*p:
    #         classes.append(1)
    #     else:
    #         classes.append(-1)
    classes = [1 if x*p >= t*p else -1 for x in X]
    # classes = list(map( lambda x: 1 if x*p >= t*p else -1, X))

    # Nsamples = len(X)
    # classes = np.zeros(Nsamples)
    # for image in range(Nsamples):
    #     classes[image] = 1 if X[image]*p >= t*p else -1

    return classes
```

2.2 Weighted classification error

This function computes the weighted classification error based on predicted and true classes, as well as the weight vector.

In [ ]:
```python
# Implement error function
# Y  - Target classes (vector)(1*Nsamples)
# Yp - Predicted classes (vector)(1*Nsamples)
# D  - Weights (vector)(1*Nsamples)
# Return a scalar for the total weighted error of all predictions.

def WeakClassifierError(Y,Yp,D):
    nY = len(Y)
    eps = 0
    for pointI in range(nY):
        if Y[pointI]*Yp[pointI] != 1:
            eps += D[pointI]

    return eps
```

2.3 Training function for a single weak classifier

Here you should implement the training of a single weak classifier, using the two functions you just implemented. Based on the training data, sample weights, and target labels, find the optimal way to separate the two classes. You should do this using brute force search, in other words, try every (reasonable) combination of parameters, measure the errors, and pick the best. Return all parameters that define the optimal weak classifier, as well as the error of said weak classifier.

Tip: Brute force search is not an efficient algorithm, so any optimization you can do is welcome. When you have a working version of this function, consider if there is any way you can decrease the amount of calls to WeakClassifier, especially by reducing the number of thresholds. Of course, you must still guarantee that you can find the optimal parameters, so for example naively discarding every other threshold will not work.

In [ ]:
```
# Implement function for training one decision stump
# X - Training data (matrix)(Nhaar*Nsamples)
# Y - Target classes (vector)(1*Nsamples)
# D - Error weights (vector)(1*Nsamples)
# Return optimal feature, threshold, polarity, and the corresponding erro
# fOpt(row of x), tOpt, pOpt, eMin

def TrainWeakClassifier(X,Y,D):

    fOpt = None
    tOpt = None
    pOpt = None
    eMin = np.inf

    # -----------------------------------------
    # === Your code here ======================
    # -----------------------------------------
    Xrows, Xcols = X.shape


    for row in range(Xrows):


        # find optimal t for each row(haar-feature)
        sortedRow = sorted(X[row])
        for pointI in range(len(sortedRow)):
            if pointI < len(sortedRow)-1:
                t = 0.5*(sortedRow[pointI] +sortedRow[pointI+1])
            else:
                t = sortedRow[pointI]+ 0.5*(sortedRow[pointI] - sortedRow
        # for t in X[row]:

            p = 1
            Yp = WeakClassifier(X[row],t,p)
            # print(Yp)
```

```
            eps = WeakClassifierError(Y, Yp, D)
            # print(eps)
            if eps > 0.5:
                p = -1
                eps = 1 - eps
            if eps < eMin:
                eMin = eps
                fOpt = row
                tOpt = t
                pOpt = p

        # test
        # print(eps, eMin)
        # print(row, fOpt, tOpt, pOpt, eMin)
        # test end




    # ============================================

    return fOpt, tOpt, pOpt, eMin
```

Test your implementation by running the following minimal test case. You should find that the optimal feature is 1 (remember that python is 0-indexed!), the optimal threshold is between 1 and 3 (depending on your implementation), and the optimal polarity is -1. The minimum error should be 0.222 repeating.

In [ ]:
```
X = np.array([[2, 2, 2, 2, 2, 3], [3, 3, 3, 1, 1, 4], [4, 2, 4, 2, 4, 2]])
Y = np.array([-1,-1, -1, 1, 1, 1])
D = np.array([1,1,1,2,2,2]) / 9

fOpt, tOpt, pOpt, eMin = TrainWeakClassifier(X,Y,D)
print(f"Optimal feature   : {fOpt}")
print(f"Optimal threshold : {tOpt}")
print(f"Optimal polarity  : {pOpt}")
print(f"Minumum error     : {eMin}")
```

```
Optimal feature   : 1
Optimal threshold : 2.0
Optimal polarity  : -1
Minumum error     : 0.2222222222222222
```

---

3. Implementing the strong classifier

You should now implement the main AdaBoost algorithm, using the training function for weak classifiers you just implemented. Return the optimal parameters for all weak classifiers. We have given you some boilerplate code here to help you get started, and to provide some useful outputs during the training.

In [ ]:
```
from time import perf_counter as tic
```

