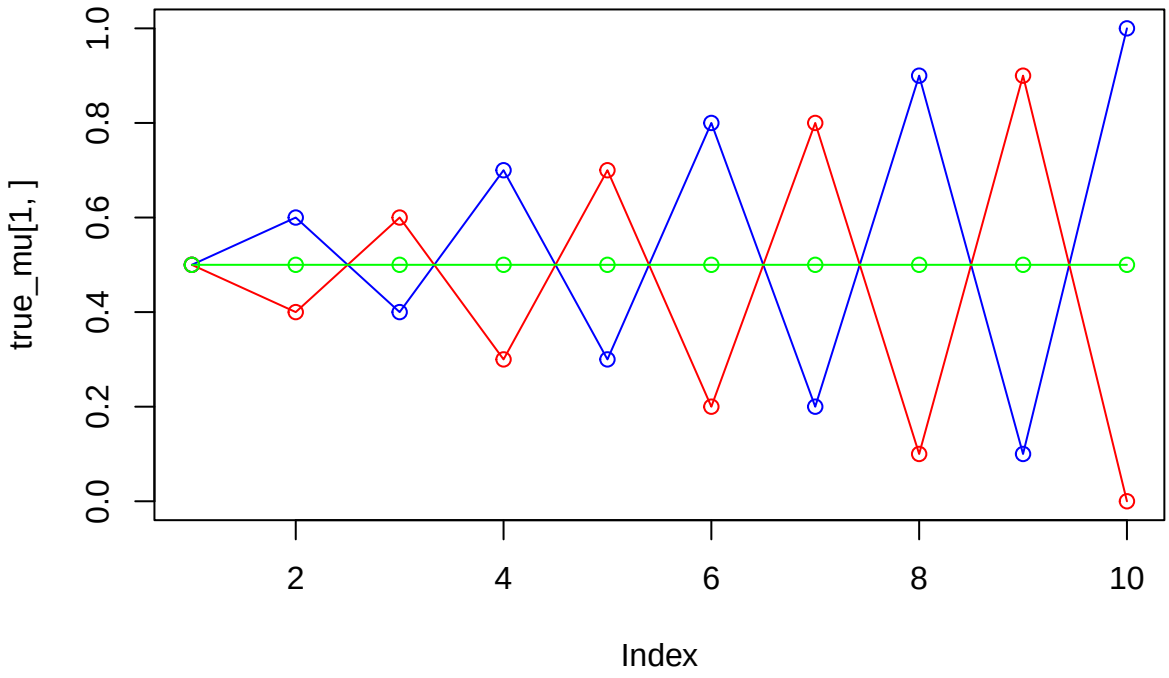
lab block2 Assginment 2

Group A20

2022-12-08

2. Mixture Models

For implementing the EM algorithm for Bernoulli mixture model First we generate a π and μ of a Bernoulli mixture model