```
from datetime import timedelta

# Implement AdaBoost training
# X - Training data (matrix)(Nhaar*Nsamples)
# Y - Target classes (vector)(1*Nsamples)
# N - Number of weak classifiers to train (scalar)
# Return the optimal features, thresholds, polarities, and alphas describ
# (1*Nclassifiers)

def TrainStrongClassifier(X,Y,N=10):

    fOpt = np.zeros(N,"int")
    tOpt = np.zeros(N)
    pOpt = np.zeros(N)
    aOpt = np.zeros(N)

    # ------------------------------------------
    # === Your code here =======================
    # ------------------------------------------

    # Initialize sample weights here
    M = len(Y)
    D = [1/M]*M
    # ==========================================

    timeStart = tic()

    for n in range(N):

        # ------------------------------------------
        # === Your code here =======================
        # ------------------------------------------

        fOpt[n], tOpt[n], pOpt[n], eMin = TrainWeakClassifier(X,Y,D)
        aOpt[n] = 0.5*np.log((1-eMin)/ eMin)

        Yp = WeakClassifier(X[fOpt[n]], tOpt[n], pOpt[n])

        classifyCorrectness = np.multiply(Yp, Y)
        D = np.multiply(D, np.exp(-np.multiply(classifyCorrectness,aOpt[n
        D = D / np.sum(D)

        # test
        # print(classifyCorrectness[:10])
        # print(D[:10])
        # print("max D", max(D))
        # test end

        # ==========================================

        # This prints the training progress in a nice format
        timeLeft = round((tic()-timeStart)*(N/(n+1) - 1))
        etaStr = str(timedelta(seconds=timeLeft))
        print(f"{n+1} of {N}: ETA {etaStr}    ", end="\r")

    return fOpt, tOpt, pOpt, aOpt
```

Finally, you should implement the StrongClassifier function. This function takes in the parameters returned from the training function and classifies the input X. If input Y is also provided, the error of the strong classifier should also be computed and returned.

Additionally, it is interesting to evaluate the strong classifier not only on the final ensemble model, but how the performance improved during the training, as more and more weak classifiers were added. As such, the error returned from this function should be a vector with the same lenght as the number of weak classifiers. The first value in the vector should only use the first weak classifier. The second value should use the first two classifiers in the ensemble, and so on. In general, the n:th entry is the error when using the first n weak classifiers as a strong classifier. This way we can plot the performance as a function of number of weak classifiers, which is important to investigate, for example, overfitting.

Tip: There is a function in numpy called `cumsum`*, which stands for cumulative sum. It is not mandatory to use this, but the code for computing the error vector can be very short if you do.*

In [ ]:
```python
# Implement strong classifier
# Params - Parameters of the weak classifiers (output from AdaBoost train
# X - Data to classify (matrix)
# Y - Target labels (vector). Optional input, see text description above.
# Return predicted classes (vector)(1*Nsample) and optionally the error (

def StrongClassifier(Params, X, Y=None):
    # Params = (fOpt([0:Nclassifier]), tOpt, pOpt, aOpt)
    fOpt, tOpt, pOpt, aOpt = Params
    H = None
    Err = None

    # -------------------------------------------
    # === Your code here ========================
    # -------------------------------------------
    Nhaar, Nsample = X.shape
    Nclassifier = len(fOpt)
    H = np.zeros(shape=(Nsample))
    Err = np.zeros(shape=(Nclassifier))

    h = np.zeros(shape=(Nclassifier, Nsample))
    for n in range(Nclassifier):
        h[n,:] = WeakClassifier(X[fOpt[n]], tOpt[n], pOpt[n])



    for n in range(Nclassifier):
        for image in range(Nsample):
            H[image] = np.sign(sum(np.multiply(aOpt[:n+1], h[:n+1,image])
        if Y is not None:
            Err[n] = 0.5*(Nsample - sum(np.multiply(H,Y))) / Nsample

    if Y is None:
        return H


    # ===========================================

    return H, Err
```

4. Training and evaluating the model

Now you will run the AdaBoost training and evaluate the results. We have provided all required code for plotting the results, but if there are additional results you wish to show you are welcome to make your own plots as well.

The following cell generates a set of random Haar-features, then precomputes the feature values and splits the data into a training set and a test set. Change the three hyperparamters; number of Haar-features, number of training samples, and number of weak classifiers to obtain the target performance 7% error or lower. Of course the error should be stable below the target as well.

In [ ]:
```
# Generate Haar feature masks
haarFeatureMasks = GenerateHaarFeatureMasks(nbrHaarFeatures = 100)

# Extract feature values and split into training and test sets
trainImages, testImages, XTrain, YTrain, XTest, YTest = GenerateTrainTest

# Train the classifier
optParams = TrainStrongClassifier(XTrain, YTrain, N = 40)
```

```
40 of 40: ETA 0:00:00
```

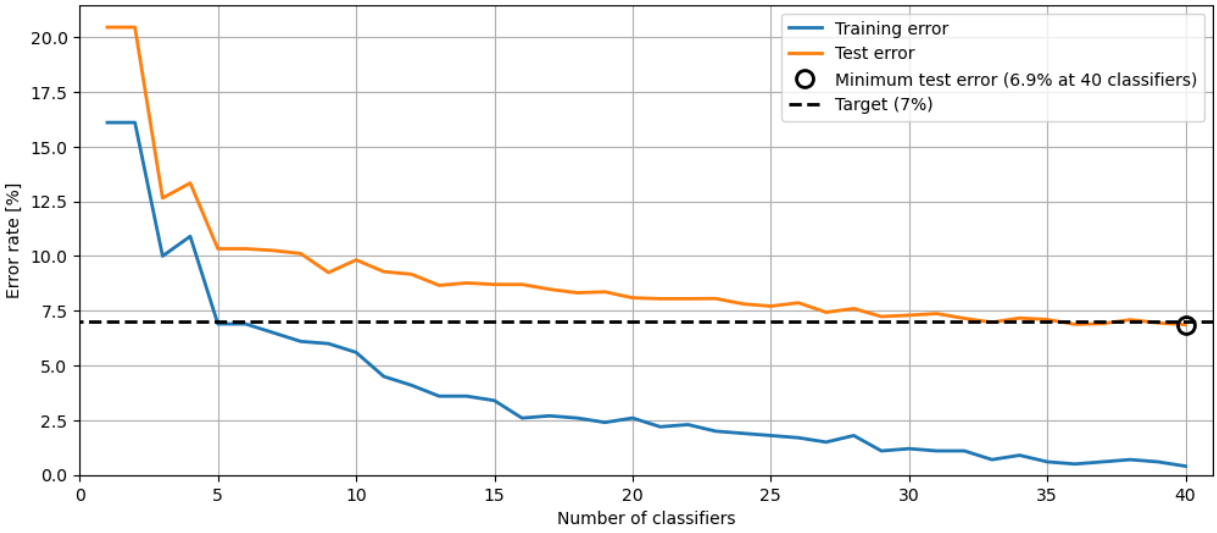
Now evaluate your classifier on the training data and test data.

In [ ]:
```
HTrain, ErrTrain = StrongClassifier(optParams, XTrain, Y = YTrain)
HTest , ErrTest  = StrongClassifier(optParams, XTest , Y = YTest)

ErrTrain *= 100
ErrTest  *= 100

print(f"Training error: {ErrTrain[-1]:.1f}%")
print(f"Test error: {ErrTest[-1]:.1f}%")
```