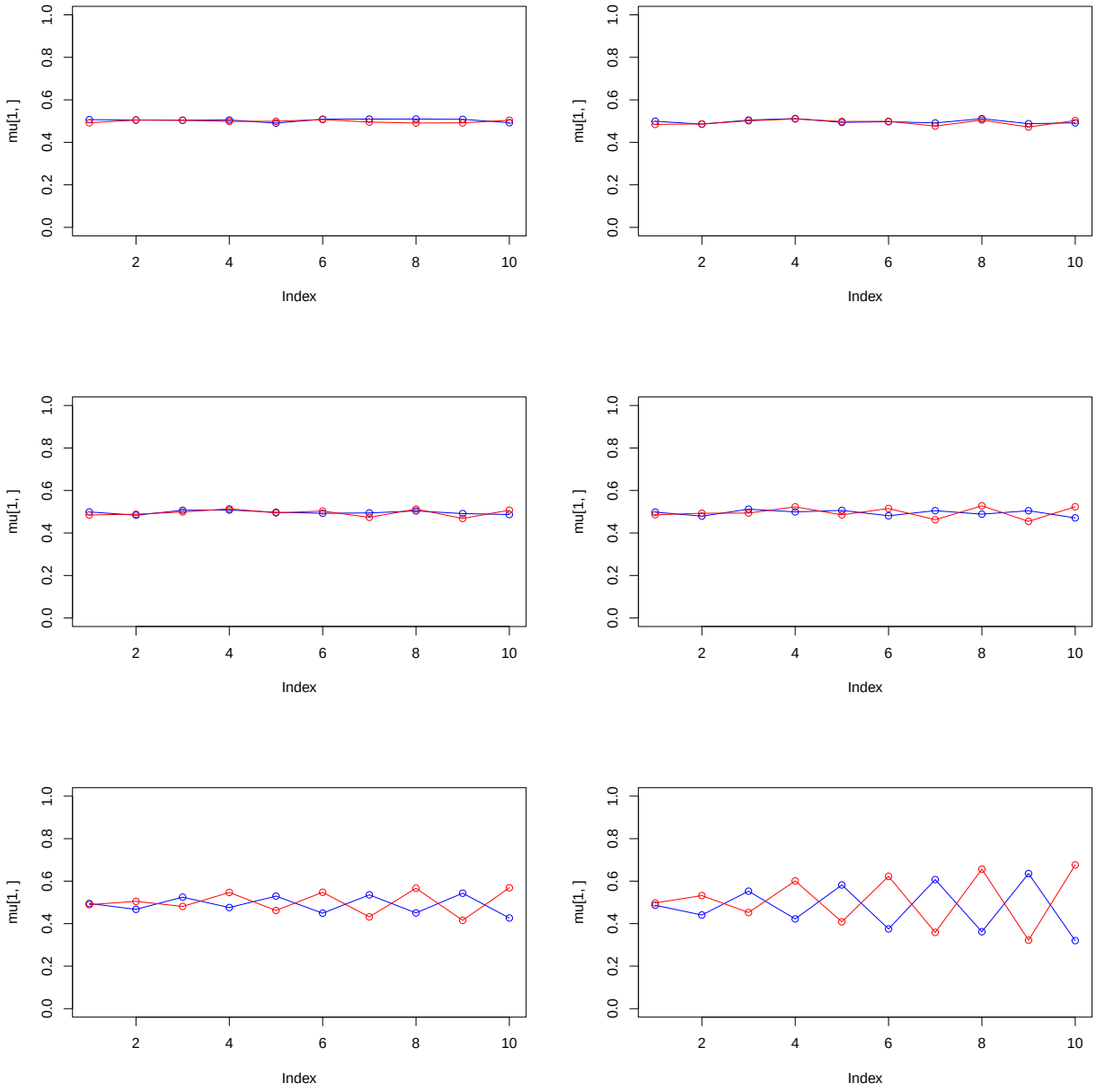


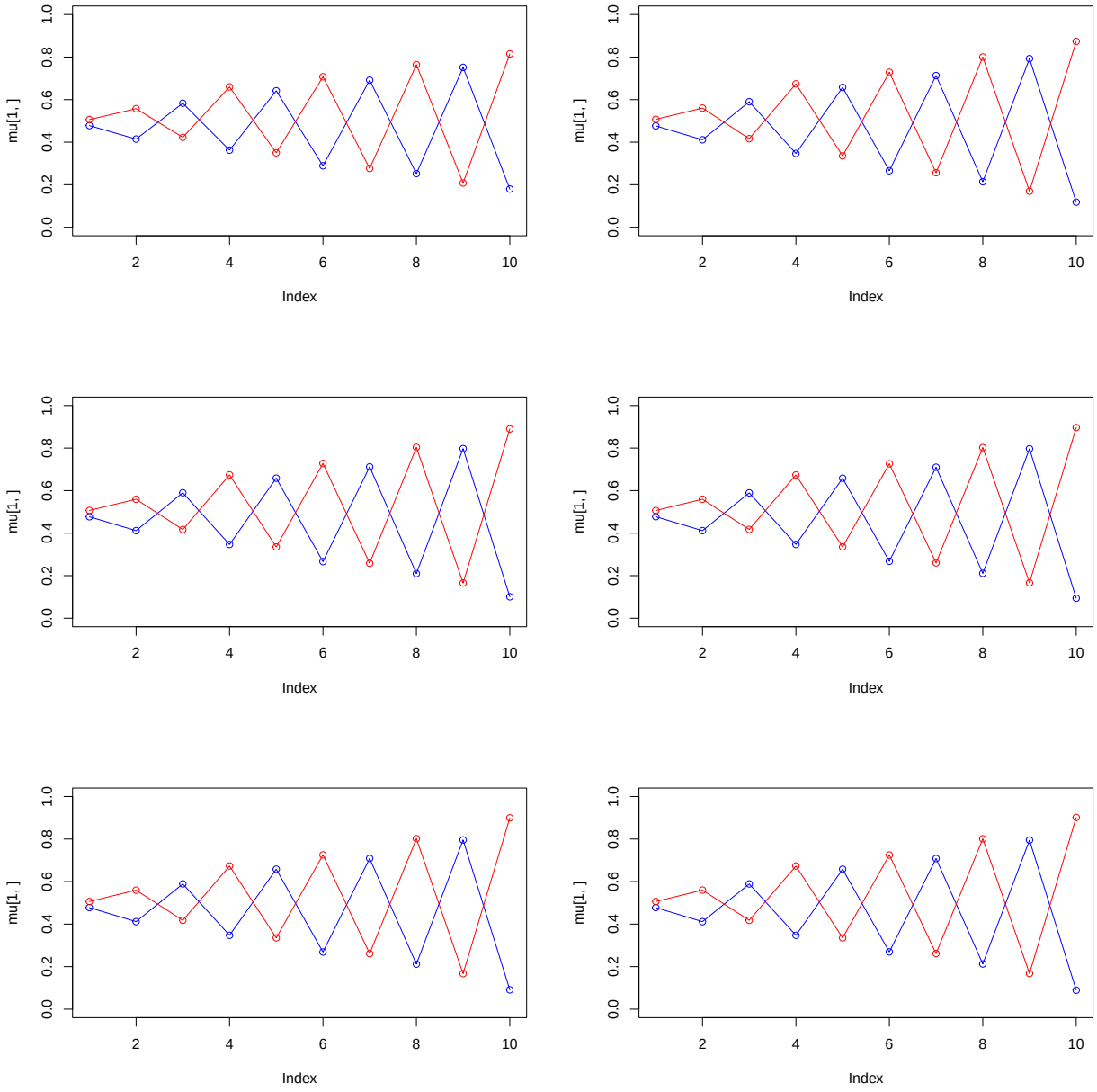
And the plot above shows the 3 μs we are using to generate data set x, with a same $\pi = \frac{1}{3}$ for each μ, and of course with a $Dimension = 10$.

```
## [1] 0.5052178 0.4947822
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]     [,7]
## [1,] 0.5068040 0.5049455 0.5041509 0.5051382 0.4905578 0.5089228 0.509453
## [2,] 0.4921113 0.5048207 0.5036886 0.4985467 0.4991731 0.5071384 0.495380
##           [,8]      [,9]     [,10]
## [1,] 0.5097480 0.5082991 0.4926313
## [2,] 0.4908757 0.4917657 0.5040657
```

First we start with cluster M = 2. And we generate a π and μ under M = 2 as the start point for our model.

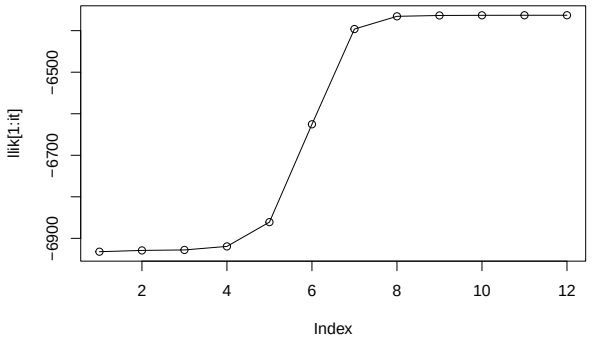





```
##  [1] "iteration:  1 log likelihood:  -6932.1625943363"
##  [2] "iteration:  2 log likelihood:  -6929.09536498845"
##  [3] "iteration:  3 log likelihood:  -6927.94380531483"
##  [4] "iteration:  4 log likelihood:  -6919.43478131895"
##  [5] "iteration:  5 log likelihood:  -6860.93456150249"
##  [6] "iteration:  6 log likelihood:  -6625.43370682142"
##  [7] "iteration:  7 log likelihood:  -6396.10695394879"
##  [8] "iteration:  8 log likelihood:  -6365.47503740155"
##  [9] "iteration:  9 log likelihood:  -6363.41093129657"
## [10] "iteration:  10 log likelihood:  -6363.02364141854"
## [11] "iteration:  11 log likelihood:  -6362.91755252171"
## [12] "iteration:  12 log likelihood:  -6362.88538329492"
```

```
## [1] 0.4979156 0.5020844
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4776009 0.4114577 0.5889962 0.3474863 0.6581537 0.2691781 0.7083954
## [2,] 0.5062796 0.5599234 0.4177178 0.6731561 0.3351926 0.7249220 0.2614677
##           [,8]      [,9]      [,10]
## [1,] 0.2125293 0.7950371 0.08913449
## [2,] 0.8010174 0.1675785 0.90147909
```