```
Training error: 0.4%
Test error: 6.9%
```

Plot the error of the strong classifier as a function of the number of weak classifiers.

In [ ]:
```
PlotErrorGraphs(ErrTrain, ErrTest)
```



Question 2:

Suppose you want to deploy your model in a real-world application, for example in a smartphone camera app that can highlight faces. Based on the above result, how many weak classifiers would you choose to include in the model? Motivate your answer.

Answer:

we should include as much weak classifier as we can, but with an early stop implemented. Because the training is done in previous so the time cost is not a major concern. And the time cost of implementing a ready-to-go model doesn't matter that much with number of weak classifiers for modern cell phones.

Question 3:

Discuss your choise of hyperparameters

- number of training images
- number of Haar-features
- number of weak classifiers

and how each affect the model performance. What are the advantages and disadvantages to low and high values for each parameter?

Answer:

- number of training images
  - the more the better, it helps reduce the overfitting and increse the accuracy, but cost a lot of time when training if it's too large. Too low value will result in overfitting and low accuracy. I would mostly like to choose a higher value but it would be too time-consuming when training.
- number of Haar-features
  - the more the better, but if it' too large the improvement of value incresing will decrease and make it not time-worthy. Too low value will result in low efficiency of training because there won't be enough haar-feature for model to choose and make the model have a poor appearance for not having satisfing enough features to use.
- number of weak classifiers
  - the more the better, especially when we will sum all weak classifier with a weight. Which makes that even when a single weakclassifier don't have any improvement when iterating, it will still help when we ensemble them into a strong classifier. But we need to take care of overfitting and the raise of test error which may make the corresponding classifiers harmful instead towards strong classifier. But it's not the case for our currentmodel, more weak classifiers is still welcome but we did't choose a higher value out of time concerning.

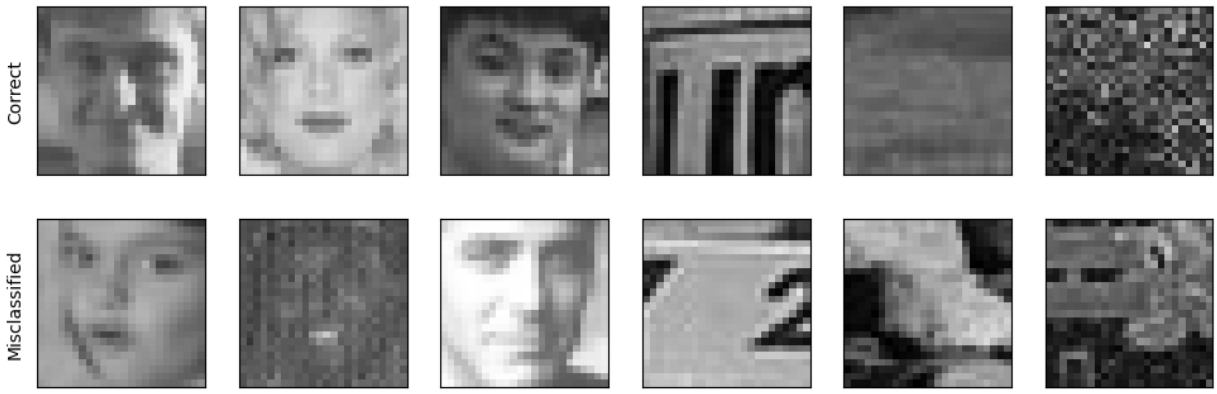
4.1 Investigate the trained model

Plot some examples of faces and nonfaces that were classified correctly and that were misclassified. To do this, we requre you to find the index in the test data of these four cases:

- Faces classified as faces (F_F)
- Faces classified as nonfaces (F_NF)
- Nonfaces classified as faces (NF_F)
- Nonfaces classified as nonfaces (NF_NF)

There are many ways to do this. It is possible to write each in a single line using numpy functions, although this is not required.

In [ ]:
```
F_F, F_NF, NF_F, NF_NF = [],[],[],[]
for i in range(len(HTest)):
    if YTest[i] == 1:
        if HTest[i] == 1:
            F_F.append(i)
        if HTest[i] == -1:
            F_NF.append(i)
    if YTest[i] == -1:
        if HTest[i] == 1:
            NF_F.append(i)
        if HTest[i] == -1:
            NF_NF.append(i)