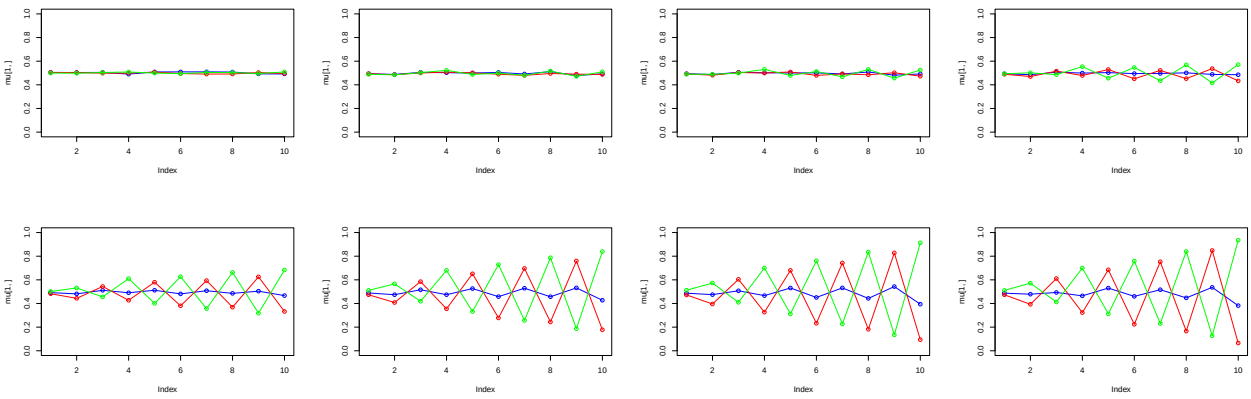


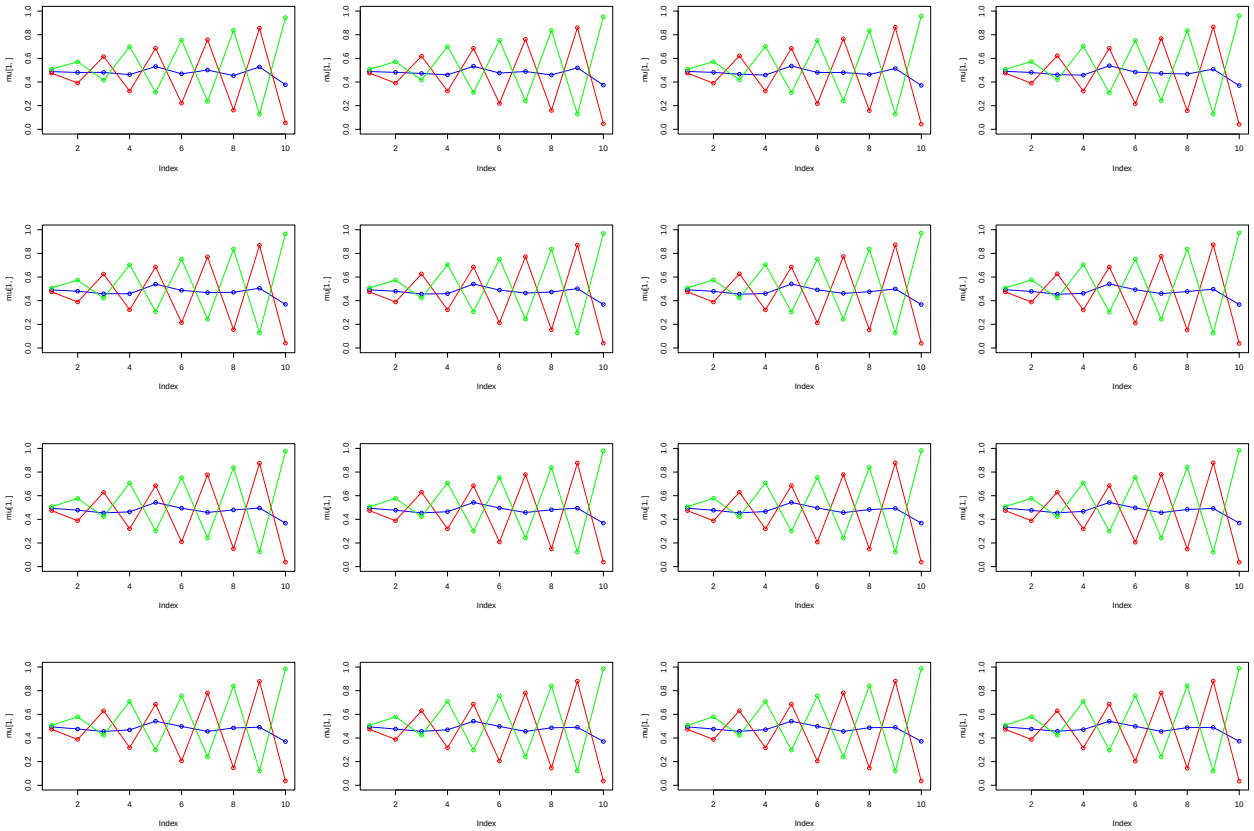
From the plots above. We can see that under this situation, μ which is equal to 0.5 every dimension seems did not being reflect. It might be that the other 2 μs are symmetrical to the 0.5 axis.
And for every iteration the predicted μ is closer and closer to the *true* μ. The rapidly raise of log-likelihood during iteration 5-7 is also reflect by the plot from significant changing of μ in corresponding iterations. The final predicted μ shows above is relatively close to the true value within around 10% differences.

```
## [1] 0.3359578 0.3290183 0.3350239
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.5049455 0.5041509 0.5051382 0.4905578 0.5089228 0.5094530 0.5097480
## [2,] 0.5048207 0.5036886 0.4985467 0.4991731 0.5071384 0.4953800 0.4908757
## [3,] 0.4996279 0.4982070 0.5043346 0.5085042 0.4994862 0.4945702 0.5041462
##           [,8]      [,9]     [,10]
## [1,] 0.5082991 0.4926313 0.4921113
## [2,] 0.4917657 0.5040657 0.4956302
## [3,] 0.5040348 0.4955050 0.5088683
```

Then we goes to M = 3, again generate a π and μ under M = 3 as the start point for our model.