PlotClassifications(testImages, F_F, F_NF, NF_NF, NF_F, N=3, selectRandom
```



Question 4:

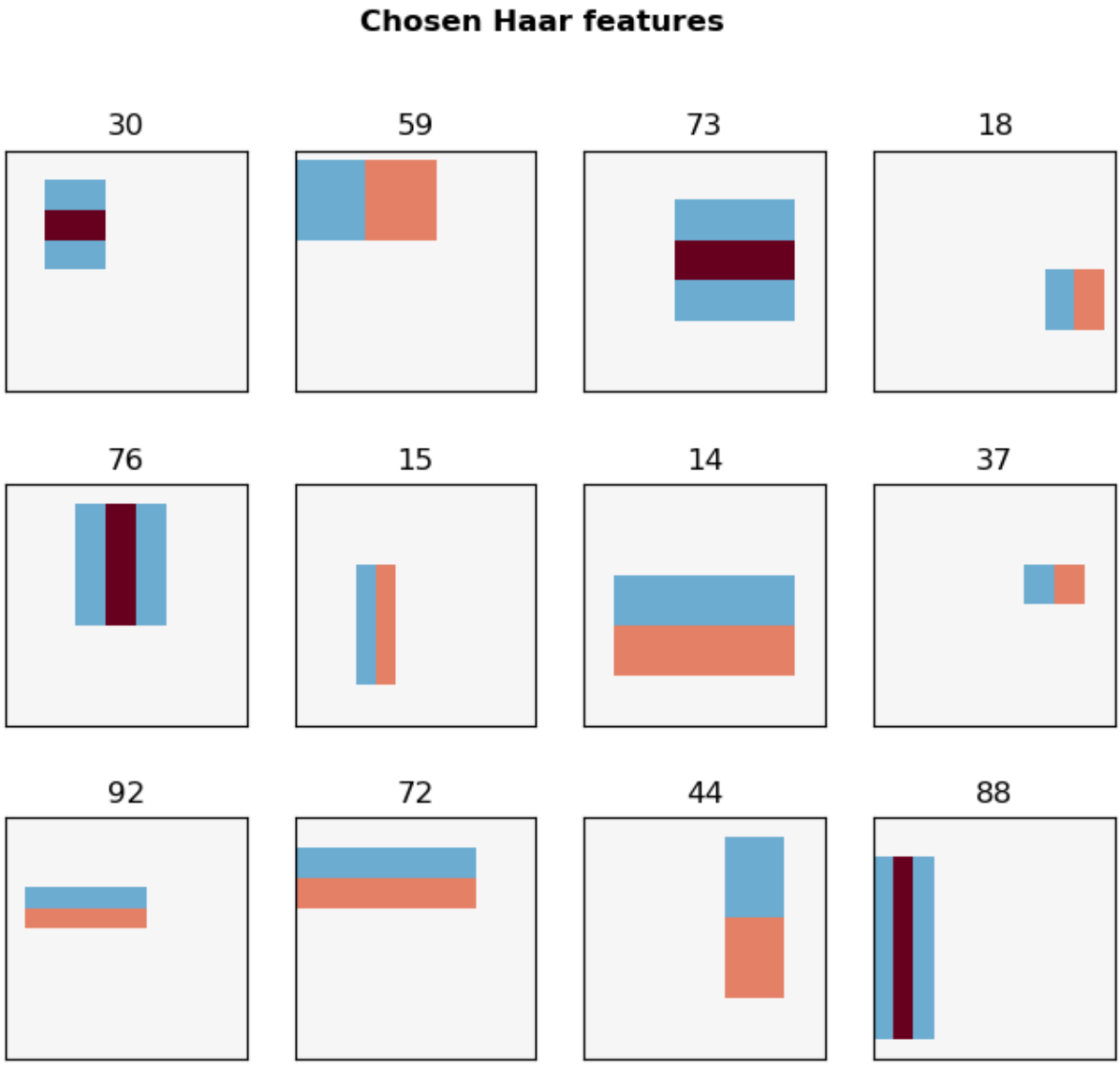
Run the above cell a few times and discuss some potential issues in the data that might explain these misclassified images. Can you think of any pre-processing that might improve the results?

Answer:

we can see that the lightning of pictures may affect the result, and there're pooly taken pictures that can't even be classified by human such as the 2nd Misclassified. we can doing some kind of normalization to make all pictures having a same mean and standard deviation of picture's pixels' grayscale value. Before that, we can dismiss pictures with a low standard deviation among pixels' grayscale value such as the 2nd Misclassified picture to rule out potentially poorly taken pictures which don't worth using.

Run the following code to plot (some) of your selected Haar-features. Feel free to change the paramters `shuffle` and `N`.

In [ ]:
```
# Inputs:
# - haarFeatureMasks: The list of all generated Haar features
# - selectedIdx: The feature numbers of Haar features selected in the tra
# - shuffle (False/True): Select in random order or as they were selected
# - N: The number of Haar features to plot