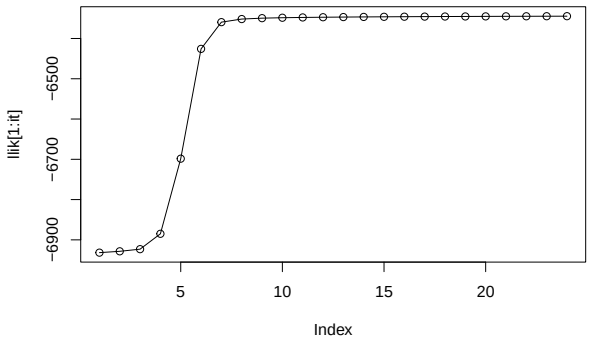





```
##  [1] "iteration:  1 log likelihood:  -6931.75122861544"
##  [2] "iteration:  2 log likelihood:  -6928.46695128332"
##  [3] "iteration:  3 log likelihood:  -6923.13091589768"
##  [4] "iteration:  4 log likelihood:  -6884.94468114899"
##  [5] "iteration:  5 log likelihood:  -6698.4237807576"
##  [6] "iteration:  6 log likelihood:  -6425.806406635"
##  [7] "iteration:  7 log likelihood:  -6359.4223002037"
##  [8] "iteration:  8 log likelihood:  -6351.74320723516"
##  [9] "iteration:  9 log likelihood:  -6349.54335699899"
## [10] "iteration:  10 log likelihood:  -6348.4434579416"
## [11] "iteration:  11 log likelihood:  -6347.73893891381"
## [12] "iteration:  12 log likelihood:  -6347.21907132148"
## [13] "iteration:  13 log likelihood:  -6346.8026280888"
## [14] "iteration:  14 log likelihood:  -6346.45328398564"
## [15] "iteration:  15 log likelihood:  -6346.15273441616"
## [16] "iteration:  16 log likelihood:  -6345.89062787433"
## [17] "iteration:  17 log likelihood:  -6345.66037682444"
## [18] "iteration:  18 log likelihood:  -6345.45729529842"
## [19] "iteration:  19 log likelihood:  -6345.2777404626"
## [20] "iteration:  20 log likelihood:  -6345.11870927259"
## [21] "iteration:  21 log likelihood:  -6344.97764443873"
## [22] "iteration:  22 log likelihood:  -6344.852335347"
## [23] "iteration:  23 log likelihood:  -6344.74086068395"
## [24] "iteration:  24 log likelihood:  -6344.64154880195"
```

```
## [1] 0.2663361 0.3438780 0.3897860
```

```
##           [,1]      [,2]     [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4938980 0.4758006 0.457040 0.4711376 0.5411178 0.4986316 0.4555200
## [2,] 0.4728756 0.3874569 0.630045 0.3163584 0.6869103 0.2039956 0.7823031
## [3,] 0.5075751 0.5799061 0.422322 0.7099547 0.2967462 0.7569457 0.2402904
##           [,8]     [,9]      [,10]
## [1,] 0.4878804 0.489488 0.37258194
## [2,] 0.1446699 0.881597 0.03501224
## [3,] 0.8422855 0.119219 0.98958938
```

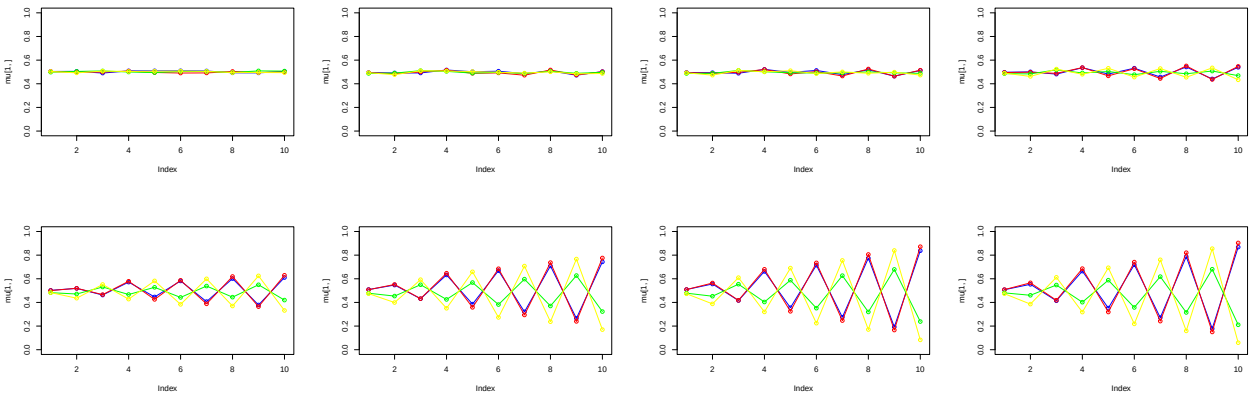


We can see that the log-likelihood rises rapidly during iteration 4-7, its also clearly shown by μ values plot. Then from final result we can see that it predict the true values well, especially in comparison with following M = 4.

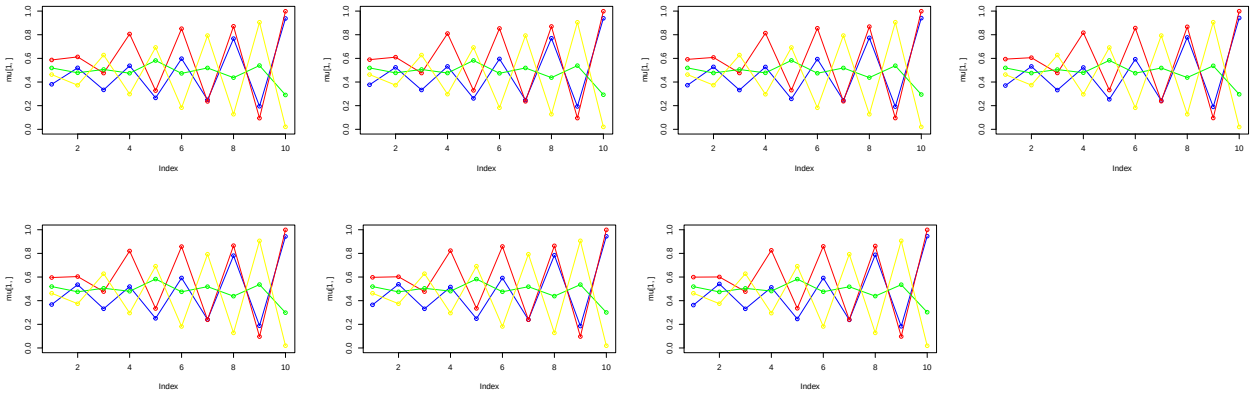
```
## [1] 0.2518811 0.2466783 0.2511809 0.2502598
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.5041509 0.5051382 0.4905578 0.5089228 0.5094530 0.5097480 0.5082991
## [2,] 0.5036886 0.4985467 0.4991731 0.5071384 0.4953800 0.4908757 0.4917657
## [3,] 0.4982070 0.5043346 0.5085042 0.4994862 0.4945702 0.5041462 0.5040348
## [4,] 0.5037389 0.4922173 0.5069624 0.5039756 0.5065369 0.5073122 0.5049473
##           [,8]      [,9]      [,10]
## [1,] 0.4926313 0.4921113 0.5048207
## [2,] 0.5040657 0.4956302 0.4996279
## [3,] 0.4955050 0.5088683 0.5072302
## [4,] 0.4943372 0.4951750 0.4940898
```

Then we change M to 4, again generate a π and μ under M = 3 as the start point for our model.