PlotSelectedHaarFeatures(haarFeatureMasks, selectedIdx = optParams[0], sh
```



Question 5:

Choose one or two of these Haar-features and speculate what feature of a face this might detect.

Answer:

for feature 76, it might detecting noses and there both side for a front view face with front lightning.
for feature 15, it might detecting noses with side lightning that will make one side of node in light and another side in shade.

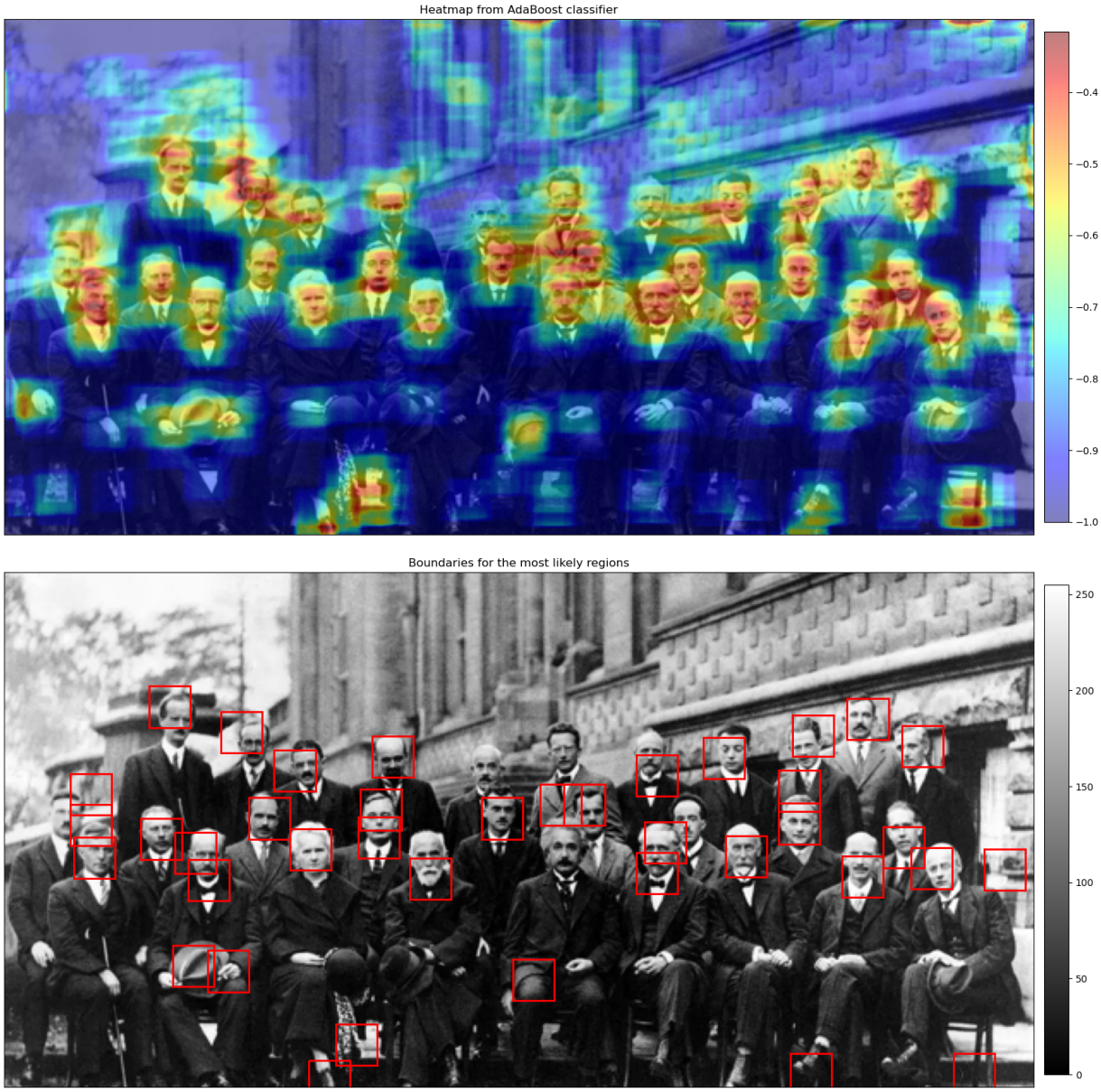
---

5. Applying the model on real data

Evaluating on the test data is, of course, a valid method to measure the objective performance of the model. However, if doesn't really give a feel for how useful the model would actually be in a real world application. This final test applies the model to every 24x24 pixel subimage in this famous photograph of the 1927 Solvay Conference on Physics, and highlights the most likely parts of the image to contain faces.

Let's see how many of these famous faces your model can find!

In [ ]:
```
solvay = sio.loadmat('Data/solvay.mat')['X'].astype("float")
xSolvay = ExtractHaarFeatures(solvay, haarFeatureMasks)

cSolvay = StrongClassifier(optParams, xSolvay)

PlotSolvayHeatmap(cSolvay)
```



Question 6:

Give a final discussion based on all the results. Do you think the model works well? Can you think of any way to improve the results?

Answer:

in general, the model is good, but there is still a lot of improvement to do.
First, the hyperparameter of TrainStrongClassifier can be improved a lot if we cost more time for a better result. In a way we discusssed above.
Second, the detail of implementation can be improved, such as the choosen of t for each weak classifier, it might be better to choose it in a different way taking the overall distribution of trainning value into consideration