mu[1,]
Index

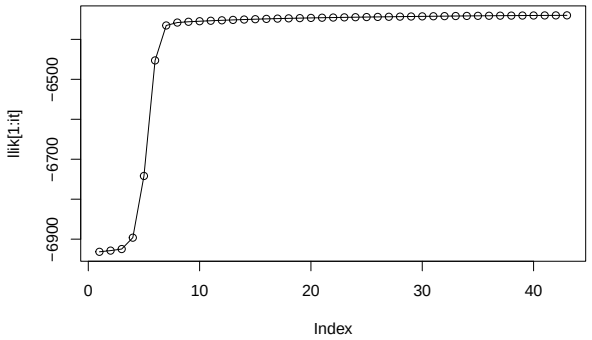



```
##  [1] "iteration:  1 log likelihood:  -6931.66006994864"
##  [2] "iteration:  2 log likelihood:  -6928.66727678128"
##  [3] "iteration:  3 log likelihood:  -6924.77054344575"
##  [4] "iteration:  4 log likelihood:  -6896.29508885115"
##  [5] "iteration:  5 log likelihood:  -6741.85787805056"
##  [6] "iteration:  6 log likelihood:  -6452.71193962151"
##  [7] "iteration:  7 log likelihood:  -6365.22915492218"
##  [8] "iteration:  8 log likelihood:  -6357.90724015069"
##  [9] "iteration:  9 log likelihood:  -6355.94977356183"
## [10] "iteration:  10 log likelihood:  -6354.59564838264"
## [11] "iteration:  11 log likelihood:  -6353.40988750751"
## [12] "iteration:  12 log likelihood:  -6352.31517546734"
## [13] "iteration:  13 log likelihood:  -6351.30092218428"
## [14] "iteration:  14 log likelihood:  -6350.36818071176"
## [15] "iteration:  15 log likelihood:  -6349.51652090179"
## [16] "iteration:  16 log likelihood:  -6348.74223300225"
## [17] "iteration:  17 log likelihood:  -6348.03936943375"
## [18] "iteration:  18 log likelihood:  -6347.40094567819"
## [19] "iteration:  19 log likelihood:  -6346.81968070248"
## [20] "iteration:  20 log likelihood:  -6346.28834868353"
## [21] "iteration:  21 log likelihood:  -6345.79994106786"
## [22] "iteration:  22 log likelihood:  -6345.34778016045"
## [23] "iteration:  23 log likelihood:  -6344.92564619259"
## [24] "iteration:  24 log likelihood:  -6344.52792780162"
## [25] "iteration:  25 log likelihood:  -6344.14978243089"
## [26] "iteration:  26 log likelihood:  -6343.78728731843"
## [27] "iteration:  27 log likelihood:  -6343.43756285041"
## [28] "iteration:  28 log likelihood:  -6343.09885115178"
## [29] "iteration:  29 log likelihood:  -6342.77053143643"
## [30] "iteration:  30 log likelihood:  -6342.4530520589"
## [31] "iteration:  31 log likelihood:  -6342.14776283848"
## [32] "iteration:  32 log likelihood:  -6341.85664504623"
## [33] "iteration:  33 log likelihood:  -6341.58196036309"
## [34] "iteration:  34 log likelihood:  -6341.32586665114"
## [35] "iteration:  35 log likelihood:  -6341.09006544297"
## [36] "iteration:  36 log likelihood:  -6340.87554389922"
## [37] "iteration:  37 log likelihood:  -6340.68245157758"
## [38] "iteration:  38 log likelihood:  -6340.51011833165"
## [39] "iteration:  39 log likelihood:  -6340.35718728741"
## [40] "iteration:  40 log likelihood:  -6340.22181690354"
```

```
## [41] "iteration:  41 log likelihood:  -6340.10190243326"
## [42] "iteration:  42 log likelihood:  -6339.99527661232"
## [43] "iteration:  43 log likelihood:  -6339.89986543919"
```

```
## [1] 0.1838186 0.2308466 0.2874187 0.2979162
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.3627106 0.5416076 0.3311785 0.5125500 0.2451860 0.5918027 0.2376350
## [2,] 0.5991155 0.6005747 0.4768083 0.8260669 0.3357255 0.8593412 0.2392769
## [3,] 0.5188276 0.4740528 0.5042778 0.4812818 0.5823257 0.4755847 0.5177702
## [4,] 0.4628904 0.3744350 0.6280787 0.2945785 0.6916641 0.1817551 0.7930593
##           [,8]       [,9]      [,10]
## [1,] 0.7878580 0.18327423 0.94572455
## [2,] 0.8624625 0.09719753 0.99920005
## [3,] 0.4387190 0.53515949 0.30265960
## [4,] 0.1275007 0.90649071 0.01848283
```



We can see that after the rapid rising of log-likelihood, the iteration 8-15 seems still have a reasonable μ value, but then it falls into overfitting and the result μ and π values seems not so favorable.